CS5002 Discrete Structures	Prof. Rachlin
Spring 2022	January 19, 2022

Homework # 1

Assigned: Wednesday January 19, 2022
Due: Tuesday January 25, 2022 @ 11:59pm ET/Boston

Instructions:

- Homework is due on Tuesday at 11:59pm ET/Boston. Homeworks received up to 15 hours late (3 pm on Wednesday) will be penalized 10 percent. *NO* assignment will be accepted after 3pm on Wednesday.
- We expect that you will study with friends and fellow students and you are welcome to verbally discuss the problems openly. However, your solution writeup should be the product of your own mind and expressed in your own words. The TAs and I will be available to answer specific questions or address speific points of confusion but we will not verify your answers.
- Assignments should be typed using Word or LateX, or hand-written *neatly.* When submitting to gradescope be sure to indicate the page containing your answer to each problem, so that the TAs don't have to search for your solution.
- *To get full credit, explain your solution and show each step!* We don't need your scratch work or draft solutions, only your final result.

Problem 1 [20 pts]: Number representations

i. Convert 200_{10} to binary and hexadecimal.

Solution: $200_{10} = 12 * 16 + 8 = C8_{16} = 11001000_2$ (using the substitution trick where each hexadecimal digit is converted to 4 bits.

ii. Convert 1010110110_2 to decimal and hexadecimal.

Solution: Make sure when grouping digits, you group them right to left, padding 0's as necessary. $1010110110_2 = (0010\ 1011\ 0110)_2 = 2B6_{16} = 2 \times 16^2 + 11 \times 16 + 6 = 694_{10}$

iii. Convert ABC_{16} to 12-bit unsigned binary. Now, treating the binary as a 12-bit two's-complement number, find the corresponding (negative) number in decimal

Solution: $ABC_{16} = (1010\ 1011\ 1100)_2$. We convert to decimal by flipping bits, adding one, and adding the negation sign out front: $1010\ 1011\ 1100 \rightarrow 0101\ 0100\ 0011 + 1 = 0101\ 0100\ 0100 = 2^{10} + 2^8 + 2^6 + 2^2 = 1348$ so the final answer is -1348.

iv. Use a substitution trick to convert 110011110_3 to base-9. (Hint $2^4 = 16$ and $3^2 = 9$)

Solution: Using the substitution trick since $3^2 = 9$, and taking care to note that the original number is base-3, not binary, $110011110_3 = (01\ 10\ 01\ 11\ 10)_3 = 13143_9$.

Problem 2 [30 pts]**: Present and Past**

i. A MAC (Media Access Control) address is a globally unique identifier assigned to network devices, and therefore it is often referred to as a hardware or physical address. MAC addresses are written in hexadecmal format like this: F0:23:9C:AA:4E:12. The first 6 hexadecimal digits identify the manufacturer, which is assigned by an Internet standards body. The second 6 hexadecimal digits are a serial number assigned by the manufacturer. How many possible devices can one manufacturer assign? How many total MAC addresses are possible? Assuming a current world population of 7.8 billion people (2021 UN estimate), how many devices could be allocated to each and every person?

<u>*Solution:*</u> Each hexadecimal digit is 4 bits, so each manufacturer can assign $2^{24} \approx 16.8$ million devices and there are $2^{48} \approx 2.8 \times 10^{14}$ possible MAC addresses, enough for about 36 thousand devices for every human being on earth.

ii. The Babylonians developed a *sexagesimal* (base 60) number system about 4000 years ago. They represented their numbers with a Cuneiform script rather than the digits 0, 1, 2... we know today, and they used the same symbol for both 1 and 60. Ignoring these subtlties, let's assume the symbols for the 60 Babylonian "digits" are (ZERO), (ONE), (TWO), ...(FIFTYEIGHT), (FIFTYNINE). How do you write 5002_{10} in sexagesimal?

<u>*Solution:*</u>

$5002_{10} = 1{\cdot}3600 + 23{\cdot}60 + 22 = (ONE)60^2 + (TWENTYTHREE)60^1 + (TWENTYTWO)60^0$.
So: (ONE)(TWENTYTHREE)(TWENTYTWO).

Problem 3 [20 pts]**: Go Huskies!**

Northeastern

address of the first byte in each row (hexadecimal) | 64 bytes of data (16 bytes per row) | data rendered as text characters

```
00000000  89 50 4e 47 0d 0a 1a 0a  00 00 00 0d 49 48 44 52  |.PNG........IHDR|
00000010  00 00 05 00 00 00 00 ec  08 06 00 00 00 2b 5a 0b  |.............+Z.|
00000020  a9 00 00 00 06 62 4b 47  44 00 ff 00 ff 00 ff a0  |.....bKGD.......|
00000030  bd a7 93 00 00 20 00 49  44 41 54 78 9c ec dd 69  |..... .IDATx...i|
```

i. The Northeastern logo (in PNG format) and a hexidecimal dump of the first 64 bytes (00_{16} to $3F_{16}$) is provided above. The dimensions of the image, width $\times$ height, measured in pixels, are encoded by 8 bytes starting at 10_{16}: four bytes for the width followed by four bytes for the height. What are the dimensions of the image?

<u>*Solution:*</u> The size is encoded in the first 8 bytes of the second row. $00000500_{16} = 1280_{10}$ and $000000EC_{16} = 236_{10}$ so the filesize is 1280×236 pixels.

ii. Suppose the image was uncompresssed and consisted of a 24-bit color encoding for each pixel. How many kilobytes of diskspace would the image consume? (1 kilobyte = 2^{10} bytes)

<u>*Solution:*</u> 1280×236 pixels $\times 24$ bits/pixel $\times$ 1 byte/8 bits $\times$ 1 kb / 1024 bytes = 885kb.

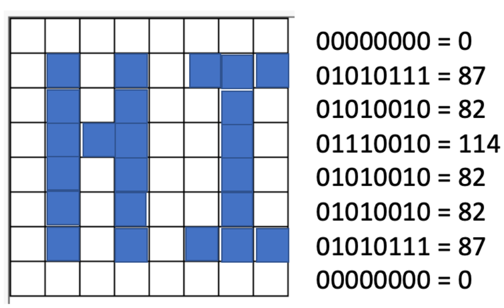
Problem 4 [30 pts]: Alien Invaders

i. While on co-op at the Very Large Array in Soccorro, New Mexico, you receive a strange transmission coming from the star Vega, 25 light-years away. It is a sequence of numbers: -35, -9, -9, -1 which keeps repeating. Everyone is stumped until you suggest converting the numbers into four 8-bit 2's-complement numbers and sequencing them together to form a single 32-bit binary sequence. Have you discovered an alien intelligence? Explain your answer by identifying the resulting pattern. (Hint, write down a sequence of numbers denoting the number of sequential ones. What is this sequence? Is it likely to be naturally occuring? Recommended movie clip from *Contact* (1997): https://www.youtube.com/watch?v=-ciK05XqlOw

<u>*Solution:*</u> Converting to 8-bit two's complement: $-35 = 11011101_2$, $-9 = 11110111_2$, and $-1 = 11111111_2$. So converting -35, -9, -9, -1 to one long 32-bit binary string gives: 11011101111101111111011111111111 where the number of ones denote prime numbers (2,3,5,7,11) separated by zeros. This is probably not a naturally occurring phenomenon. Hooray! You've discovered intelligent life on another planet. In another 25 years, the Vegan's will finally receive your response.

ii. While on your next assignment at the Arecibo Radio Observatory in Puerto Rico, you get the following message coming from Proxima Centauri, the closest star (other than the Sun) to Earth at 4.22 light-years. The message reads: 0, 87, 82. 114, 82, 82, 87, 0. This time a single long binary sequence doesn't work. Try stacking the 8-bit binary representations to form an 8x8 pixel array (1=ON, 0=OFF). What is the message?

<u>*Solution:*</u> They are just saying HI!



iii. Your fame as an exobiologist is secured! At Roswell, New Mexico, you are asked to examine a technical journal from an alien crash site. One strange equation reads: $412 + 156 = 601$. Assuming this equation is correct, and the alients learned to count with their fingers, how many fingers do our aliens probably have? For full credit derive your result algebraically rather than just guessing and verifying. Let b = the base, with the digit places representing powers of b: b^0, b^1, b^2, etc. Now solve for b.

Solution: Humans probably use base-10 because we have 10 fingers. So what we're really asking is, what number system makes the equation valid? To determine this, we must have $4b^2 + b + 2 + 1b^2 + 5b + 6 = 6b^2 + 1$ which simplifies to $b^2 - 6b - 7 = (b - 7)(b + 1) = 0$. So $b = 7$ or $b = -1$. We can ignore the -1 solution. Our aliens use a septenary (base-7) number system and so presumably have 7 fingers.