Problem #1: *the input-output measurements of a SISO dynamic system S_1 to be modeled for* *<u>simulation</u>* *through a transfer function $G(z)$ of* <u>*degree* $n \leq 10$</u> *have been collected in the* MATLAB **`data1a.mat`** *file.*

1.1) *Identify ARX, ARMAX and OE models of different orders and delays (assume $n_a=n_b=n_c=n_f \leq 10$, $n_k \leq 3$), using the first* 5000 *samples of the experimental data and looking for models guaranteeing satisfactory characteristics of whiteness of the residuals associated to this first dataset.*

1.1.a) Candidate models with satisfactory residual characteristics (specify class, orders, delay, residual characteristics, test threshold):

Candidate models with satisfactory residual characteristics (specify class, orders, delay, residual characteristics, test threshold):

Threshold = 4; Max_Lag = 30;

ARX

nk = 1, na >= 4
nk = 2, na >= 4
nk = 3, na >= 4

ARMAX

nk = 1, na >= 2
nk = 2, na >= 2
nk = 3, na >= 2

OE

nk = 1, nf >= 4
nk = 2, nf = 4,5,6,7,9,10
nk = 3, nf >= 3

1.1.b) Clear report including the reasoning behind the computations and the possible critical analysis of the main numerical results:

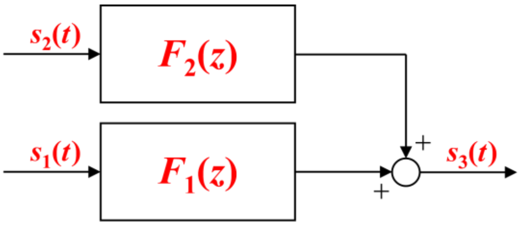
Since all the models show very good behaviour, I decided to use a reasonably small threshold equal to 4. In addition to that I consider as "out of the interval" the points that were mostly out of the boundaries, instead I consider as "in the interval" the points that were tangent the boundaries.

1.2) *Compare only the models that guarantee satisfactory residual characteristics in the previous step, by considering the following two criteria, both applied to the remaining dataset not used for identification and assuming* N_0*=15: RMSE and AIC.*

1.2.a) Candidate models with satisfactory residual characteristics (specify class, orders, delay, values of *RMSE* and *AIC*):

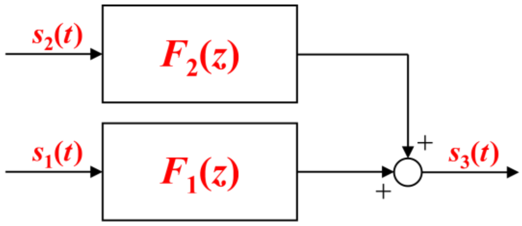
```
Candidate models with satisfactory residual characteristics
(specify class, orders, delay, values of RMSE and AIC):

ARX(9,9,1) -> AIC = -3.0719 ; RMSE = 0.2146; n = 9

ARMAX(3,3,3,1) -> AIC = -4.5967; RMSE = 0.1003; n = 3

OE(4,4,1) -> AIC = -4.5959; RMSE = 0.1003; n = 4
```

1.2.b) Best trade-off between *RMSE* and transfer function degree *n*, with: $n = n_a$ (ARX and ARMAX models) or $n = n_f$ (OE models). Report the corresponding model in simulation mode, specifying the transfer functions $F_1(z)$ and $F_2(z)$ and the effective signals $s_1(t)$, $s_2(t)$ and $s_3(t)$ of the following block diagram representation:




```
Best trade-off between RMSE and transfer function degree n
(write the corresponding model in simulation mode,
 specifying the block diagram transfer functions and signals):

best model = ARMAX(3,3,3,1)

s3(t) = y(t)
s2(t) = u(t)
s1(t) = e(t)


F2(z) = B(z)/A(z) =   0.1029 z^-1 - 0.09155 z^-2 - 0.006172 z^-3

                      ------------------------------------------

                      1 - 2.727 z^-1 + 2.487 z^-2 - 0.7589 z^-3


F1(z) = C(z)/A(z) =  1 - 2.727 z^-1 + 2.4874 z^-2 - 0.7584 z^-3

                      ------------------------------------------

                     1 - 2.727 z^-1 + 2.4869 z^-2 - 0.7589 z^-3
```

1.2.c) Best trade-off between *AIC* and transfer function degree n, with: $n = n_a$ (ARX and ARMAX models) or $n = n_f$ (OE models). Report the corresponding model in simulation mode, specifying the transfer functions $F_1(z)$ and $F_2(z)$ and the effective signals $s_1(t)$, $s_2(t)$ and $s_3(t)$ of the following block diagram representation:




```
Best trade-off between AIC and transfer function degree n
(write the corresponding model in simulation mode,
 specifying the block diagram transfer functions and signals):


best model = ARMAX(3,3,3,1)

s3(t) = y(t)
s2(t) = u(t)
s1(t) = e(t)


F2(z) = B(z)/A(z) =   0.1029 z^-1 - 0.09155 z^-2 - 0.006172 z^-3

                      ------------------------------------------

                      1 - 2.727 z^-1 + 2.487 z^-2 - 0.7589 z^-3


F1(z) = C(z)/A(z) =  1 - 2.727 z^-1 + 2.4874 z^-2 - 0.7584 z^-3

                      -----------------------------------------

                     1 - 2.727 z^-1 + 2.4869 z^-2 - 0.7589 z^-3
```

1.2.d) Clear report including the reasoning behind the computations and the possible critical analysis of the main numerical results:

```
In order to choose the best model, among the ones that passed the
whiteness test, I relay on their complexity and values of AIC and RMSE.
Therefor I immediately discard the ARX model, since it was the worst in
terms of complexity and values of RMSE and AIC, so I needed to pick
between the ARMAX and the OE, they both have very similar values of RMSE
and AIC, but the complexity of the ARMAX was lower, so I choose as best
model the ARMAX(3,3,3,1).
```

1.3.a) *Using all the experimental data, estimate the parameters of an ARX*(2,3,2) *model by means of the Recursive Least-Squares algorithm named* *RLS–3**, showing by plots their asymptotic dynamic convergence to the Standard Least-Squares estimates as the number of considered data increases.*

Write the numerical values of the RLS–3 estimates (final value) and the standard LS estimates:

Numerical value of the RLS-3 estimates (final value):

```
theta_RLS3_f =

   -0.8383

   -0.0380

    0.1989

    0.1216

    0.1934
```

Numerical value of the standard LS estimates:

```
theta_LS =

   -0.8384

   -0.0380

    0.1989

    0.1216

    0.1934
```

1.3.b) *Compute the Estimate Uncertainty Intervals* EUI^2 *and, if possible, the Parameter Uncertainty Intervals* PUI^2*, assuming that the output measurements are corrupted by an energy-bounded noise whose* 2*-norm is less than* 1.

Write the numerical values of the EUI^2 and, if possible, the PUI^2:

Numerical value of the EUI^2**:**

```
EUI2 =

   -0.9079   -0.7688

   -0.1007    0.0247

    0.1184    0.2794
```

```
    0.0144    0.2287

    0.1049    0.2818
```

Numerical value of the PUI²:

PUI2 cannot be computed

1.3.c) *Compute the Estimate Uncertainty Intervals EUI*$^\infty$*, assuming that the output measurements are corrupted by an amplitude-bounded noise whose* ∞*-norm is less than* 0.05.

Write the numerical value of the *EUI*$^\infty$:

Numerical value of the EUI$^\infty$**:**

```
EUIinf =

   -1.0758   -0.6010

   -0.2510    0.1750

    0.0411    0.3566

   -0.1275    0.3706

   -0.0046    0.3914
```

1.3.d) Clear report including the reasoning behind the computations and the possible critical analysis of the main numerical results (in particular, write the difference equation corresponding to the identified *ARX*(2,3,2) model):

Difference equation :
y(t) = -a1*y(t-1)-a2*y(t-2)+b1*u(t-2)+b2*u(t-3)+b3*u(t-4)+e(t)

Regressor vector: phi(t) = [-y(t-1) -y(t-2) u(t-2) u(t-3) u(t-4)]'

As we can see from the results, we obtain a very good convergence between the Theta_LS and the recursive Theta_RLS3. In order to obtain this convergence, I set alpha = 100, knowing that as we increase the value of alpha, we increase the convergence but we also increase the overshoot, so we have to pick a good tradeoff between the two parameters.
For what is concerning the evaluation of PUI2, since we have that alpha^2 is greater than eps^2, we obtain that FPS2 is empty, so we cannot compute PUI2.
Finally, as expected EUIinf is larger than EUI2.