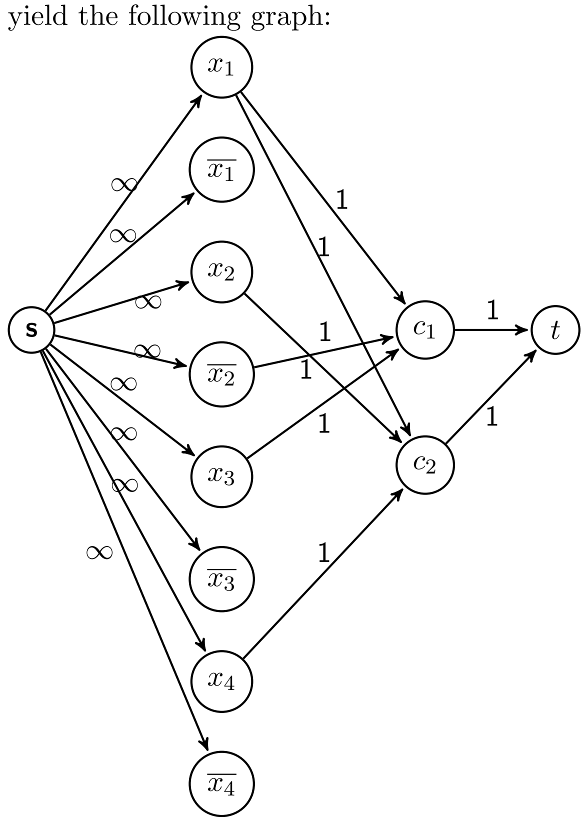
CSCI 270 Homework #8

Due Date: Wednesday, April 23rd, 11:59pm

1. In the k-Flow problem, you are given a graph $G = (V, E)$, an integer k, a source $s \in V$, and a sink $t \in V$. You are wondering if there is a flow from s to t of value $\geq k$.
   (a) Show that k-Flow is in NP.
   (b) Barry Bruin claims to have a proof that k-Flow is NP-Complete. If correct, this would mean that Network Flow is NP-Complete. What would be the implications if Barry's proof is correct?
   (c) Barry Bruin shows you his proof, and it is a (correct) reduction from k-Flow to 3-SAT. What are the implications of this proof?
   (d) Barry Bruin modifies his reduction in the following manner: Given a 3-SAT instance, create the following k-Flow instance: For each literal x_i, create two nodes labeled x_i and $\overline{x_i}$. For each clause c_j, create a node c_j. Create two more nodes s and t. Connect s to every x_i and $\overline{x_i}$ with an edge of infinite capacity. Add an edge of capacity 1 from every c_j to t. For each clause c_j, add an edge of capacity 1 from each literal in c_j to the node c_j. The 3-SAT instance is satisfiable iff there is a flow from s to t equal to the number of clauses.

   Is this a correct proof that k-Flow is NP-Complete? Explain your answer.

   For example, a two clause instance $c_1 = (x_1 \vee \overline{x_2} \vee x_3)$ and $c_2 = (x_1 \vee x_2 \vee x_4)$ would yield the following graph:



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2. Answer the following true/false questions, and explain your answers.

   (a) If A is **NP**-Complete, and $A \leq_p B$, then B is **NP**-Complete

   (b) The following problem is in **NP**: given an n-bit positive integer, determine if it is composite.

   (c) If someone proves $\mathbf{P} = \mathbf{NP}$, then every problem in $\mathbf{P}$ is **NP**-Complete.

   (d) Determining whether a Network Flow graph has a flow of value k is in **NP**.

3. Prove that Subset Sum can be solved in polynomial time if the target W is expressed in unary.

4. The subgraph-isomorphism problem takes two undirected graphs G_1 and G_2, and asks whether G_1 is isomorphic to a subgraph of G_2. Show that the subgraph-isomorphism problem is NP-complete.