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CS 420
10/29/14

HW 2

1) (10 points) For each of the following triplets of points, find the normal vectors manually to the plane (if it exists) that passes through the triplet. Show your steps.

a. (1, 1, 1), (1, 2, 1), (3, 0, 4)
b. (6, 3, -4), (0, 0, 0), (2, 1, -1)

(You may check your answer using the program you wrote in the lab.)

Answer: a) P2 – P1 = (1, 2, 1) – (1, 1, 1) = (0, 1, 0) A
P3 – P1 = (3, 0, 4) – (1, 1, 1) = (2, -1, 3) B
cross product: n = (0, 1, 0) x (2, -1, 3) = [(1*3) – (0*-1)]i + [(0*2) – (0*3)]j + [(0*-1) – (1*2)]k
= (3 – 0)i + (0 – 0)j + (0 – 2)k
=3i + 0j – 2k
3i + 0j – 2k is the normal vector to the plane passing through the points (1, 1, 1), (1, 2, 1), and (3, 0, 4).

Answer: b) P2 – P1= (0, 0, 0) – (6, 3, -4) = (-6, -3, 4) A
P3 – P1 = (2, 1, -1) – (6, 3, -4) = (-4, -2, 3) B
cross product: n = (-6, -3, 4) x (-4, -2, 3) = [(-3*3) – (4*-2)]i + [(4*-4) - (-6*3)]j + [(-6*-2) - (-3*-4)]k
= (-9 + 8)i + (-16 + 18)j + (12 - 12)k
= -1i + 2j + 0k
-1i + 2j + 0k is the normal vector to the plane passing through the points (6, 3, -4), (0,0,0), and (2, 1, -1)

Find the normalized normal to the plane 5x - 3y + 6z = 7 and determine if the points P1 = (1, 5, 2) and P2 = (-3, -1, 2) are on the same side of the plane.

Answer:
Magnitude of n = √ [(5 * 5)+ (-3 * -3) + (6 * 6)] = √(25 + 9 + 36) = √70
Normalized normal = (5/√70, -3/√70, 6/√70)
f(P1) = 5(1) – 3(5) + 6(2) - 7 = 5 – 15 + 12 - 7 = -5
f(P2) = 5(-3) - 3(-1) + 6(2) - 7 = -15 + 3 + 12 - 7 = -7
Since both points result in the same sign, both points are on the same side of the plane.

2) (10 points) Find the normalized normal at the point (1, 2, 3) for each of the following two cases:
a) A sphere: $x^2 + y^2 + z^2 = 14$
b)A plane: 3x - 4y + 2z - 1 = 0

Answer:
a) Since the sphere is centered at (0, 0, 0), the normal vector n = (1, 2, 3) - (0, 0, 0) = (1, 2, 3)
The magnitude of n: |n| = √[(1 * 1) + (2 * 2) + (3 * 3)] = √(1 + 4 + 9) = √14
The normalized normal at the point (1, 2, 3) is (1/√14, 2/√14, 3/√14)

b) The normal is the same for every point on a plane, so the normal vector n = (3, -4, 2)
The magnitude of n: |n| = √[(3 * 3) + (-4 * -4) + (2 * 2)] = √(9 + 16 + 4) = √29
The normalized normal at the point (1, 2, 3) is (3/√29, -4/√29, 2/√29)

3) (10 points) A robust method to find the normal to any polygon with N vertices is called the Newell's method. It computes the components of the normal **n** according to the following formulas:

$$n_x = \sum_{i=0}^{n-1} (y_i - y_{i+1})(z_i + z_{i+1})$$

$$n_y = \sum_{i=0}^{n-1} (z_i - z_{i+1})(x_i + x_{i+1})$$

$$n_z = \sum_{i=0}^{n-1} (x_i - x_{i+1})(y_i + y_{i+1})$$

a) Apply the Newell's method to a. of Question 1, and see whether you get the same answer.
b) Find the normal to the polygon (1, 1, 2), (2, 0, 5), (5, 1, 4), (6, 0, 7).

Answer:
a) Question 1: (1, 1, 1), (1, 2, 1), (3, 0, 4), so vertex 0(v0) = (1, 1, 1), vertex 1(v1) = (1, 2, 1), vertex 2(v2) = (3, 0, 4).

nx = (1 – 2)(1 + 1) + (2 – 0)(1+ 4) + (0 – 1)(4 + 1) = -2 + 10 - 5 = 3
ny = (1 – 1)(1 + 1) + (1 – 4)(1 + 3) + (4 -1)(3 + 1) = 0 – 12 + 12 = 0
nz = (1 – 1)(1 + 2) + (1 – 3)(2 + 0) + (3 – 1)(0 + 1) = 0 – 4 + 2 = -2
So the normal is 3i + 0j – 2k

b) vertex 0 (v0) = (1, 1, 2); vertex 1 (v1) = (2, 0, 5); vertex 2 (v2) = (5, 1, 4); vertex 3 (v3) = (6, 0, 7)

nx=(1 – 0)(2 + 5) + (0 – 1)(5 + 4) + (1 – 0)(4 + 7) + (0 – 1)(7 + 2) = 7 - 9 + 11 - 9 = 0
ny= (2 – 5)(1 + 2) + (5 – 4)(2 + 5) + (4 – 7)(5 + 6) + (7 – 2)(6 + 1) = -9 + 7 - 33 + 35 = 0
nz=(1 - 2)(1 + 0) + (2 - 5)(0 + 1) + (5 - 6)(1 + 0) + (6 - 1)(0 + 1) = -1 -3 -1 + 5 = 0
So the normal vector is 0i + 0j + 0k

4) (10 points) Let vectors **A** = $(2, -1, 1)^T$ and **B** = $(1, 1, -1)^T$. Find

a. the angle between **A** and **B**,
b. a unit vector perpendicular to both **A** and **B**.

Answer:

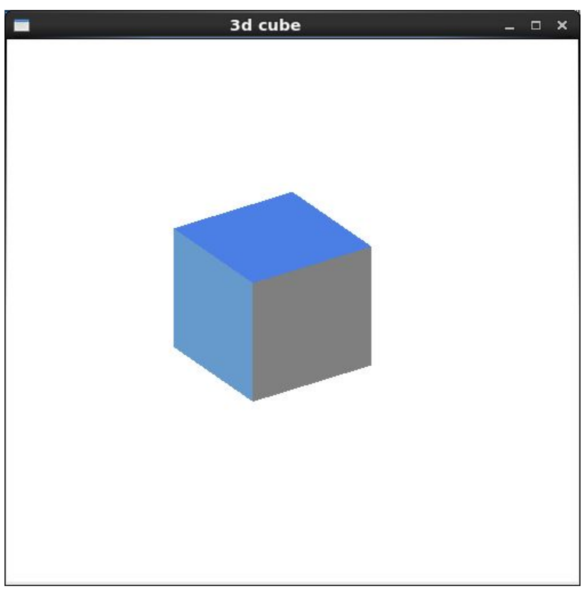
a)|A||B| cos θ = A.B, so we must find the dot product of A and B, the magnitude, and solve for theta of cos

A.B = 2(1) - 1(1) + 1(-1) = 0
|A| = √[(2 * 2) + (-1 * -1) + (1 * 1)] = √6
|B| = √[(1 * 1) + (1 * 1) + (-1 * -1)] = √4
cos θ = 0 / (√6 *√4)
cos θ = 0
θ = cos-1 0
θ = 90 °

b) To find a unit vector perpendicular to A and B, use the cross product to find the normalized normal.

A x B = [(-1 * -1) -(1 * 1)]i + [(1 * 1) -(2 * -1)]j+ [(2 * 1) -(-1 * 1)]k=0i +3j +3k
|A x B| = √ [(0 * 0) + (3 * 3) + (3 * 3)]= √18
So the unit vector perpendicular to A and B is (0, 3/√18, 3/√18)

5) (10 points) Use **glDrawElements()** to draw the following cube with each face having a different color.

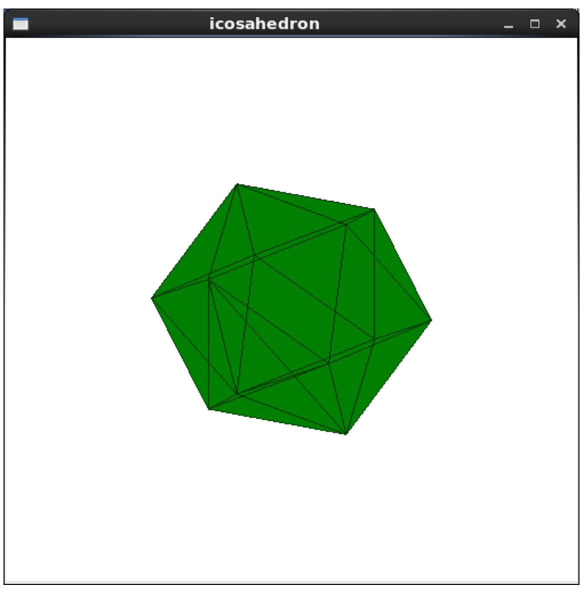


Code for the cube:

```
static GLint vertices[] = {
      0, 0, 0,
      1, 0, 0,
      1, 1, 0,
      0, 1, 0,
      0, 0, 1,
```

```
      1, 0, 1,
      1, 1, 1,
      0, 1, 1 };
   glEnableClientState (GL_VERTEX_ARRAY);
   glVertexPointer (3, GL_INT, 0, vertices);
   static GLubyte frontIndices[]= {4, 5, 6, 7};
   static GLubyte rightIndices[]= {1, 2, 6, 5};
   static GLubyte bottomIndices[] = {0, 1, 5, 4};
   static GLubyte backIndices[]= {0, 3, 2, 1};
   static GLubyte leftIndices[]= {0, 4, 7, 3};
   static GLubyte topIndices[] = {2, 3, 7, 6};
   glEnable(GL_CULL_FACE);
   glCullFace (GL_BACK);
   glColor3f(0.5, 0.5, 0.5);
   glDrawElements(GL_QUADS, 4, GL_UNSIGNED_BYTE, frontIndices);
   glColor3f(0.2, 0.3, 0.6);
   glDrawElements(GL_QUADS, 4, GL_UNSIGNED_BYTE, rightIndices);
   glColor3f(0.6, 0.2, 0.8);
   glDrawElements(GL_QUADS, 4, GL_UNSIGNED_BYTE, bottomIndices);
   glColor3f(0.9, 0.5, 0.2);
   glDrawElements(GL_QUADS, 4, GL_UNSIGNED_BYTE, backIndices);
   glColor3f(0.4, 0.6, 0.8);
   glDrawElements(GL_QUADS, 4, GL_UNSIGNED_BYTE, leftIndices);
   glColor3f(0.3, 0.5, 0.9);
   glDrawElements(GL_QUADS, 4, GL_UNSIGNED_BYTE, topIndices);
```

6) (10 points) Write a program or programs to reproduce one of the following figures of icosahedron and dodecahedron (extra credit for both).



Code for the icosahedron:

```
void icosahedron() {
   #define a .525731112119133606
   #define b .850650808352039932
```

```
    static GLfloat vdata[12][3] = {
        {-a, 0.0, b}, {a, 0.0, b}, {-a, 0.0,-b}, {a, 0.0,-b},
        {0.0, b, a}, {0.0, b,-a}, {0.0,-b, a}, {0.0,-b,-a},
        {b, a, 0.0}, {-b, a, 0.0}, {b,-a, 0.0}, {-b,-a, 0.0}
    };
    static GLuint tindices[20][3] = {
        {0,4,1}, {0,9,4}, {9,5,4}, {4,5,8}, {4,8,1},
        {8,10,1}, {8,3,10}, {5,3,8}, {5,2,3}, {2,7,3},
        {7,10,3}, {7,6,10}, {7,11,6}, {11,0,6}, {0,1,6},
        {6,1,10}, {9,0,11}, {9,11,2}, {9,2,5}, {7,2,11}
    };
    glColor4f(0.0, 0.5, 0.0, 0.5);
    glBegin(GL_TRIANGLES);
    for (int i = 0; i < 20; i++) {
        glVertex3fv(&vdata[tindices[i][0]][0]);
        glVertex3fv(&vdata[tindices[i][1]][0]);
        glVertex3fv(&vdata[tindices[i][2]][0]);
    }
    glEnd();
    glColor4f(0.0, 0.2, 0.0, 0.5);
    for (int i = 0; i < 20; i++) {
        glBegin(GL_LINE_LOOP);
        glVertex3fv(&vdata[tindices[i][0]][0]);
        glVertex3fv(&vdata[tindices[i][1]][0]);
        glVertex3fv(&vdata[tindices[i][2]][0]);
        glVertex3fv(&vdata[tindices[i][0]][0]);
    }
    glEnd();
}
```

Summary:

I have completed all the parts of the homework and did my best to show my work for all the problems. I also learned about using the “gluLookAt” function and how to use it to change the angle of the camera. So since I have completed all the parts of the homework, I am giving myself 60 points.