End of the Term Test
26th April 2022
Duration: 2 hours

Answer the following:

Question 1: (3 Marks)

Let a = 8316 and b = 10920.
(a) Find d = GCD (a, b), the greatest common divisor of a and b.
(b) Find integers m and n such that d = ma + nb

Question 2: (6 Marks)

Which of the following are true?

(*a*) $446 \equiv 278 \pmod 7$, (*c*) $269 \equiv 413 \pmod{12}$, (*e*) $445 \equiv 536 \pmod{18}$
(*b*) $793 \equiv 682 \pmod 9$, (*d*) $473 \equiv 369 \pmod{26}$, (*f*) $383 \equiv 126 \pmod{15}$

Question 3: (4 Marks: 1 Mark each)

Find: (*a*) $\varphi(81)$, $\varphi(7^6)$; (*b*) $\varphi(72)$, $\varphi(3000)$.

Question 4: (4 Marks: 2 Marks each)

Find a^{-1} in $\mathbf{Z}_m$ where: (*a*) $a = 37$ and $m = 249$; (*b*) $a = 15$ and $m = 234$.

Question 5: (8 Marks: 2 Marks each)

Solve each linear congruence equation:

(*a*) $3x \equiv 2 \pmod 8$; (*b*) $6x \equiv 5 \pmod 9$; (*c*) $4x \equiv 6 \pmod{10}$

Question 6: (6 Marks)

Solve the congruence equation $1092x \equiv 213 \pmod{2295}$.

Question 7: (8 Marks: 2 Marks each)

Consider the following five relations on the set $A = \{1, 2, 3, 4\}$:

$R1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$

$R2 = \{(1,1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

$R3 = \{(1, 3), (2, 1)\}$

$R4 = \varnothing$, the empty relation

$R5 = A \times A$, the universal relation

a) Determine which of the relations are reflexive.
b) Determine which of the relations are symmetric.
c) Determine which of the relations are antisymmetric.
d) Determine which of the relations are transitive.

Question 8: (8 Marks: 2 Marks each)

Let R and S be the following relations on A = {1, 2, 3}:

R = {(1, 1), (1, 2), (2, 3), (3, 1), (3, 3)}, S= {(1, 2), (1, 3), (2, 1), (3, 3)}

Find $(a)\ R \cup S,\ R \cap S,\ R^{C};\quad (b)\ R \circ S;\quad (c)\ S^2 = S \circ S.$

Question 9: (6 Marks: 2 Marks each)

Give an example of a relation R on $A = \{1, 2, 3\}$ such that:

(a) R is both symmetric and antisymmetric.

(b) R is neither symmetric nor antisymmetric.

(c) R is transitive but $R \cup R^{-1}$ is not transitive.

Question 10: (6 Marks: 2 Marks each)

Consider the relation R = {(a, a), (a, b), (b, c), (c, c)} on the set A = {a, b, c}. Find:
(a) reflexive(R).
(b) symmetric(R); (c) transitive(R).

Question 11: (4 Marks)

Let R be the following equivalence relation on the set A = {1, 2, 3, 4, 5, 6}:
R = {(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)}
Find the partition of A induced by R, i.e., find the equivalence classes of R.

Question 12 (4 Marks)

For each pair of integers, a and b, find integers q and r such that a = bq + r and 0 < r < |b|
(a) a = −381 and b = 14

********** END OF THE PAPER **********