CSCI 270 Homework #5

Due Date: Wednesday, March 12th, 11:59pm

1. You are given a set $S = \{a_1, a_2, \ldots, a_n\}$ of tasks, where task a_i requires p_i units of processing time to complete, once it has started. You have one computer on which to run these tasks, and the computer can run only one task at a time. Let c_i be the completion time of task a_i, that is, the time at which task a_i completes processing. Your goal is to minimize the average completion time, that is, to minimize $(1/n)\sum_{i=1}^{n} c_i$. For example, suppose there are two tasks, a_1 and a_2, with $p_1 = 3$ and $p_2 = 5$, and consider the schedule in which a_2 runs first, followed by a_1. Then $c_2 = 5, c_1 = 8$, and the average completion time is $(5 + 8)/2 = 6.5$. If task a_1 runs first, however, then $c_1 = 3, c_2 = 8$, and the average completion time is $(3 + 8)/2 = 5.5$

   Give a greedy algorithm that schedules the tasks so as to minimize the average completion time. Each task must run non-preemptively, that is, once task a_i starts, it must run continuously for p_i units of time. Prove that your algorithm minimizes the average completion time.

   One of the two following problems can be solved using a greedy algorithm, the other cannot. Recall that greedy algorithms are faster than dynamic programming algorithms, so you should always try to solve with a greedy algorithm when possible.

2. You have homework due in n classes. It will take you t_i hours to complete your homework for class i, which will earn you g_i points (otherwise you earn no points for that homework). All of your homework is due in T hours. Give an efficient algorithm (either Dynamic Programming or Greedy) to maximize the number of points you earn, and analyze the runtime. If you solve this problem using Greedy, then give an exchange argument to prove the correctness of your algorithm.

3. Alice is participating in a game where she shoots balloons and minimizes her total penalty. The balloons are numbered from 1 to n, and the balloon $i(1 \leq i \leq n)$ is at an altitude of H_i at the start of the game with the altitude increasing by S_i every second. She can break one balloon at the beginning of the competition, and can break one balloon every second thereafter. The penalty associated with a balloon is an integer value equal to the altitude at which the balloon was broken. Her total penalty is the maximum of the penalties of the n balloons she shot. Alice has a personal-best total penalty of $X + 1$. Design an efficient algorithm (either Dynamic Programming or Greedy) that will net Alice a total penalty of X or less, or assert that no such solution exists. If you solve this problem using Greedy, then give an exchange argument to prove the correctness of your algorithm.