Homework # 2

Assigned: Wednesday January 26, 2022
Due: Tuesday February 1, 2022 @ 11:59pm ET/Boston

Instructions:

- Homework is due on Tuesday at 11:59pm ET/Boston. Homeworks received up to 15 hours late (3 pm on Wednesday) will be penalized 10 percent. *NO* assignment will be accepted after 3pm on Wednesday.
- We expect that you will study with friends and fellow students and you are welcome to verbally discuss the problems openly. However, your solution writeup should be the product of your own mind and expressed in your own words. The TAs and I will be available to answer specific questions or address speific points of confusion but we will not verify your answers.
- Assignments should be typed using Word or LateX, or hand-written *neatly*. When submitting to gradescope be sure to indicate the page containing your answer to each problem, so that the TAs don't have to search for your solution.
- *To get full credit, explain your solution and show each step!* We don't need your scratch work or draft solutions, only your final result.

Problem 1 [30 pts] (5pts each): Logical Equivalence

Prove each of the following by applying equivalence rules. State the equivalence law you are applying with each step in your proof.

i. $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Solution:

expression	*reason*
$\neg(p \vee (\neg p \wedge q))$	*start*
$\neg p \wedge \neg(\neg p \wedge q)$	*DeMorgan's*
$\neg p \wedge (\neg(\neg p) \vee \neg q)$	*DeMorgan's*
$\neg p \wedge (p \vee \neg q)$	*double negation*
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	*distributive*
$F \vee (\neg p \wedge \neg q)$	*complement*
$\neg p \wedge \neg q$	*identity*

ii. $p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$

Solution:

$expression$	$reason$
$p \to (q \to r)$	$start$
$p \to (\neg q \lor r)$	$definition\ of\ implication$
$\neg p \lor \neg q \lor r$	$definition\ of\ implication$
$\neg q \lor \neg p \lor r$	$rearranging\ terms$
$\neg q \lor (p \to r)$	$definition\ of\ implication$
$q \to (p \to r)$	$definition\ of\ implication$

iii. $\neg(p \to q) \equiv p \land \neg q$

Solution:

$expression$	$reason$
$\neg(p \to q)$	$start$
$\neg(\neg p \lor q)$	$definition\ of\ implication$
$\neg\neg p \land \neg q$	$DeMorgan's\ Law$
$p \land \neg q$	$removing\ double\ negative$

iv. $p \to q \equiv \neg q \to \neg p$

Solution:

$expression$	$reason$
$p \to q$	$start$
$\neg p \lor q$	$definition\ of\ implication$
$q \lor \neg p$	$rearranging\ terms$
$\neg q \to \neg p$	$definition\ of\ implication$

v. $p \to (q \land r) \equiv (p \to q) \land (p \to r)$

Solution:

$expression$	$reason$
$p \to (q \land r)$	$start$
$\neg p \lor (q \land r)$	$definition\ of\ implication$
$(\neg p \lor q) \land (\neg p \lor r)$	$distributive\ law$
$(p \to q) \land (p \to r)$	$definition\ of\ implication\ (2X)$

vi. $((p \to q) \land (q \to r)) \to (p \to r) \equiv T$ (a tautology)

Solution:

$expression$	$reason$
$((p \to q) \wedge (q \to r)) \to (p \to r)$	$start$
$\neg((p \to q) \wedge (q \to r)) \vee (p \to r)$	$definition\ of\ implication$
$\neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)$	$definition\ of\ implication\ (3X)$
$\neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee \neg p \vee r$	$DeMorgan's\ Law$
$(p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$	$DeMorgan's\ Law\ (2X)$
$\neg p \vee (p \wedge \neg q) \vee r \vee (q \wedge \neg r)$	$Rearranging\ terms$
$[(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee [(r \vee q) \wedge (r \vee \neg r)]$	$Distributive\ law\ (2X)$
$(\neg p \vee \neg q) \vee (r \vee q)$	$a \vee \neg a \equiv T\ (2X)$
$\neg p \vee r \vee (\neg q \vee q)$	$Distributive\ Law$
$\neg p \vee r \vee T$	$a \vee \neg a \equiv T$
T	$a \vee \neg a \equiv T$

Problem 2 [25 pts (5 pts each)]**: Converting to Logic**

In this problem, the letters J, P, and R denote simple statements:

J	"She likes Java"
P	"She likes Python"
R	"She likes Racket"

Use these letters and logic connectives $\neg$, $\vee$, $\wedge$ (and parentheses where needed) to write the following compound statements in symbolic form.

i. She likes Java but neither Python nor Racket.

Solution: $J \wedge \neg P \wedge \neg R$

ii. She likes at least one of these programming languages.

Solution: $J \vee P \vee R$

iii. She likes at least two of these programming languages.

Solution:

$$(J \wedge P \wedge R) \vee (J \wedge P \wedge \neg R) \vee (J \wedge \neg P \wedge R) \vee (\neg J \wedge P \wedge R)$$

or simplifying:

$$(J \wedge P) \vee (J \wedge R) \vee (P \wedge R)$$

iv. She likes *at most* one of these programming languages.

Solution:

$$(J \wedge \neg P \wedge \neg R) \vee (\neg J \wedge P \wedge \neg R) \vee (\neg J \wedge \neg P \wedge R) \vee (\neg J \wedge \neg P \wedge \neg R)$$

or simplifying:

$$(\neg J \wedge \neg P) \vee (\neg J \wedge \neg R) \vee (\neg P \wedge \neg R)$$

v. She likes all of these programming languages if she likes any of them.(Remember to remove the implication!)

Solution:

$$(J \vee P \vee R) \to (J \wedge P \wedge R)$$

simplifies to

$$(\neg J \wedge \neg P \wedge \neg R) \vee (J \wedge P \wedge R)$$

Problem 3 [25 pts]**: Where is Spock when you need him?**

As Captain of the USS Enterprise, your mission is to travel at warp speed to the planet Zorg to rescue some Vulcans whose vessel recently crashed. But beware – Romulan spies may be trying to infiltrate your ship! When you arrive, you find three aliens named Aravik, Balev, Chu'lak. They offer up the following statements:

- Aravik: *I am a Vulcan or Balev is a Romulan.*
- Balev: *Aravik is a Vulcan and Chu'lak is a Romulan.*
- Chu'lak: *Balev and I are different.*

Knowing as you do that Vulcans *always* tell the truth and Romulans *always* lie, which of the aliens, if any, do you rescue and which do you leave stranded on the planet? In your answer, clearly demonstrate that there is only one possible solution by enumerating all possible combinations of Romulans and Vulcans. Explain why each combination except one cannot be valid either because a Vulcan is lying or a Romulan is telling the truth.

Solution: You should rescue Aravik and Chu'lak and leave Balev behind. The truth table shows why.

Aravik	*Balev*	*Chu'lak*	*Does this make sense?*
R	R	R	*NO, then Aravik is an honest Romulan*
R	R	V	*NO, then Aravik is an honest Romulan*
R	V	R	*NO, then Balev is a lying Vulcan*
R	V	V	*NO, then Balev is a lying Vulcan*
V	R	R	*NO, then Balev is an honest Romulan*
V	R	V	*YES, Save Aravik and Chu'lak only!*
V	V	R	*NO, then Chu'lak is an honest Romulan*
V	V	V	*NO, then Chu'lak is a lying Vulcan*

Problem 4 [20 pts]**: Circular Argument**

Find all truth value combinations for a, b, c, d that satisfy $(a \to b) \wedge (b \to c) \wedge (c \to d) \wedge (d \to a)$, without using a truth table.

Solution: If a is true, then by first implication b must be true, by second implication c is true, then $c \to d$ makes d is true which also verifies the last implication. So we found a solution $(a = b = c = d = 1)$

If a is false, we work backwards: $(d \to a)$ can only be satisfied when d is false, then $(c \to d)$ makes c false, and $(b \to c)$ implies b false. So we have another solution $(a = b = c = d = 0)$

These two cases are complement to each other (a must be either T or F), so there is no other satisfying quadruplet.