CS5002 Fall 2021 Midterm Exam

Instructions: PLEASE READ!

1. The midterm exam is a take-home exam, open book and open notes. You will submit your exam to GradeScope just as you would a regular homework. You may use a calculator but it isn't required. *Show your work and explain your answers for credit.* Unless otherwise specified, you may leave your answer in the form of a fraction or mathematical expression that clearly shows your thought process.

2. We recommend you take the exam during the scheduled class period. However, you have a 12 hour window during which you must start and complete your exam. The 24-hour window is from Wednesday October 27th at 12:00 noon (ET/Boston) to 11:59pm (ET/Boston). Once started, you have 3 hours and 15 minutes to complete *and submit* the exam. So make sure you start before 8:45pm (ET/Boston) on Wednesday and give yourself 15-20 minutes to correctly submit your exam.

3. If you have a question about the exam, post a PRIVATE question on Piazza. The instructors and TAs will be monitoring Piazza throughout the period and will respond to you as soon as we can.

4. *IMPORTANT:* You may *not* look up answers on the Internet. You may *not* discuss the exam with other students. Even comparing a single answer will be regarded as cheating and you will be given a zero for the entire exam and possibly an automatic F for the course. Please take us seriously on this point. Giving *or* receiving help on the midterm of any kind will have very unfortunate consequences for everyone involved.

5. Do your best and best of luck!

Section 1 [25 pts (5 each)]**: Number Representations**

1. Convert -17_{10} to 8-bit 2's complement. (Show your work.)

   <u>*Solution:*</u> $+17_{10} = 00010001_2$. Flipping bits and adding one: $00010001_2 \rightarrow 11101110_2 + 1 = 11101111_2$.

2. Convert 5002_6 to decimal. (Show your work.)

   <u>*Solution:*</u> $5 \cdot 6^3 + 0 \cdot 6^2 + 0 \cdot 6^1 + 2 \cdot 6^0 = 1082_{10}$.

3. What is the largest 3 digit number we can represent in Octal? Give the octal representation and convert to decimal.

   <u>*Solution:*</u> $777_8 = 7 \cdot 64 + 7 \cdot 8 + 7 \cdot 1 = 511_{10}$

4. Convert 10110111011_2 to hexadecimal.

   <u>*Solution:*</u> $10111111011_2 = 0101\ 1011\ 1011_2 = 5BB_{16}$

5. A and B are 8-bit two's complement integers. $A = 11110001_2$ and $B = 11111111_2$. Using binary arithmetic, compute $A - B$. Give your result as an 8-bit two's complement integer.

<u>*Solution:*</u> $A - B = A + (-B)$. Convert B to its negation by flipping the bits and adding one. This gives 00000001_2 for B. So we have simply: $11110001_2 + 00000001_2 = 11110010_2$. (Verifying: $-79 - (-1) = -78$)

Section 2 [25 pts (10,10,5)]**: Logic**

Given the domains $x \in NortheasternStudents$ and $y \in Jobs$ and the following predicates:

- K(x): x is a Khoury student.
- G(x): x is a student who graduates.
- O(x, y): x gets offered job y.
- A(x, y): x applies for job y.

1. Express the following English sentence with predicate logic using Existential and Universal quantifiers:
   Any Northeastern student who graduates from Khoury college gets at least two job offers

   Solution: $\forall x \exists y \exists z K(x) \wedge G(x) \rightarrow O(x, y) \wedge O(x, z) \wedge y \neq z$

2. The English statement *There is a job that every Koury graduate applies for.* can be represented as: $\exists y \forall x \ K(x) \wedge G(x) \rightarrow A(x, y)$. Apply laws of logical equivalence to negate this predicate expression. In your final answer, negations should only occur before single predicates if and when negations occur and parentheses should be minimized. *Apply one step at a time and identify the equivalence rule being applied with each step.*

   Solution:
   $\neg\exists y \forall x \ K(x) \wedge G(x) \rightarrow A(x, y)$ Start
   $\forall y \neg\forall x \ K(x) \wedge G(x) \rightarrow A(x, y)$ Negation of existential quantifier
   $\forall y \exists x \ \neg(K(x) \wedge G(x) \rightarrow A(x, y))$ Negation of universal quantifier
   $\forall y \exists x \ \neg(\neg(K(x) \wedge G(x)) \vee A(x, y))$ Defn. of Implication (conditional identity)
   $\forall y \exists x \ \neg\neg(K(x) \wedge G(x)) \wedge \neg A(x, y)$ DeMorgan's
   $\forall y \exists x \ (K(x) \wedge G(x)) \wedge \neg A(x, y)$ Double Negation
   $\forall y \exists x \ K(x) \wedge G(x) \wedge \neg A(x, y)$ Associative

3. Convert your final expression from problem (2) back into English.

   Solution: For every job there is a Khoury graduate who does not apply.

Section 3 [25 pts (5 each)]**: Sets**

1. List the elements in the set $S = \{n | n \in \mathbb{N} \text{ and } n! = 7\}$

   Solution: $\emptyset$ or $\{\}$

2. State whether this statement is TRUE or FALSE (justify your answer).$13 \in \{3x + 10 | \ x \in \mathbb{N}\}$

   Solution: True. $x = 1$ is in the natural numbers.

3. Let A=$\{X, Y, Z\}$ and B=$\{1, 2, 3, 4, 5\}$. What is $|\mathcal{P}(A \times B)|$? ($\mathcal{P}$ denotes the power set.)

   Solution: $|\mathcal{P}(A \times B|) = 2^{|A \times B|} = 2^{|A| \cdot |B|} = 2^{15} \approx 32$ thousand.

4. Let A, B, and C be sets, using the laws of set theory show that: $(B - A) \cup (C - A) = (B \cup C) - A$

   Solution: Recall that $A - B = A \cap \overline{B}$
   $(B - A) \cup (C - A)$
   $= (B \cap \overline{A}) \cup (C \cap \overline{A})$
   $= (B \cup C) \cap \overline{A}$ Distributive law
   $= (B \cup C) - A$

5. Pavement Coffeehouse conducted a survey to see what kinds of coffee its customers like most. Given the following responses, determine how many people responded to the survey:

   - 40 like espresso
   - 37 like lattes
   - 75 like iced coffee
   - 19 like both espresso and iced coffee
   - 13 like both espresso and lattes
   - 10 like both iced coffee and lattes
   - 4 like all three options
   - 16 don't like of any of the options

   Solution: Let's start by defining sets: E is the set of people who like espresso, L for lattes, and I for iced coffee.
   The total number of people surveyed is:
   $|E \cup L \cup I| = |E| + |L| + |I| - |E \cap L| - |E \cap I| - |L \cap I| + |E \cap L \cap I|$
   $|E \cup L \cup I| = 40 + 37 + 75 - 13 - 19 - 10 + 4 = 114$. Finally, there are 16 who don't like any of the coffee options, so we get $114 + 16 = 130$

Section 4 [25 pts (5 each)]**: Counting**

It is October when thoughts turn to baseball and the Red Sox winning the World Series! (Maybe next year.)

1. The world series is played by the final two teams not eliminated in the playoffs. The team that wins the world series is the first team to win 4 games. As soon as one team or the other has won 4 games, the series is over and no more games are played. Explain why the world series can not go more than 7 games. What mathematical principle are you appealing to in your answer?

   Solution: By the pigeon hole principle at least one team will have won four games (concluding the series) in 7 games or fewer. The games are the pigeons, and the holes are the two teams who receive a WIN whenever they win the game and

   $$\lceil \frac{7}{2} \rceil = \lceil 3.5 \rceil = 4,$$

   so one of the teams will have won 4 games by the 7th game (or possibly sooner.)

2. If the Red Sox play in the World Series, in how many ways can they win the World Series? Each *way* is defined be a sequence of Wins (W) or Losses (L). Possible winning sequences include WWWW, WLWWLW, and LLLWWWW. Remember that the World Series can go at most 7 games and no more games are played after one of the two teams wins 4 games. For full credit, state your answer in the form of a concise mathematical expression and then reduce your answer to a single integer. (No credit is given for simply enumerating all possibilities.)

   Solution: Remembering that the series ends immediately after the Red Sox win their 4th game, we add up all the ways to have won exactly 3 games in the prior 3, 4, 5, or 6 games.

   $$\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \binom{6}{3} = 1 + 4 + 10 + 20 = 35$$

   ways for the Red Sox to win (or lose) the World Series.

3. On a baseball team there are 9 player positions, designated 1-9. The manager decides the batting order of the nine players. This is sometimes called the batting lineup or simply the *lineup*. How many lineups have the 6 infielders $\{1, 2, 3, 4, 5, 6\}$ batting together (in any order) and the 3 outfielders $\{7, 8, 9\}$ batting together in any order. One possible batting sequence is $8 \to 7 \to 9 \Rightarrow 1 \to 6 \to 2 \to 5 \to 3 \to 4$ and another is $5 \to 6 \to 3 \to 4 \to 1 \to 2 \Rightarrow 9 \to 8 \to 7$. A concise mathematical expression suffices for an answer but be sure to *explain* your answer.

   Solution: $2 \cdot 3! \cdot 6! = 8,640$
   There are 3! ways to order the outfielders and 6! ways to order the infielders. We add a factor of 2 because the infielders may bat before or after the outfielders.

4. How many lineups have player 1 batting before player 2 and player 7 batting before player 8, and player 8 before player 9. In each case, *before* does not necessarily mean *immediately before.* For example, one possible valid lineup is $3 \to ① \to 4 \to \boxed{7} \to \boxed{8} \to ② \to 5 \to 6 \to \boxed{9}$. A concise mathematical expression suffices for an answer.

   <u>*Solution:*</u> This is like the BANANA problem. Treat players 1 and 2 as identical because there is only one valid ordering of these players, and treat players 7, 8, and 9 as identical because there is only one valid ordering of these three players. So we reduce the problem to permutations on A, A, 3, 4, 5, 6, B, B, B which gives $\frac{9!}{2! \cdot 3!}$

5. You go to the Fenway park snack bar to buy lunch. They offer five drinks (Coke, Pepsi, beer, water, lemonade), five hotdog condiments (ketchup, mustard, relish, onions, hot sauce), and five snacks (peanuts, popcorn, ice cream, cracker jack, candy). If a lunch consists of 2 (possibly identical) drinks, a hotdog with 0 to 5 distinct condiments on top, and two *distinct* snacks, how many possible lunches could you buy? (Note: hotdogs with different sets of condiments are distinct but the order in which condiments are added to the hotdog makes no difference. The order in which the drinks are selected makes no difference. The order in which the two snacks are chosen makes no difference.)

   <u>*Solution:*</u>

   1. Drink selection is balls and bins: $\binom{2+5-1}{5-1} = \binom{6}{2} = 15$.
   2. Pick a subset of hotdog condiments in 2^5 ways.
   3. Pick two distinct snacks in $\binom{5}{2}$ ways.

   Altogether the number of possible lunches is $15 \cdot 32 \cdot 10 = 4800$