CSCI 270 Homework #4

Due Date: Wednesday, February 26th, 11:59pm

0. You **may not** use late days on this assignment.

1. Provide a dynamic programming algorithm that, given n, calculates the number of distinct binary search trees you can construct with exactly n nodes, using all the values from 1 to n, and analyze the runtime. If $n = 2$, then you should return 2, since you can either have 2 as the root with 1 as its left child, or 1 as the root with 2 as its right child. If $n = 3$, then you should return 5.

2. Granger wants to create a magic potion. In order to make the magic potion, she needs to mix powder A and powder B in the ratio of $M_a : M_b$. The pharmacy sells n bottles, where bottle i costs c_i, contains a_i grams of powder A, and b_i grams of powder B. Provided that Granger has to use all the powder she bought, design a dynamic programming algorithm to determine the minimum budget needed to construct the potion, or assert that no such formulation is possible, and analyze the runtime. You may assume that M_a, M_b, a_i and b_i are all integers.

3. This summer, you plan to take a road trip from Los Angeles to Boston. Your car can go p miles when the tank is full, and you have a map that indicates the distances between gas stations along the route. You are given $\mathrm{d}_1 < \mathrm{d}_2 < \ldots < \mathrm{d}_\mathrm{n}$, where d_i is the distance from Los Angeles to the ith gas station. You may assume that the distance between neighboring gas stations is at most p miles.

   You want to drop by as few gas stops as possible. Give a greedy algorithm to determine which gas stations you should stop at, and prove that your strategy yields an optimal solution.

4. You are given an infinite supply of coins of type A, B, and C with values 10, 5 and 1 respectively. Given a target sum s, you are asked to find the minimum number of coins required to generate the total value s. Provide a greedy algorithm to solve the problem, and prove its correctness.