Homework #11

**Problem 1** [20 pts (10, 10)]**: Graph Theory**

# Nine people are at a party. Explain why it is impossible for each of the nine party goers to have shaken hands with exactly five other party goers.

1. If a graph with 1 million vertices consists of 3 connected components, what is the minimum number of edges it might have? Hint: How do we ”minimally connect” a graph?

Solution:

1. Consider 9 party goers to stand in a circle and construct a k9 graph.

To disconnect one of the vertices, at least 8 edges need to be removed to make it disconnected from the k9 graph.

So each of the nine party goers must have shaken hands with exactly 8 other party goers rather that 5.

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1. If we are looking for the minimum number of edges a group of vertices that are connected. These vertices must line up and each vertex must have degree of 2 except for vertices on both ends which only have a degree of 1.

To get minimal edges for a graph, each vertex must have least edges as well.

The form of a graph that has minimal edges is a line.

When a graph is a line, vertices at both ends of the line have only degree of 1, and other vertices have a degree of 2. To disconnect the graph and split it into three connected components, only two edges need to be removed to make it.

A graph with 1million vertices consists of 1 million – 1 edges, in this sense, the minimum number of edges must be 1,000,000 – 1 - 2= 999,997 edges to form 3 connected components.

**Problem 2** [20 pts (10,10)**: Graph Representations**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* |
| *A* 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| *B* 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| *C* 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| *D* 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| *E* 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| *F* 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| *G* 0 | 1 | 0 | 0 | 0 | 1 | 0 |

# For the above adjacency matrix representation of an unweighted undirected graph, create an equivalent adjacency list representation and draw the resulting graph.

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# Deg(A) = 2

# Deg(B) = 2

# Deg(C) = 2

# Deg(D) = 3

# Deg(E) = 2

# Deg(F) = 5

# Deg(G) = 2

**Problem 3** [20 pts (10,10)**: Graph Traversal**

# For the 9-vertex graph below, highlight the edges that would be traversed using Depth-First Search (left) and Breadth-First Search (right) starting at the middle vertex labeled *X*. Add a number, 1 to 9 next to each vertex to show the order in which each vertex is visited, starting with a 1 next to *X*. In cases of a tie, the traversal should proceed alphabetically.

# 

# Solution:

# DFS: BFS:

# X -> B -> A -> D -> F ->G ->H ->E -> C X -> B -> D -> G -> E -> A -> C -> F -> H

# 

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