CSCI 270 Lecture 25: Packing Problems

Independent Set

We know IS $\in$ NP. Now we want to show that 3-SAT $\leq_p$ IS. Turn an arbitrary 3-SAT instance into an IS problem.

$(\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \neg x_3)$

1. Add 3 nodes in a triangle for each clause.
2. What's missing in our reduction?
3. How many problems have we shown are NP-Complete?

Remember, this example is only an illustration to clarify my proof. My proof must work on all 3-SAT instances, not just this one!

There are a lot of NP-complete problems. This is why we highly suspect that P $\neq$ NP. All we would have to do to prove P=NP is come up with a polynomial time algorithm for one of these problems. With how much effort has been expended, we probably would have done so by now if it were possible. Proving that no polynomial algorithm exists for a problem is much much harder.

Set Packing

Given n elements $U = \{u_1, u_2, ..., u_n\}$, m subsets $S_1, S_2, ..., S_m \subseteq U$, and an integer k. Are there k sets which don't intersect?

Think about the similarities between the problems. In Independent Set we are packing as many nodes as we can such that no edge is represented twice. In Set Packing we are packing as many subsets as we can such that no element is represented twice.

k-Clique

Given a graph and an integer k, is there a set S of $\geq k$ nodes such that every pair of nodes in S have an edge between them?

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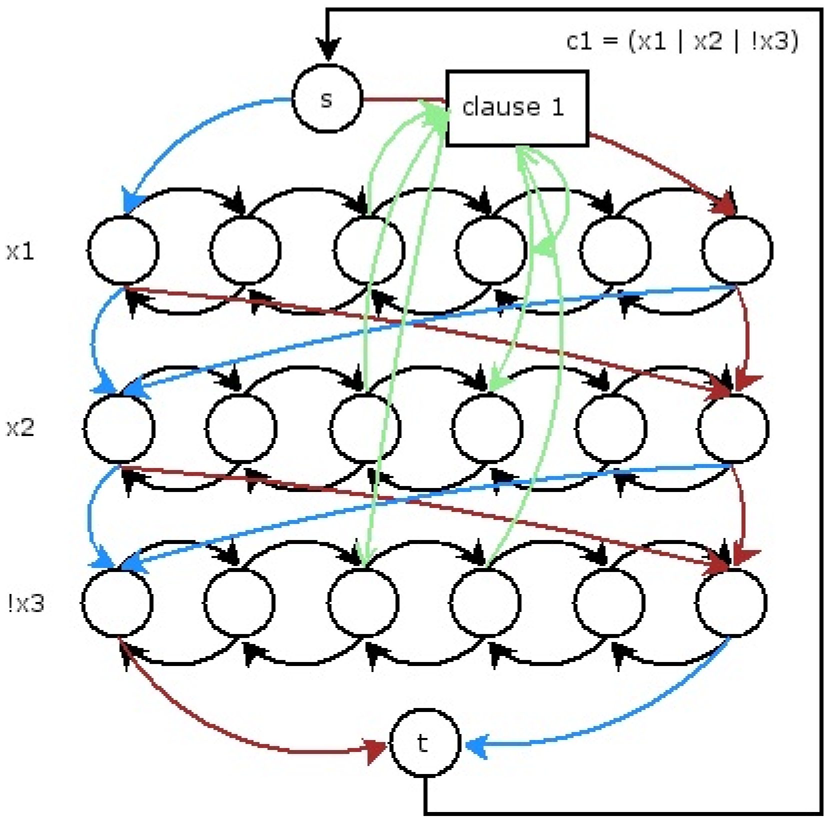
Directed Hamiltonian Cycle

Given a directed graph $G = (V, E)$, is there a simple directed cycle C that visits every node?



Undirected Hamiltonian Cycle

Given an undirected graph $G = (V, E)$, is there a simple cycle C that visits every node?