Problem 1:

i. $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Start w/ left side expression.

$\neg(p \vee (\neg p \wedge q))$ 1.

$\neg(p \vee \neg p \wedge p \vee q)$ 2. distributive law

$\neg(\underline{T} \wedge \underline{p \vee q})$ 3. Complement law

$\neg(\underline{p \vee q} \wedge \underline{T})$ 4. commutative law

$\neg(p \vee q)$ 5. Identity law

$\boxed{\neg p \wedge \neg q}$ 6. De Morgan's law.

which equals to right side expression.

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ii. $p \to (q \to r) \equiv q \to (p \to r)$

Start from left side

$p \to (q \to r)$ 1.

$\neg p \vee (\neg q \vee r)$ 2. conditional identity laws twice.

$(\neg p \vee \neg q) \vee r$ 3. associative law

$(\neg q \vee \neg p) \vee r$ 4. Distributive law

$\neg q \vee (\neg p \vee r)$ 5. associative

$\boxed{q \to (p \to r)}$ 6. backward conditional identities

which equals to the right side.

Problem 1.

iii. $\neg(p \to q) \equiv p \land \neg q$

$\neg(p \to q)$ — 1. from left side.

$\neg(\neg p \lor q)$ — 2. conditional identities

$\neg\neg p \land \neg q$ — 3. De Morgan's

$\boxed{p \land \neg q}$ — 4. Double negation.

Equals righ-hand side.

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iv. $p \to q \equiv \neg q \to \cancel{q}\neg p$

$\neg q \to \neg p$ — 1. from righ-hand side.

$\neg\neg q \lor \neg p$ — 2. conditional identities.

$q \lor \neg p$ — 3. Double negation

$\neg p \lor q$ — 4. commutative

$\boxed{p \to q}$ — 5. backward conditional identities.

Equals left-hand side.

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v. $p \to (q \land r) \equiv (p \to q) \land (p \to r)$

$p \to (q \land r)$ — 1. left-hand side.

$\neg p \lor (q \land r)$ — 2. conditional identities.

$(\neg p \lor q) \land (\neg p \lor r)$ — 3. Distributive laws.

$\boxed{(p \to q) \land (p \to r)}$ — 4. backwards conditional identities.

Equals right-hand side.

Problem 1:

vi. $((p \to q) \land (q \to r)) \to (p \to r) \equiv T$ (a tautology).

$((p \to q) \land (q \to r)) \to (p \to r)$ 1. left-hand side

$\neg((\neg p \lor q) \land (\neg q \lor r)) \lor (\neg p \lor r)$ 2. Conditional identities four times.

$\neg(\neg p \lor q) \lor \neg(\neg q \lor r) \lor (\neg p \lor r)$ 3. De Morgan's.

$(p \land \neg q) \lor (q \land \neg r) \lor (\neg p) \lor (r)$ 4. De Morgan's

~~$p \land \neg q \lor q \land \neg r \lor r \lor \neg p$ 5. Commutative~~

~~$p \land T \land T \lor \neg p$ 6. complement law~~

$\neg p \lor (p \land \neg q) \lor (q \land \neg r) \lor r$ 5. associative

$(\neg p \lor p) \land (\neg p \lor \neg q) \lor (q \lor r) \land (\neg r \lor r)$ 6. distributive

$T \land (\neg p \lor \neg q) \lor (q \lor r) \land T$ 7. complement.

$\neg p \lor (\neg q \lor q) \lor r$ 8. Identity

$\neg p \lor (T \lor r)$ 9. complement

$\neg p \lor T$ 10. domination

$\boxed{T}$ 11 domination.

Equals right-hand side.

problem 2.

J : "She likes java"

P : "She likes Python"

R : "She likes Racket".

i. she likes java but neither Python nor Racket.

$$J \wedge \neg(\cancel{B}P \wedge R)$$

ii. she likes at least one of these programming languages.

$$J \vee P \vee R$$

iii. she likes at least two of these programming languages.

$$((J \wedge P) \vee R) \vee ((J \wedge R) \vee P) \vee ((P \wedge R) \vee J)$$

iv. she likes at most one of these programming languages.

$$(J \wedge \neg(P \wedge R)) \vee (P \wedge \neg(J \wedge R)) \vee (R \wedge \neg(J \wedge P))$$

v. she likes all of these programming languages if she likes any of them.

$$(J \vee P \vee R) \longrightarrow (J \wedge P \wedge R)$$

problem 3:

Aravik: I am a Vulcan or Balev is an Romulan

Balev: Aravik is a Vulcan and Chu'lak is a Romulan

Chu'lak: Balev and I are different.

	A	B	C	
1	R	R	R	×
2	R	R	V	×
3	R	V	R	×
4	R	V	V	×
5	V	R	R	×
6	V	R	V	×
7	V	V	R	✓
8	V	V	V	×

check:

if A were true (Vulcan),
A is V or B is R.
7 qualifies,

$\boxed{1}$ if C were true (Vulcan),
B and C are different.
Eliminate 2, 8

if C were false (Romulan),
B and C are the same,
Eliminate 3, 5

$\boxed{2}$ if B were true (Vulcan).
A is V and C is R.
Eliminate 4

if B were false (Romulan),
A is R and C is V.
Eliminate 1 6

$\boxed{3}$ The only one possible solution is 7:
A and B are Vulcans and C is Romulan.

Problem 4

$(a \to b) \land (b \to c) \land (c \to d) \land (d \to a)$

Find all truth value combo w/o using a truth table.

Apply conditional identities to the long expression

$E \Rightarrow (\neg a \lor b) \land (\neg b \lor c) \land (\neg c \lor d) \land (\neg d \lor a)$

Each element appears twice in the expression E. but in different forms. one in normal, one is negation.

To find truth values, each short expressions in each parenthesis needs to be true.

which

$(\neg a \lor b) = (\neg b \lor c) = (\neg c \lor d) = (\neg d \lor a) = T$

~~if there were one or more False term~~

if both elements inside each parenthesis are false. will eventually result a False for expression E.

if $(\neg a \lor b)$ or $(\neg b \lor c)$ or $(\neg c \lor d)$ or $(\neg d \lor a)$ is false

$E = F$.

Only if $a = b = c = d = T$ or $a = b = c = d = F$,

$E = T$.

so

a	b	c	d	E
T	T	T	T	T
F	F	F	F	T