HW 7 Midterm problems and recursive algorithms
Due: March 22, 2022 at 5pm PT

Deadline shows as **8pm ET on Gradescope, note the timezone**!

Instructions

- Please see previous homework assignments for the instructions. By now it's expected you are familiar with the expectations.
- This homework assignment is to be done in pairs. You must work with someone else in the class and submit one single submission on Gradescope. To learn how to add group members at the time of your submission, see: https://help.gradescope.com/article/m5qz2xsnjy-student-add-group-members

Problems

1. **[6 points]** Define a function $f : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Q}$ as $f(a, b) = \frac{a}{b}$. Answer each question below, and **justify your answers**; a simple answer without justification will not earn credit.
   (a) Is f one-to-one?
   (b) Is f onto?
   (c) Is f a bijection?

2. **[6 points]** Let $f : A \to B$ and $g : B \to C$ be functions, prove that if $g \circ f$ is onto, then g is onto, but f may not be.

3. **[8 Points: 3, 5]** Given the recurrence relation: $T(n) = 9T(n/3) + n$, where $T(1) = 1$, (a) use the Master Method to to find a tight bound and (b) find the closed form of this recurrence iteratively (either forward or backward substitution, up to you).

4. **[8 points (2, 2, 4)]** Given the algorithm below, determine what it does and analyze it to find the worst-case time complexity based on the number of operations performed. More specifically, answer each of the following questions. Be sure to show your work, an answer (e.g. a summation) without proper justification will earn minimal points.

```
MysteryProcess

 Input: a1, a2,…, an
        n, the length of the sequence

 Output: ???

 m = 0
 r = a1
 for i = 1 to n
   c = 0
   for j = i+1 to n
     if ai == aj
       c += 1
   if c > m
     m = c
     r = ai

 return r
```

(a) **summarize concisely in "natural language"** (in just a few words) what the algorithm does. Avoid explaining what each instruction does. Instead, focus on what the algorithm does by the time it's done — in this case, what does the value returned represent?

(b) What is the worst-case input for the algorithm?

(c) Characterize the asymptotic growth of the worst-case time complexity of the algorithm (that is, find the upper bound and the lower bound for the worst-case time complexity). Justify your answer.

5. **[20 points]** These three problems are related to reading assignment 7 on recursive algorithms and divide and conquer.
   (a) **Design a recursive algorithm** whose input is a sequence of sorted numbers, the length of the sequence, and two numbers (say x and y). Its output is the number of values in the sequence that are in the inclusive range `[x, y]`.

   - For example, if the sequence is `[-8, -1, 6.2, 9, 10.5, 17, 18]` then the algorithm should give as result 0 when x=0 and y=5, and 2 when x=5 and y=10.
   - The algorithm should return 0 when the range is undefined, e.g.: x=3 and y=0
   - The algorithm should return the length of the sequence when x is at most the value of the first element in the sequence and y is at least the value of the last element in the sequence.

   - The algorithm should be as efficient as possible. To do this, adapt the recursive binary search algorithm.

   (b) Use induction to **prove that your algorithm outputs the correct result**. See relevant participation activities in zyBooks like Proofs 10.11.1 and 10.11.2

   (c) Describe the worst-case for the algorithm.

   (d) Give and solve the recurrence relation to describe the asymptotic time complexity of your algorithm.