Problem 1

5 cards, 3 black $\{1, 2, 3\}$, 2 red $\{1, 2\}$

randomly pick 2.

- (R_1, B_1), (R_2, B_1), (B_1, B_2)
- (R_1, B_2), (R_2, B_2), (B_1, B_3)
- (R_1, B_3), (R_3, B_3), (B_2, B_3)
- (R_1, R_2),

i. Both cards are red.

$$= \frac{2}{5} \times \frac{1}{4} \quad (\text{1st} \times \text{2nd})$$

$$= \frac{2}{20}$$

$$= \frac{1}{10}$$

ii. Known at least 1 red, & both are red.

At least 1 red = Total − No red

$$= c(5, 2) - c(3, 2)$$

$$= 10 - 3$$

$$= 7$$

$$\frac{\text{Both Red case}}{\text{At least 1 red}} = \frac{1}{7}$$

iii. Known one is red $= \{1\}$, & both are red

$$\frac{\text{Favorable case}}{\text{Total sample}} = \frac{1}{4}$$

Problem 2.

i. $\frac{1}{4}$ th time go in every pot

$$\text{Winning per ball} = \frac{1}{4} \times 1 + \frac{1}{4} \times 2 + \frac{1}{4} \times 3 + \frac{1}{4} \times 4$$
$$= \frac{10}{4}$$

$$\text{Total 4 balls} = \frac{10}{4} \times 4$$
$$= \underline{10}$$

ii. each probability $= \frac{1}{4} \times \frac{2}{3}$
$= \frac{1}{6}$

$$\text{Winning per ball} = \frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \frac{1}{6} \times 4$$
$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6}$$
$$= \frac{10}{6}$$
$$= \frac{5}{3}$$

$$\text{Total 4 balls} = \frac{5}{3} \times 4$$
$$= \underline{\frac{20}{3}}$$

#of1 Problem 3

1 $1 = 00001$, $2 = 00010$, $4 = 00100$, $8 = 01000$, $16 = 10000$.

2 $3 = 00011$, $5 = 00101$, $6 = 00110$, $9 = 01001$, $10 = 01010$, $24 = 11000$
$12 = 01100$, $17 = 10001$, $18 = 10010$, $20 = 10100$,

3 $7 = 00111$, $11 = 01011$, $13 = 01101$, $14 = 01110$, $19 = 10011$ $21 = 10101$
$22 = 10110$, $25 = 11001$, $26 = 11010$, $28 = 11100$. $n = 10$

4 $15 = 01111$, $23 = 10111$, $27 = 11011$, $29 = 11101$, $30 = 11110$,

5 $31 = 11111$,

1. $S = 32$, $n = 10$

$$P(W=3) = \frac{n}{S} = \frac{10}{32} = 0.3125$$

2. $n = 6$

$$P(W=3 \cap Odd) = \frac{n}{S} = \frac{6}{32} = 0.1875$$

3. $n = 6$, $S = 10$

$$P(Odd \mid W=3) = \frac{n}{S} = \frac{6}{10} = 0.6$$

4. $$\text{Expected per card} = \frac{\text{Possible of 1 in 5 Places}}{\text{sample space}}$$

$$= \frac{C(5,1)\times 1 + C(5,2)\times 2 + C(5,3)\times 3 + C(5,4)\times 4 + C(5,5)\times 5}{32}$$

$$= \frac{5 + 20 + 30 + 20 + 5}{32} = \frac{80}{32} = \frac{5}{2}$$

$$\text{Total 3 cards} = \text{Expected per card} \times 3$$

$$= \frac{5}{2} \times 3 = \frac{15}{2} = 7.5$$

5. Sum = 13 {(5, 5, 3), (5, 4, 4)}, 5 can not occur 2 times

$$P(\text{sum}=13) = \frac{\text{Possible of sum of 13}}{\text{Sample space}}$$

$$= \frac{6}{32} = 0.1875$$

Problem 4

infected		$P(I) = 0.2$
infected	positive	$P(P/I) = 0.9$
Uninfected	positive	$P(P/U) = 0.5$
Uninfected		$P(U) = 1 - 0.2 = 0.8$

1. $P(P) = P(P/I) \times P(I) + P(P/U) \times P(U)$
$= 0.9 \times 0.2 + 0.5 \times 0.8$
$= 0.18 + 0.4$
$= \underline{0.58}$

2. $P(V/P) = \frac{P(I)}{P(P)}$
$= \frac{0.2}{0.58}$
$= \underline{0.345}$

3. $P(N) = 1 - P(P)$
$= 1 - 0.58$
$= 0.42$

$P(T_1) = P(P) = 0.58$

$P(T_2) = P(I) \times [1 - P(P/I)]$
$= 0.2 \times (1 - 0.9)$
$= 0.02$

Total probability $= P(T_1) \cdot P(T_2)$
$= 0.58 \times 0.02$
$= \underline{0.0116}$