Haozhe(Jason) Zhang
CS 5002 Discrete Math

Homework #04

1. Problem 1 [25 pts]: Convert each set S to the listing method. It is ok to include ellipses where appropriate. Remember that the set N includes 0, which is an even number.

   i. S = {x: x ∈ N and |x|<5}
   ii. S = {x ∈ R: 2x∈R} where R = {2,3,5,8,13}.

   iii. S = {x: x ∈ P and 2x−1∈ P} where P = {x ∈ N: x ≤ 20and x is prime}

   iv. S= {2x: x ∈ T} where T ={y:3y∈W}and W = {1,3,7}.
   v. S = {(x, y) ∈ Z×Z: d((x, y),(0,0))= √2 and d is the Euclidian distance function}

   (N : natural numbers, Z : all integers, Q : rational numbers, R : real numbers)

   N ⊂ Z ⊂ Q ⊂ R

   Solution:

   i. S = {x: x ∈ N and |x|<5}

      S = {1, 2, 3, 4}, natural numbers are 1, 2, 3, 4…….

   ii. S = {x ∈ R: 2x∈R} where R = {2,3,5,8,13}.

      S = {}, there is no such set that both itself and twice of the value exists in the set R.

   iii. S = {x: x ∈ P and 2x−1∈ P} where P = {x ∈ N: x ≤ 20and x is prime}

      S = {2, 3, 7}, 2 * 11 = 22 where is out of range, the possible answers are 2, 3, 5, 7, and when x = 5, 2x-1 = 9 which can be excluded from the S set.

   iv. S= {2x: x ∈ T} where T = {y: 3y ∈ W}and W = {1,3,7}.

      S = {2/3, 2, 14/3}, W / 3 * 2 = S.

   v. S = {(x, y) ∈ Z×Z: d((x, y),(0,0))= √2 and d is the Euclidian distance function}

      S = {(1,1), (-1,-1)}, d(euclidian) = sqrt (x^2 + y^2) where x,y belongs to Z.

      x=y=-1 or x=y=1 satisfy the condition.

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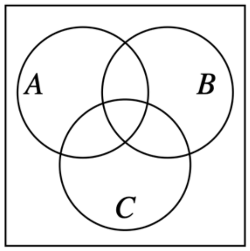
Problem 2 [25 pts]: Venn Diagrams

Draw Venn diagrams for the following expressions using the layout indicated below.



i. $(A \cup B) \cap \overline{C}$
ii. $(B - A) \cap (C - A)$

iii. $(C - B) \triangle A$

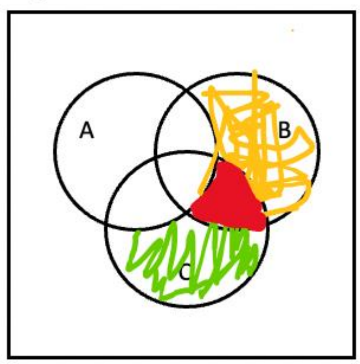
iv. $A \triangle \overline{B}$

v. $(A \triangle B \triangle C) \cup (A - B)$

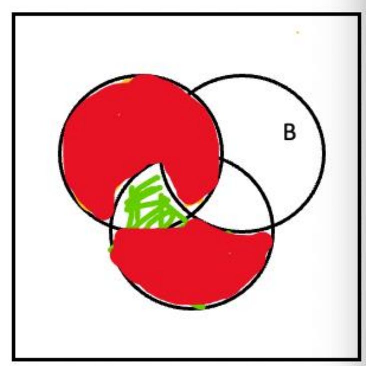
Solution: (solution in red)

i

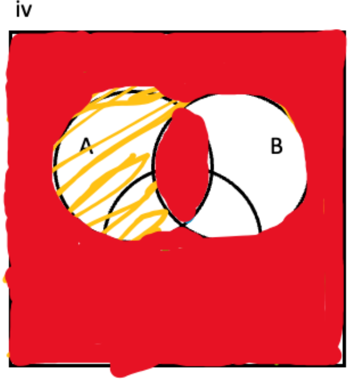
ii



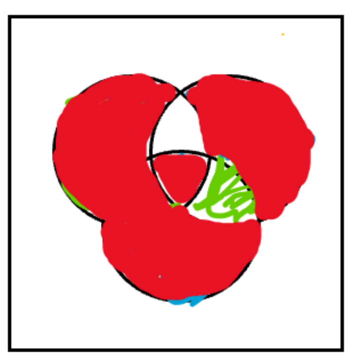
iii



iv



v



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Problem 3 [25 pts]: Programming Languages

We asked 100 CS majors whether they liked Python, Java, and/or Racket.
• 15 students didn't respond.
• 10 liked all three languages.
• 13 liked Java and Racket, 25 liked Python and Racket, and 15 liked Java and Python.

• Half the surveyed students indicated they like Python.

• Twice as many students indicated they liked Racket as liked Java.

Note that when we say half liked Python, we mean that they liked Python and maybe other languages as well. Similarly, the 13 students that liked Java and Racket includes the 10 that indicated they like all three languages. (a) How many students liked Java and how many liked Racket? (b) How many students indicated they liked exactly one language?

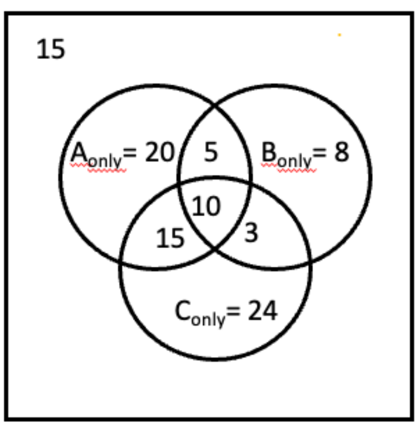
Solution:

Let A: Student who likes Python, B: Student who likes Java, C: Student who likes Racket

$A \cup B \cup C = 100 - 15 = 85$

$A \cap B \cap C = 10$

$B \cap C = 13$, $A \cap C = 25$, $B \cap A = 15$, $A = 50$, $C = 2B$;



a) There are 26 students liked Java and 52 students liked racket.
b) 20 + 8 + 24 = 52 students indicated that they liked exactly one language.

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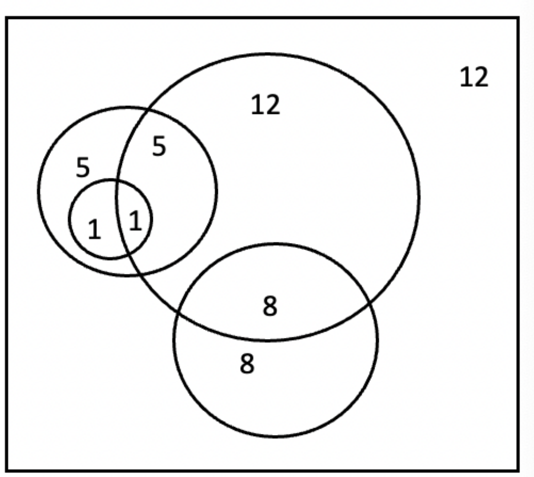
Problem 4 [25 pts]: Playing Cards Let C represent a set of 52 playing cards with four suits (♥,♦,♣,♠) each having 13 ranks (Ace,2,3,4,5,6,7,8,9,10,Jack,Queen,King). The ♥'s and ♦'s are red cards. The ♣'s and ♠'s are black. We define the following additional sets. F =Face cards (Jack, Queen, and King). R =Red cards. P =Cards having a rank that is a prime number (2,3,5,7). J =One-eyed Jacks (Jack of Hearts, Jack of Spades).

1. Depict these sets as a Venn Diagram and show the cardinality of each distinct region. The regions don't have to be perfectly to scale - this can be hand-drawn. Only overlap sets that truly overlap. Disjoint sets, those sharing no members, should be rendered as non-overlapping.
2. Using set notation, give an expression for the set of cards that are red or face cards or prime numbered or one-eyed Jacks and compute its cardinality.
3. Give a set expression and compute the cardinality for the set of cards that are not face cards or not prime-numbered cards.
4. Give a set expression and compute the cardinality for the complement of the set of cards that are red or prime-numbered but not one-eyed Jacks.
5. Give a set expression and compute the cardinality for the set of cards that are either red non-prime cards or one-eyed Jacks, but not both.

Solution:

F ∩ R = 6, P ∩ R = 8, J ∩ R = 1, J ∩ F = 2;



1.

2. R ∨ F ∨ P ∨ J = 52 – 12 = 40, cardinality is 40.

3. Not F or not P = ¬F ∨ ¬P = ¬ (F ∧ P) = $\overline{F \cap P}$ = $\overline{F} \cup \overline{P}$, cardinality = 52

4. Not (F or P and not J) = ¬((F ∨ P) ∧ ¬J) = $(\overline{(F \cup P) \cap \overline{J}})$ = $\overline{F} \cap \overline{P} \cup J$, cardinality = 26

5. (R and not P) XOR J = (R ∧ ¬P) ⊕ J = $R \cap \overline{P} \,\triangle\, J$, cardinality = 16