CS5002 Discrete Structures	Prof. Rachlin
Spring 2022	February 9, 2022

Homework # 4

Assigned: Wednesday February 9, 2022
Due: Tuesday February 15, 2022 @ 11:59pm ET/Boston

Instructions:

- Homework is due on Tuesday at 11:59pm ET/Boston. Homeworks received up to 24 hours late (11:59pm ET on Wednesday) will be penalized 5 percent. Homeworks received up to 48 hours late (11:59pm ET on Thursday) will be penalized 10 percent. *NO* assignment will be accepted after 48 hours.
- We expect that you will study with friends and fellow students and you are welcome to verbally discuss the problems openly. However, your solution writeup should be the product of your own mind and expressed in your own words. The TAs and I will be available to answer specific questions or address speific points of confusion but we will not verify your answers.
- Assignments should be typed using Word or LateX, or hand-written *neatly*. When submitting to gradescope be sure to indicate the page containing your answer to each problem, so that the TAs don't have to search for your solution.
- *To get full credit, explain your solution and show each step of the solution process!* Simply writing down a correct answer will receive little or no credit. We don't need your scratch work or draft solutions, only your final solution explaining your step-by-step reasoning. Recommendation: try to imagine you need to explain your solution to someone not in this class.
- If you think the TA made a clerical error in grading your assignment, you may submit a regrade request on Gradescope within 1 week of the publication of the grades. After 1 week of publication, ALL GRADES ARE FINAL.

Problem 1 [25 pts]**:** Convert each set S to the listing method. It is ok to include ellipses where appropriate. Remember that the set $\mathbb{N}$ includes 0, which is an even number.

i. $S = \{x : x \in \mathbb{N} \; and \; |x| < 5\}$

<u>*Solution:*</u> $S = \{0, 1, 2, 3, 4\}$ (We'll accept $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ due to an error in the original solutions.)

ii. $S = \{x \in R : 2x \in R\}$ where $R = \{2, 3, 5, 8, 13\}$.

<u>*Solution:*</u> $S = \varnothing$

iii. $S = \{x : x \in P \text{ and } 2x - 1 \in P\}$ where $P = \{x \in \mathbb{N} : x \leq \text{ 20 and x is prime}\}$

<u>*Solution:*</u> $P = \{2, 3, 5, 7, 11, 13, 17, 19\}$ So $S = \{2, 3, 7\}$ since $2 \cdot 2 - 1 = 3 \in P$, $3 \cdot 2 - 1 = 5 \in P$ and $7 \cdot 2 - 1 = 13 \in P$, but $5 \cdot 2 - 1 = 9 \notin P$ and $2 \cdot x - 1 > 20$ for all $x \geq 11$.

iv. $S = \{2x : x \in T\}$ where $T = \{y : 3y \in W\}$ and $W = \{1, 3, 7\}$.

Solution:

$T = \{\frac{1}{3}, 1, \frac{7}{3}\}$ So: $S = \{\frac{2}{3}, 2, \frac{14}{3}\}$

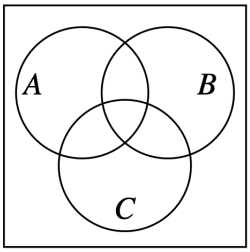
v. $S = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \ : d((x, y), (0, 0)) = \sqrt{2}$ and d is the Euclidian distance function$\}$

Solution:

$S = \{(1, 1), (-1, 1), (1, -1), (-1, -1)\}$

Problem 2 [25 pts]**: Venn Diagrams**

Draw Venn diagrams for the following expressions using the layout indicated below.



i. $(A \cup B) \cap \bar{C}$

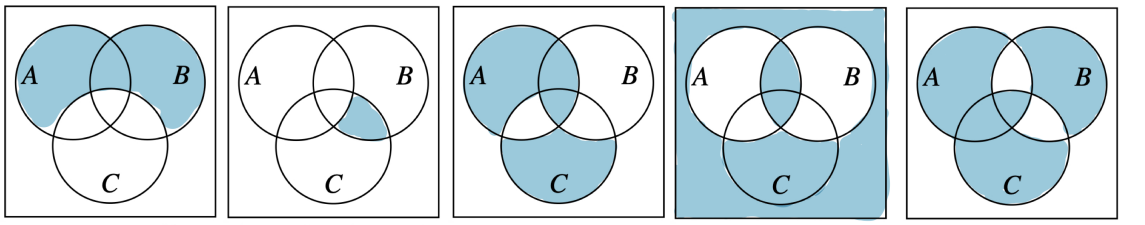
ii. $(B - A) \cap (C - A)$

iii. $(C - B) \triangle A$

iv. $A \triangle \bar{B}$

v. $(A \triangle B \triangle C) \cup (A - B)$

Solution:



$(A \cup B) \cap \bar{C}$ $(B - A) \cap (C - A)$ $(C - B) \triangle A$ $A \triangle \bar{B}$ $(A \triangle B \triangle C) \cup (A - B)$

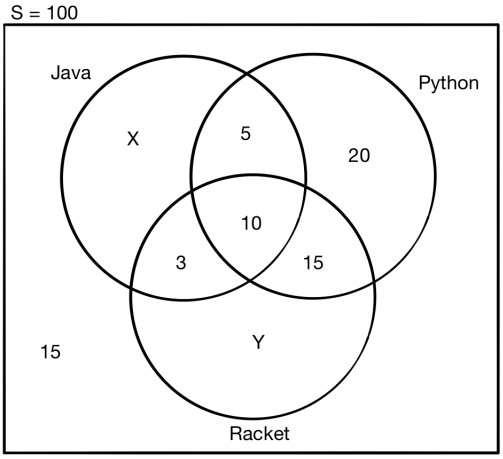
Problem 3 [25 pts]**: Programming Languages**

We asked 100 CS majors whether they liked Python, Java, and/or Racket.

- 15 students didn't respond.
- 10 liked all three languages.
- 13 liked Java and Racket, 25 liked Python and Racket, and 15 liked Java and Python.
- Half the surveyed students indicated they like Python.
- Twice as many students indicated they liked Racket as liked Java.

Note that when we say half liked Python, we mean that they liked Python and maybe other languages as well. Similarly, the 13 students that liked Java and Racket includes the 10 that indicated they like all three languages. (a) How many students liked Java and how many liked Racket? (b) How many students indicated they liked exactly one language?

Solution:

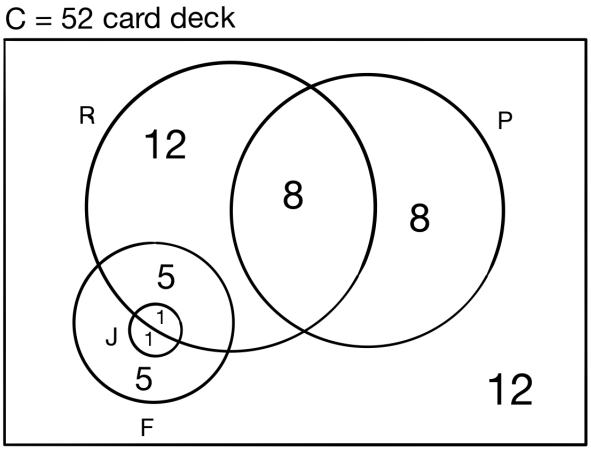


Set up the problem as shown in the diagram. Since 15 students didn't respond, we have $|Java \cup Python \cup Racket| = 85$. We also know that $2 \cdot |Java| = |Racket|$. So $X + Y + 53 = 85$ and $2 \cdot (X + 18) = (Y + 28)$. Solving for X and Y gives $X = 8$ and $Y = 24$. So 26 students liked Java and 52 liked Racket. The number of students that liked exactly one language is $X + Y + 20 = 52$.

Problem 4 [25 pts]**: Playing Cards** Let C represent a set of 52 playing cards with four suits ($\heartsuit, \diamondsuit, \clubsuit, \spadesuit$) each having 13 ranks (Ace,2,3,4,5,6,7,8,9,10,Jack,Queen,King). The $\heartsuit$'s and $\diamondsuit$'s are red cards. The $\clubsuit$'s and $\spadesuit$'s are black. We define the following additional sets. F =Face cards (Jack, Queen, and King). R =Red cards. P =Cards having a rank that is a prime number (2,3,5,7). J =One-eyed Jacks (Jack of Hearts, Jack of Spades).

1. Depict these sets as a Venn Diagram and show the cardinality of each distinct region. The regions don't have to be perfectly to scale - this can be hand-drawn. Only overlap sets that truly overlap. Disjoint sets, those sharing no members, should be rendered as non-overlapping.

   Solution:

2. Using set notation, give an expression for the set of cards that are red or face cards or prime numbered or one-eyed Jacks and compute its cardinality.

   Solution:

   $A = R \cup F \cup P \cup J$. $|A| = 40$

3. Give a set expression and compute the cardinality for the set of cards that are not face cards or not prime-numbered cards.

   Solution:

   $A = \bar{F} \cup \bar{P} = \overline{(F \cap P)} = C$. $|A| = |C| = 52$

4. Give a set expression and compute the cardinality for the *complement* of the set of cards that are red or prime-numbered but not one-eyed Jacks.

   Solution:

   $|(R \cup P)| = 26 + 16 - 8 = 34$
   $|(R \cup P) - J| = 34 - 1 = 33$ (Only one of the one-eyed Jacks is red, the Jack of Hearts.)
   $A = \overline{(R \cup P) - J}$. So $|A| = 52 - 33 = 19$

5. Give a set expression and compute the cardinality for the set of cards that are either red non-prime cards or one-eyed Jacks, but not both.

   Solution:

   $A = (R \cap \bar{P}) \triangle J$. $|A| = 12 + 5 + 2 - 1 = 18$