Homework # 3

Assigned: Wednesday February 2, 2022
Due: Tuesday February 8, 2022 @ 11:59pm ET/Boston

Instructions:

- Homework is due on Tuesday at 11:59pm ET/Boston. Homeworks received up to 15 hours late (3 pm on Wednesday) will be penalized 10 percent. *NO* assignment will be accepted after 3pm on Wednesday.
- We expect that you will study with friends and fellow students and you are welcome to verbally discuss the problems openly. However, your solution writeup should be the product of your own mind and expressed in your own words. The TAs and I will be available to answer specific questions or address speific points of confusion but we will not verify your answers.
- Assignments should be typed using Word or LateX, or hand-written *neatly*. When submitting to gradescope be sure to indicate the page containing your answer to each problem, so that the TAs don't have to search for your solution.
- *To get full credit, explain your solution and show each step!* We don't need your scratch work or draft solutions, only your final result.

Problem 1 [30 pts (5 pts each)]: Predicate Logic

For this problem, the domain is the set of all solar system objects:

P(x) means x is a Planet

M(x) means x is a Moon

O(x, y) means x orbits y

Formulate the following statements using predicate logic. You may use $x \neq y$ to indicate that the x and y are different.

1. All planets orbit the sun and all moons orbit a planet.

   Solution: $\forall x : (P(x) \to O(x, Sun)) \wedge (M(x) \to \exists y : P(y) \wedge O(x, y))$

2. Some planets have no moon.

   Solution: $\exists x \forall y : P(x) \wedge (M(y) \to \neg O(y, x))$

3. Some planets have two or more moons.

   Solution: $\exists x, y, z : P(x) \land M(y) \land M(z) \land O(y, x) \land O(z, x) \land y \neq z$

4. Some objects orbit the sun that are not planets

   Solution: $\exists x : O(x, Sun) \land \neg P(x)$

5. Everything that orbits the sun is a planet.
   (Also prove that this statement is the negation of the previous statement)

   Solution:

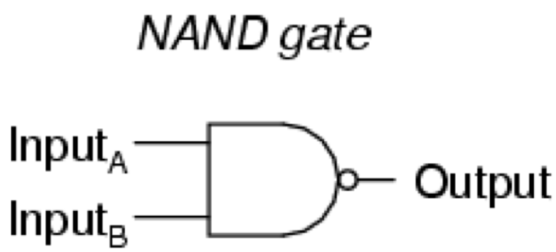
   $\forall x : O(x, Sun) \rightarrow P(x)$
   $\forall x : \neg O(x, Sun) \lor P(x)$
   $\forall x : \neg\neg(\neg O(x, Sun) \lor P(x)$
   $\forall x : \neg(\neg\neg O(x, Sun) \land \neg P(x)$
   $\forall x : \neg(O(x, Sun) \land \neg P(x)$
   $\neg\exists x : O(x, Sun) \land \neg P(x)$

6. Pluto is a planet!

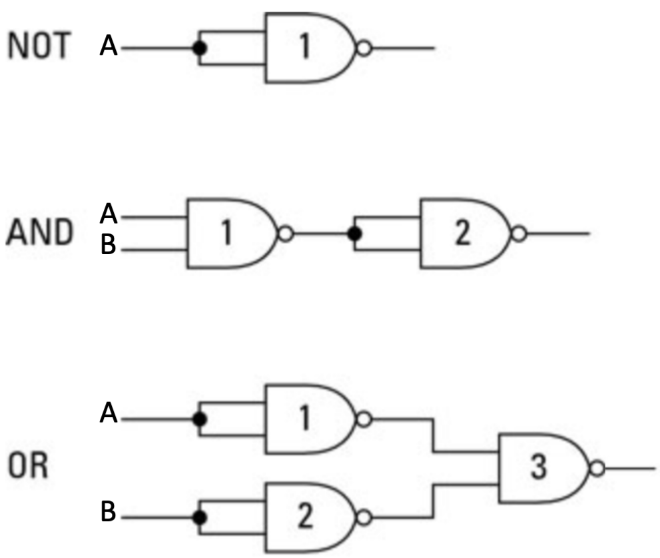
   Solution: $P(Pluto)$

Problem 2 [20 pts] (10, 10)**: NAND: The universal gate**

In this problem we'll explore the fact that all logical circuits can be implemented using just NAND gates. The figure below shows you the symbol for a NAND gate and its truth table. We then show you how NAND gates can be wired together to perform the equivalent of a NOT gate, an AND gate, and an OR gate.



A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0



i. Let's denote p NAND q as $p \barwedge q$. Write a logical expression for the three circuits corresponding to AND, OR, and NOT.

Solution:

- NOT: $p \barwedge p$
- AND: $(p \barwedge q) \barwedge (p \barwedge q)$
- OR: $(p \barwedge p) \barwedge (q \barwedge q)$

ii. Validate your three logical expressions with three truth tables. For clarity and full credit, show each variable and distinct sub-clause in a separate column, culminating in your final formula.

Solution:

- NOT:

p	$p \barwedge p$
0	1
1	0

- AND:

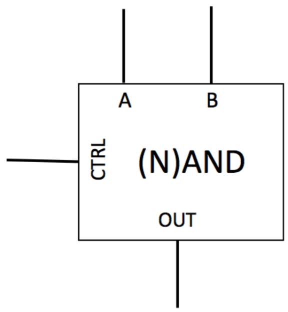
p	q	$p \barwedge q$	$p \barwedge q$	$(p \barwedge q) \barwedge (p \barwedge q)$
0	0	1	1	0
0	1	1	1	0
1	0	1	1	0
1	1	0	0	1

- OR:

p	q	$p \barwedge p$	$q \barwedge q$	$(p \barwedge p) \barwedge (q \barwedge q)$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

Problem 3 [25 pts (8,8,9)]: **More fun with NAND**

In the figure below we depict an unusual little device that we'll call a (N)AND gate. It takes two inputs A and B and a control input labelled CTRL, while producing one output labelled OUT. If the CTRL input is 0, the device works like an AND gate. If the CTRL input is 1, the device works like a NAND gate.



1. Create a truth table for the (N)AND device.

   Solution:

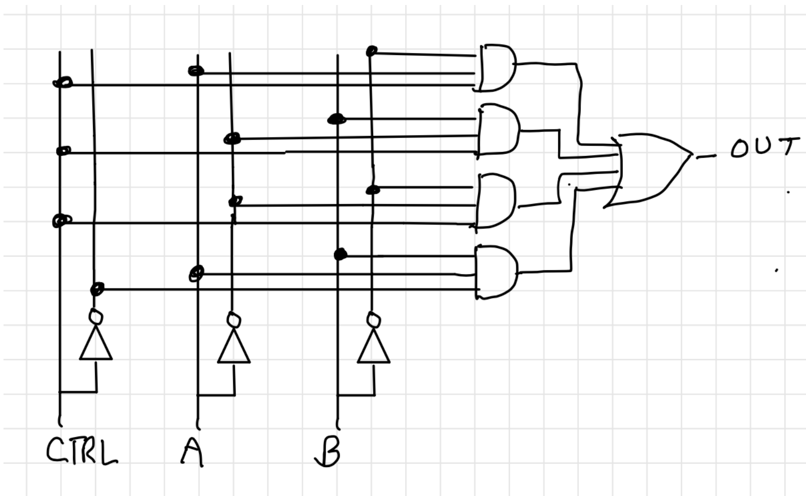
$CTRL$	A	B	OUT
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

2. Write down an unsimplified disjunctive normal form (DNF) expression and explain why a conjunctive normal form (CNF) expression would have the same number of clauses. (You may use C for the control input.)

   Solution: $(\neg C \wedge A \wedge B) \vee (C \wedge \neg A \wedge \neg B) \vee (C \wedge \neg A \wedge B) \vee (C \wedge A \wedge \neg B)$ Since the output has an equal number of 0 (False) and 1 (True) output values, the CNF and DNF representation both have four clauses.

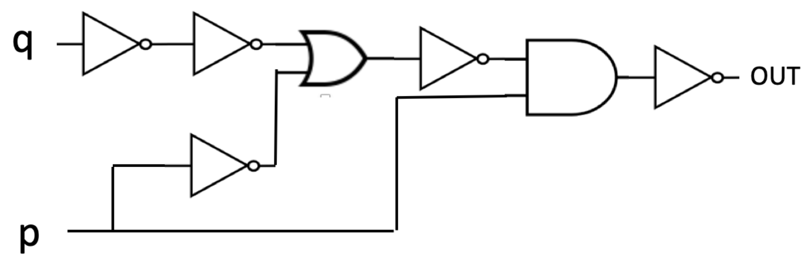
3. draw the circuit using the canonical format shown in class.

Solution:



Problem 4 [25 pts (10,10,5)]: **Circuits**

i. Write the logic expression that represents the output of the following circuit as a function of its inputs. (Do not simplify your expression).



Solution: $\neg(\neg(\neg\neg q \vee \neg p) \wedge p)$

ii. Use logic rules to simplify your expression. Show each step and state the law of logical equivalence that are applying at each step.

Solution:

$$
\begin{aligned}
\neg(\neg(\neg\neg q \vee \neg p) \wedge p) &\equiv \neg(\neg(q \vee \neg p) \wedge p) && \text{Double Negation} \\
&\equiv \neg((\neg q \wedge \neg\neg p) \wedge p) && \text{DeMorgan's} \\
&\equiv \neg((\neg q \wedge p) \wedge p) && \text{Double Negation} \\
&\equiv \neg(\neg q \wedge p) && \\
&\equiv \neg\neg q \vee \neg p && \text{DeMorgan's} \\
&\equiv q \vee \neg p && \text{double negation}
\end{aligned}
$$

iii. Draw the circuit corresponding to your simplified expression.

Solution: