Calculating constant acceleration to stop at a given point, p_{fin}.

First we make some basic assumptions to simplify the math.

1. $p_{init} = 0.0$ m (problem is set up in the car x-y frame)
2. $t_{init} = 0.0$ sec (convention for current time t)
3. $v_{fin} = 0.0$ m/sec (goal is come to a complete stop)
4. $a(t) = a_c$ (constant acceleration)

Below are the fundamental calculations of position and velocity as a function of acceleration.

$$p_{fin} = p_{init} + \int_{t_{init}}^{t_{fin}} v(\tau)d\tau \quad (1)$$

$$v_{fin} = v_{init} + \int_{t_{init}}^{t_{fin}} a(\tau)d\tau \quad (2)$$

Let a_c denote the constant acceleration:

$$a(t) = a_c \quad (3)$$

Evaluate (2) using assumption in (3), then apply base assumptions:

$$v(t) = v_{init} + (t - t_{init})a_c \quad (4)$$

$$v_{fin} = v_{init} + (t_{fin} - t_{init})a_c$$

$$0 = v_{init} + (t_{fin} - 0)a_c$$

$$0 = v_{init} + t_{fin}a_c \quad (5)$$

Evaluate (1), substituting (4), then apply base assumptions:

$$p_{fin} = p_{init} + \int_{t_{init}}^{t_{fin}} v_{init} + (\tau - t_{init})a_c d\tau$$

$$p_{fin} = p_{init} + (t_{fin} - t_{init})v_{init} + \frac{1}{2}\left[\left(t_{fin}^2 - t_{init}\right) - \left(t_{init}^2 - t_{init}\right)\right]a_c$$

$$p_{fin} = 0 + (t_{fin} - 0)v_{init} + \frac{1}{2}\left[\left(t_{fin}^2 - 0\right) - (0 - 0)\right]a_c$$

$$0 = -p_{fin} + t_{fin}v_{init} + \frac{1}{2}t_{fin}^2 a_c \quad (6)$$

Equations (5) and (6) give two equations and two unknowns. Use substitution to solve for a_c:

$$0 = v_{init} + t_{fin}a_c$$

$$t_{fin} = \frac{-v_{init}}{a_c} \quad (7)$$

$$0 = -p_{fin} - \frac{v_{init}^2}{a_c} + \frac{1}{2}\frac{v_{init}^2}{a_c} \quad (8)$$

$$0 = -p_{fin} - \frac{1}{2}\frac{v_{init}^2}{a_c}$$

$$a_c = -\frac{1}{2}\frac{v_{init}^2}{p_{fin}} \quad (9)$$

Substituting (6) into (4), we can calculate the velocity at point, $v(x)$

$$0 = -p_{fin} + t_{fin}v_{init} + \frac{1}{2}t_{fin}^2 a_c$$

$$t_{fin}^2\frac{a_c}{2} + t_{fin}v_{init} - p_{fin} = 0 \quad (10)$$

Apply the quadratic formula to (10), replace p_{fin} with any distance x, and substitute back into (4):

$$t_{fin} = \frac{-v_{init} - \sqrt{v_{init}^2 + 2a_c p_{fin}}}{a_c} \tag{11}$$

$$v(x) = v_{init} + \left(\frac{-v_{init} - \sqrt{v_{init}^2 + 2a_c x}}{a_c} \right) a_c$$

$$v(x) = -\sqrt{v_{init}^2 + 2a_c x} \tag{12}$$