CSCI 270 Homework #9

Due Date: Friday, May 2nd, 11:59pm

0. Please fill out the online evaluations for this course. Every time a student doesn't fill out course evaluations, a Bruin kicks a puppy.

1. In the Traveling Hulk (TH) problem, you are given a weighted undirected graph G (self loops and multi-edges are allowed, and all weights are positive), a vertex v, and a positive integer l. Traversing an edge of cost x makes Bruce Banner angrier by x units. You must determine if there exists a route that starts at vertex v and ends at v, such that Bruce will have an anger level of exactly l (the total weight of all edges traveled) without repeating any edges. The route can be non-simple (you are allowed revisit nodes). Prove that TH is NP-Complete.

2. In the Frequency Allocation (FA) problem, Tony Stark has n wireless transmitters. Each transmitter T_i has a list of frequencies $L_i = \{f_1, f_2, ..., f_x\}$ at which it can send signals. Different transmitters could have different frequency lists. It is okay for two transmitters to use the same frequency, as long as they are not too close together. For each transmitter i, there is a set of transmitters $S_i \subset \{1, ...n\}$ that are too close together. You need to determine if you can choose one frequency for each of Tony's transmitters, such that there is no interference at all. Prove that FA is NP-Complete.

3. In the Star Lord (SL) problem, you are given an undirected graph G of n planets (nodes), a starting planet s, and an ending planet t. Each edge (u, v) has a positive weight $w_{(u,v)}$ which is the amount of Units (money) you can obtain when you travel from planet u to planet v. You only get the units the first time you traverse the edge. Edges can start and end at the same planet (self-loops are allowed). Each planet u has a positive integer value c_u which is the amount of Units you need to pay to visit this planet (docking fees, taxes, fuel, customs, etc). Every time you revisit a planet (either from another planet or taking a self-loop), you need to pay c_u again. You do not need to pay for s unless you revisit it.

   An expedition (cycle) is said to have profit p if p is the total amount of units collected on the edges traversed, minus the total cost of visiting planets along the way. You need to determine if you can plan your expedition in such a way that the profit is at least k. Prove that SL is NP-Complete.

4. In the Triangle-Free Vertex Cover (TFVC) problem, the Avengers are given an undirected graph G with no triangles. That is, for any three vertices, u, v and w, at least one of (u, v), (u, w) and (v, w) does not exist. Each of the k Avengers need to position themselves at a node in the graph so that they form a Vertex Cover, thereby cutting off Thanos' escape route. Prove that TFVC is NP-Complete.