

## CS52002 Midterm Exam

### Instructions (for midterm exam):

1. The exam is open book and open notes. You may use a calculator but it should not be necessary.
2. The exam window starts at 12:00pm ET/Boston (Noon) and all exams must be received by 11:59pm ET/Boston. Once started you will have 3 hours and 15 minutes to complete the exam AND submit your exam to gradescope.
3. Discussing the exam with others is strictly prohibited. Giving or receiving help from fellow students earns all parties involved an automatic F for the class. Do not test us on this.
4. *Unless otherwise specified*, you may leave your answer in the form of a fraction or mathematical expression that clearly shows your thought process. For credit you must show your scratch work and clearly explain how you got to your final answer. If your expression isn't clear or obvious you won't earn full credit.
5. If you have a question, post a PRIVATE question on Piazza. An instructor will respond as soon as possible.
6. When submitting your exam you must specify, for each problem, the page or pages where the solution to that problem is to be found. If we can't see your solution you will not receive credit for the problem.
7. You have been granted an extra 15 minutes solely for the purpose of uploading your exam. Exams emailed to instructors will receive a 5 percent penalty, so be sure to give yourself enough time to submit your exam! Exams received via email more than a few minutes beyond the deadline will not be graded.
8. May the force be with you!

PRINT FULL NAME: \_\_\_\_\_ STUDENT ID: \_\_\_\_\_

I have read the instructions above and understand that I may not discuss the exam with other students at any time. SIGN HERE: \_\_\_\_\_

## Section 1 [25 pts (5 each)]: Number Representations

1. Convert  $BB8_{16}$  to octal. (Show your work.)

$$BB8(16) = 11 * 16^2 + 11 * 16 + 8 = 3,000(10)$$

In hexadecimal, B = 11, number inside the parenthesis represents which numeral system the number is using.

2. Convert  $D2_{16}$  to binary. Now, treating the number as an 8-bit two's-complement number, convert to decimal (base-10).

D = 13 in hexadecimal

$$D2(16) = 11010010(2) \leftarrow 8\text{-bit 2's complement number}$$

Flipping bits, 11010010  $\rightarrow$  00101101

Adding one, 00101101  $\rightarrow$  00101110

$$00101110(2) = 46$$

Adding the negation sign = - 46

3. The droid C3PO from Star Wars knows 6 million forms of communication. We assign each form of communication a unique identifier in the form of a fixed-length unsigned binary number. (By fixed-length we mean that the number of bits allocated to each identifier is the same.) How many bits must be allocated for each identifier? Give your answer in the form of an integer, and explain.

Let  $n$  be the fixed-length of bits.

Then the maximum unsigned  $n$ -bits integer has  $2^n - 1$  bits

$$\text{Let } 2^n - 1 = 6,000,000$$

$$2^{22} = 4,194,304$$

$$2^{23} = 8,388,608$$

In this case, we need to have a fixed-length of  $n = 23$  bits unsigned binary to include 6,000,000.

4.  $2^{15} = 32,768$ . How many numbers between 0 and  $2^{15}$  are the sum of exactly three powers of 8? Each power must be a natural number. Natural numbers are the non-negative integers (0, 1, 2, 3, ...). The powers need not be distinct. For example, 10 is one such number because  $10 = 8^1 + 8^0 + 8^0 = 8 + 1 + 1$  and 521 is another such number because  $521 = 8^3 + 8^1 + 8^0 = 512 + 8 + 1$ . Reduce your answer to an integer and explain your answer.

5. Droids on the planet Tatooine perform calculations in base-6. What is the largest base-6 number that can be represented with three distinct digits? (Each digit in your base-6 number must be different.) Convert your answer to decimal (base-10).

The largest base-6 number that is represented with three distinct digits is 543

$$5 * 6^2 + 4 * 6^1 + 3 = 207$$

## Section 2 [25 pts]: Logic

Given the domains  $m \in MOVIES$  and  $c \in CHARACTERS$  and the following predicates:

- $SW(m)$ :  $m$  is a Star Wars movie.
  - $A(c, m)$ :  $c$  is a character who appears in the movie  $m$
  - $D(c)$ :  $c$  is a Droid.
  - $L(m)$ : I liked  $m$ .
  - $Q(c, q)$ :  $c$  says  $q$  ( $c$  is quoted as saying  $q$ .)
  - $jarjar$  is the character Jar Jar Binks
  - $bad$  is the quote: "*I've got a bad feeling about this!*"
1. Express the following English sentence with predicate logic using Existential and/or Universal quantifiers: *I didn't like any Star Wars movie if Jar Jar Binks was in it.*

x

2. Express the following English sentence with predicate logic using Existential and/or Universal quantifiers: *At least two droids appear in every Star Wars movie.* You may use the notation  $x = y$  and  $x \neq y$  as needed.



**Section 3** [25 pts (15,10)]: **Sets**

1. In this problem  $A = \{R2D2, C3PO, BB8\}$  and  $B = \{BB8, R5D4\}$ .  $|A|$  denotes the cardinality (number of members) of the set  $A$ . Answer each question with an integer or an expression involving exponents. You do *not* need to show each step.

i.  $|A \times B| = ?$

ii.  $|A - B| = ?$

iii.  $|(A - B) \cap (B - A)| = ?$

iv.  $|\mathcal{P}(A \triangle B)| = ?$

v.  $|\{x : x \in \mathcal{P}(A \cup B) \wedge |x| = 2\}| = ?$

2. Some rebel pilots were surveyed to determine what they thought of different star fighters. The three kinds of star fighters are the X-Wing, the Y-Wing, and Tie Fighters.

- Every pilot liked at least one kind of star fighter
- 1 liked all three. (The force is strong with this pilot!)
- 1 liked X-Wings and Y-Wings
- 2 liked X-Wings and Tie Fighters
- 3 liked Y-Wings and Tie Fighters
- 5 liked X-Wings
- 8 liked Y-Wings
- 13 liked Tie Fighters.

Use the Principle of Inclusion and Exclusion to determine how many pilots answered the survey. Draw a Venn Diagram depicting the above data. Note: when we say, for example, that  $n$  pilots liked a particular kind of Star Fighter, it means they might have liked other types as well. For example, some of the 13 pilots that like Tie Fighters liked X-Wings, Y-Wings or maybe both.

**Section 4 [25 pts (5 each)]: Counting**

1. Suppose droids in the Star Wars universe follow the following naming conventions:

- Names must be 3-5 characters including uppercase letters and digits. (No dashes.)
- All names must start with a letter and contain at least 1 digit.

How many droid names are possible? You can leave your answer as an expression as long as you clearly explain each term in the expression. Some valid names include BB8, R2D2, C3PO, and H2SO4.

2. Seventeen Jedi are fighting for their lives in an arena. Each Jedi uses a light saber with one of 4 colors.

- (a) Explain why five or more Jedi must be using the same color light saber. What mathematical principle are you invoking in your explanation?
- (b) How many Jedi would there need to be in the arena in order to guarantee that seven or more Jedi are using the same color light saber?



3. Nine Star Wars movies make up the three Star Wars trilogies:

- (a) Episodes 1-3 (The prequels with Jar Jar Binks)
- (b) Episodes 4-6 (The originals starring Mark Hamill as Luke Skywalker)
- (c) Episodes 7-9 (The ones starring Daisy Ridley as Rey)

My weekend movie marathon consists of an ordered sequence of five distinct movies selected from these nine. How many movie marathons are possible? (The same five movies watched in a different sequence constitute a different marathon.)

4. What if each marathon consists of four to six movies and each marathon is distinguished by the particular movies I select and not the order in which I watch them. Furthermore, I'm allowed to pick the same movie more than once. Now how many movie marathons are possible?

5. What if my movie marathon consists of exactly five distinct movies: two movies from one of the trilogies, 2 movies from a 2nd trilogy, and 1 movie from a 3rd. The movies can be watched in whatever order I choose and the same five movies watched in a different order constitute a different marathon. Now how many movie marathons are possible?