Final answers are highlighted in yellow.
Problem 1: p1-p2
Problem 2: p2-p3
Problem 3: p4
Problem 4: p4-p6

Problem 1: Number representations

i. Convert 20010 to binary and hexadecimal.

Binary conversion:
200_{10} => 2^7 * 1 + 2^6 * 1 + 2^3 * 1

200_{10} => 11001000_2

Hexadecimal conversion:
20010 => 16^1 * 12 + 16^0 * 8

200_{10} => $C8_{16}$

ii. Convert 1010110110_2 to decimal and hexadecimal.

Decimal conversion:
1010110110_2 =>
By using bitwise operation,
$1_2 = 1_{10}$
$10_2 = 1 * 2 = 2_{10}$
$101_2 = 2 * 2 + 1 = 5_{10}$
…
$101011011_2 = 173 * 2 + 1 = 347_{10}$

$1010110110_2 = 694_{10}$

Hexadecimal conversion:
1010110110_2 =>
By splitting binary numbers to 0010|1011|0110
$0010_2 = 2_{16}$
$1011_2 = B_{16}$
$0110_2 = 6_{16}$

$1010110110_2 = 2B6_{16}$

iii. Convert ABC_{16} to 12-bit unsigned binary. Now, treating the binary as a 12-bit two's complement number, find the corresponding (negative) number in decimal

ABC_{16} =>
A_{16} => 10 * 16^1 => 1010_2
B_{16} => 11 * 16^1 => 1011_2
C_{16} => 12 * 16^1 => 1100_2

ABC_{16} => 101010111100_2
101010111100_2, the first digit represents the negative sign, the number actual number is -2^11 + 2^9 + 2^7 + 2^5 + 2^4 + 2^3 + 2^2 = -1348.

iv. Use a substitution trick to convert 110011110_3 to base-9. (Hint $2^4 = 16$, $3^2 = 9$)

110011110_3 =>
By splitting the number to 01|10|01|11|10
$01_3 = 1_9$
$10_3 = 3_9$
$01_3 = 1_9$
$11_3 = 4_9$
$10_3 = 3_9$
11001110_3 => 13143_9

Problem 2: Present and Past

i. A MAC (Media Access Control) address is a globally unique identifier assigned to network devices, and therefore it is often referred to as a hardware or physical address. MAC addresses are written in hexadecimal format like this: F0:23:9C:AA:4E:12. The first 6 hexadecimal digits identify the manufacturer, which is assigned by an Internet standards body. The second 6 hexadecimal digits are a serial number assigned by the manufacturer. How many possible devices can one manufacturer assign? How many total MAC addresses are possible? Assuming a current world population of 7.8 billion people (2021 UN estimate), how many devices could be allocated to each and every person?

Manufacturer number: F0:23:9C
Serial number: AA:4E:12

Since there are 16 possible inputs for each hexadecimal digit numbers, the possible devices that one manufacturer assign is 16^6 -1 = 16,777,215.
The total MAC addresses, based on the similar calculation step, are possible to be 16^12 -1 = 281,474,976,710,655, each individual would be allocated with (6^12 - 1) / (7.8* 10^9) = 36086.5≈ 36,086 devices.

ii. The Babylonians developed a sexagesimal (base 60) number system about 4000 years ago. They represented their numbers with a Cuneiform script rather than the digits 0, 1, 2... we know today, and they used the same symbol for both 1 and 60. Ignoring these subtlties, let's assume the symbols for the 60 Babylonian "digits" are (ZERO), (ONE), (TWO), ...(FIFTYEIGHT), (FIFTYNINE). How do you write 500210 in sexagesimal?

5002_{10}=>

If we were calculated a sexagesimal conversion, we can treat it as we were calculating a binary conversion.

60^0 = 1
60^1 = 60
60^2 = 3,600
60^3 = 216,000
60^4 = 12,960,000

5002 is less than 60^3, therefore the high bit will not exist on the fourth digit counting from left to right, the total digits of the converted sexagesimal number are only three.

1.
5002 div 60 = 83
5002 mod 60 = 22

2.
83 div 60 = 1
83 mod 60 = 23

3.
1 div 60 = 0
1 mod 60 = 1

The converted sexagesimal number is ONE TWENTYTHREE TWENTYTWO.

Problem 3: Go Huskies!

i. The Northeastern logo (in PNG format) and a hexadecimal dump of the first 64 bytes (00_{16} to $3F_{16}$) is provided above. The dimensions of the image, width × height, measured in pixels, are encoded by 8 bytes starting at 10_{16}: four bytes for the width followed by four bytes for the height. What are the dimensions of the image?

Starting at 10_{16} :

Width: $0000\ 0500_{16}$ => 1280
Height: $0000\ 00ec_{16}$ => 236

The dimension of the image is 1280 in width and 236 in height, and the total is 302,080 pixels.

ii. Suppose the image was uncompressed and consisted of a 24-bit color encoding for each pixel. How many kilobytes of diskspace would the image consume? (1 kilobyte = 2^{10} bytes)

8 bits = 1 byte;
The uncompressed image in bit = 1280 * 236 * 24 = 7,249,920 bits;
The uncompressed image in bytes = 7249920/8 = 906,240 bytes;
The uncompressed image in kilobytes = 906240/1024 = 885 kilobytes.

Problem 4: Alien Invaders

i. While on co-op at the Very Large Array in Soccorro, New Mexico, you receive a strange transmission coming from the star Vega, 25 light-years away. It is a sequence of numbers: -35, -9, -9, -1 which keeps repeating. Everyone is stumped until you suggest converting the numbers into four 8-bit 2's-complement numbers and sequencing them together to form a single 32-bit binary sequence. Have you discovered an alien intelligence? Explain your answer by identifying the resulting pattern. (Hint, write down a sequence of numbers denoting the number of sequential ones. What is this sequence? Is it likely to be naturally occuring? Recommended movie clip from Contact (1997): https://www.youtube.com/watch?v=-ciK05XqlOw

$-35_{10} \underset{abs}{\Longrightarrow} 35_{10} \underset{bin}{\Longrightarrow} 00100011_2 \underset{flip}{\Longrightarrow} 11011100 \underset{-1}{\Rightarrow} 11011101$

$-9_{10} \underset{abs}{\Longrightarrow} 9_{10} \underset{bin}{\Longrightarrow} 00001001_2 \underset{flip}{\Longrightarrow} 11110110 \underset{-1}{\Rightarrow} 11110111$

$-9_{10} \underset{abs}{\Longrightarrow} 9_{10} \underset{bin}{\Longrightarrow} 00001001_2 \underset{flip}{\Longrightarrow} 11110110 \underset{-1}{\Rightarrow} 11110111$

$-1_{10} \underset{abs}{\Longrightarrow} 1_{10} \underset{bin}{\Longrightarrow} 00000001_2 \underset{flip}{\Longrightarrow} 11111110 \underset{-1}{\Rightarrow} 11111111$

1101110111110111111101111111111

2 3 5 7 11(ones separated by zero)

The sequential ones are all prime numbers starting at 2, and it's likely to be naturally occurring.

ii. While on your next assignment at the Arecibo Radio Observatory in Puerto Rico, you get the following message coming from Proxima Centauri, the closest star (other than the Sun) to Earth at 4.22 light-years. The message reads: 0, 87, 82. 114, 82, 82, 87, 0. This time a single long binary sequence doesn't work. Try stacking the 8-bit binary representations to form an 8x8 pixel array (1=ON, 0=OFF). What is the message?

$0_{10} \underset{bin}{\Longrightarrow} 00000000_2$

$87_{10} \underset{bin}{\Longrightarrow} 01010111_2$

$82_{10} \underset{bin}{\Longrightarrow} 01010010_2$

$114_{10} \underset{bin}{\Longrightarrow} 01110010_2$

$82_{10} \underset{bin}{\Longrightarrow} 01010010_2$

$82_{10} \underset{bin}{\Longrightarrow} 01010010_2$

$87_{10} \underset{bin}{\Longrightarrow} 01010111_2$

$0_{10} \underset{bin}{\Longrightarrow} 00000000_2$

1	0	0	0	0	0	0	0	0
2	0	1	0	1	0	1	1	1
3	0	1	0	1	0	0	1	0
4	0	1	1	1	0	0	1	0
5	0	1	0	1	0	0	1	0
6	0	1	0	1	0	0	1	0
7	0	1	0	1	0	1	1	1
8	0	0	0	0	0	0	0	0
	1	2	3	4	5	6	7	8

The message is “HI”.

iii. Your fame as an exobiologist is secured! At Roswell, New Mexico, you are asked to examine a technical journal from an alien crash site. One strange equation reads: 412 + 156 = 601. Assuming this equation is correct, and the aliens learned to count with their fingers, how many fingers do our aliens probably have? For full credit derive your result algebraically rather than just guessing and verifying. Let b = the base, with the digit places representing powers of b: b^0 ,b^1 ,b^2 ,etc. Now solve for b.

Let b = the base,

Then the equation becomes:
4 * b^2 + 1* b^1 + 2*b^0 + 1 * b^2 + 5* b^1 + 6*b^0 = 6 * b^2 + 0* b^1 + 1*b^0
After simplifying the equation, the equation becomes:
b^2 - 6b - 7 = 0
(b-7)(b+1) = 0
b = 7 or -1, since the aliens do have figures, b = 7.
There are 7 figures that aliens have.