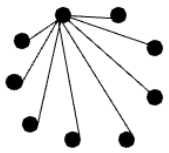
Homework #11

Problem 1 [20 pts (10, 10)]**: Graph Theory**

1. Nine people are at a party. Explain why it is impossible for each of the nine party goers to have shaken hands with exactly five other party goers.
2. If a graph with 1 million vertices consists of 3 connected components, what is the minimum number of edges it might have? Hint: How do we "minimally connect" a graph?

Solution:

1. Consider 9 party goers to stand in a circle and construct a k9 graph.
   To disconnect one of the vertices, at least 8 edges need to be removed to make it disconnected from the k9 graph.
   So each of the nine party goers must have shaken hands with exactly 8 other party goers rather that 5.



2. If we are looking for the minimum number of edges a group of vertices that are connected. These vertices must line up and each vertex must have degree of 2 except for vertices on both ends which only have a degree of 1.

   To get minimal edges for a graph, each vertex must have least edges as well.
   The form of a graph that has minimal edges is a line.

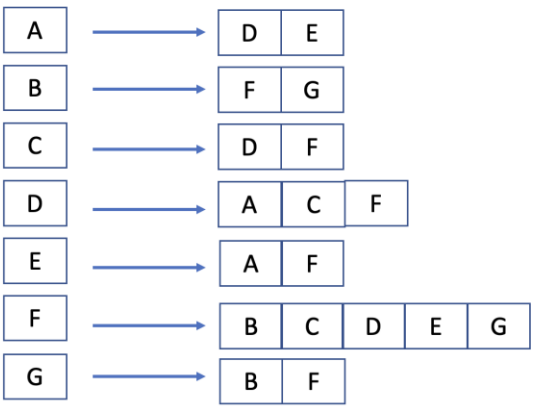
   When a graph is a line, vertices at both ends of the line have only degree of 1, and other vertices have a degree of 2. To disconnect the graph and split it into three connected components, only two edges need to be removed to make it.

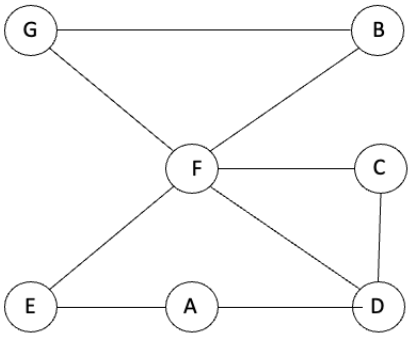
   A graph with 1million vertices consists of 1 million – 1 edges, in this sense, the minimum number of edges must be 1,000,000 – 1 - 2= 999,997 edges to form 3 connected components.

Problem 2 [20 pts (10,10)**: Graph Representations**

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	0	0	0	1	1	0	0
B	0	0	0	0	0	1	1
C	0	0	0	1	0	1	0
D	1	0	1	0	0	1	0
E	1	0	0	0	0	1	0
F	0	1	1	1	1	0	1
G	0	1	0	0	0	1	0

For the above adjacency matrix representation of an unweighted undirected graph, create an equivalent adjacency list representation and draw the resulting graph.





Deg(A) = 2

Deg(B) = 2

Deg(C) = 2

Deg(D) = 3

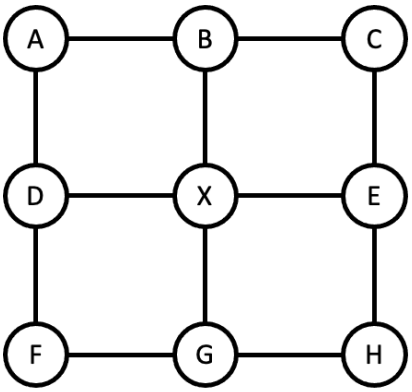
Deg(E) = 2

Deg(F) = 5

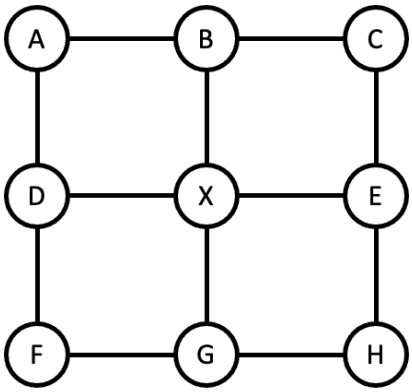
Deg(G) = 2

Problem 3 [20 pts (10,10)**: Graph Traversal**

For the 9-vertex graph below, highlight the edges that would be traversed using Depth-First Search (left) and Breadth-First Search (right) starting at the middle vertex labeled *X*. Add a number, 1 to 9 next to each vertex to show the order in which each vertex is visited, starting with a 1 next to *X*. In cases of a tie, the traversal should proceed alphabetically.



Depth-First Search



Breadth-First Search

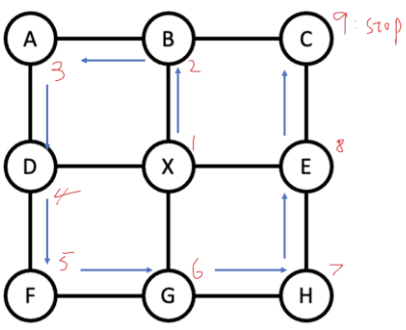
Solution:

DFS:

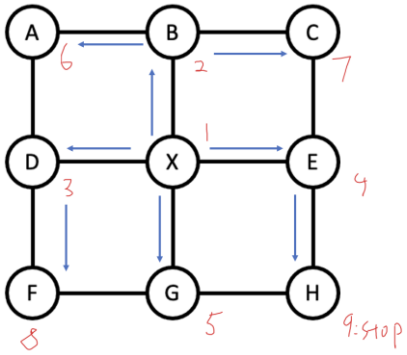
X -> B -> A -> D -> F ->G ->H ->E -> C

BFS:

X -> B -> D -> G -> E -> A -> C -> F -> H



Depth-First Search



Breadth-First Search