Problem 3 — Given

$$G_p(s) = \frac{100}{s^2 + 5.5s + 4.5}$$

$G_s = 1$
$G_a = 0.014$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ | D_{a0} | \leq 1.5 \cdot 10^{-3};$
$d_p(t) = a_p \sin(\omega_p t), \quad | a_p | \leq 16 \cdot 10^{-2}, \ \ \omega_p \leq 0.03$ rad s^{-1}.
$d_s(t) = a_s \sin(\omega_s t), \quad | a_s | \leq 2 \cdot 10^{-1}, \ \ \omega_s \geq 60$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 1$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $| e_r^\infty | \leq 1.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $| e_{d_a}^\infty | \leq 4.5 \cdot 10^{-3}$
(S4) Steady-state output error in the presence of d_p: $| e_{d_p}^\infty | \leq 2 \cdot 10^{-3}$.
(S5) Steady-state output error in the presence of d_s: $| e_{d_s}^\infty | \leq 8 \cdot 10^{-4}$.
(S6) Rise time: $t_r \leq 2$ s
(S7) Settling time: $t_{s,\,5\%} \leq 8$ s
(S8) Step response overshoot: $\hat{s} \leq 12\%$

Requirement 1

$$G_p(s) = \frac{100}{s^2 + 5.5s + 4.5}$$

$$\Downarrow$$

$$p = 0 \Rightarrow p + \nu \geq 1 \Rightarrow \nu = 1$$

$$K_d = \frac{1}{G_f G_s} \Rightarrow G_f = 1$$

Requirement 2

$$\frac{K_d^2 R_0}{K_p K_c G_a} = 1.5 \cdot 10^{-1}$$

$$\Uparrow$$

Input order / System type	Step input (order 0)	Ramp input (order 1)	Parabola input (order 2)
0	$\frac{K_d^2 R_0}{K_d + K_p K_c G_a}$	∞	∞
1	0	$\frac{K_d^2 R_0}{K_p K_c G_a}$	∞
2	0	0	$\frac{K_d^2 R_0}{K_p K_c G_a}$

$\nu + p = 1$

$\nu = 1$

$$K_p \lim_{s \to 0} s^p \, L_p = 22{,}22$$

$$K_c \geq \frac{K_d^2 R_0}{K_p \cdot G_a \cdot 1{,}5 \cdot 10^{-1}} = 21{,}43$$

Requirement 3

$$|e_{da}^{\infty}| \triangleq |Y_{da}^{\infty}| = \lim_{t \to +\infty} Y_{da}(t) = \lim_{s \to 0} s \, G_{da}(s) = \lim_{s \to 0} \cancel{s} \frac{G_p}{1+L} \frac{D_a}{\cancel{s}}$$

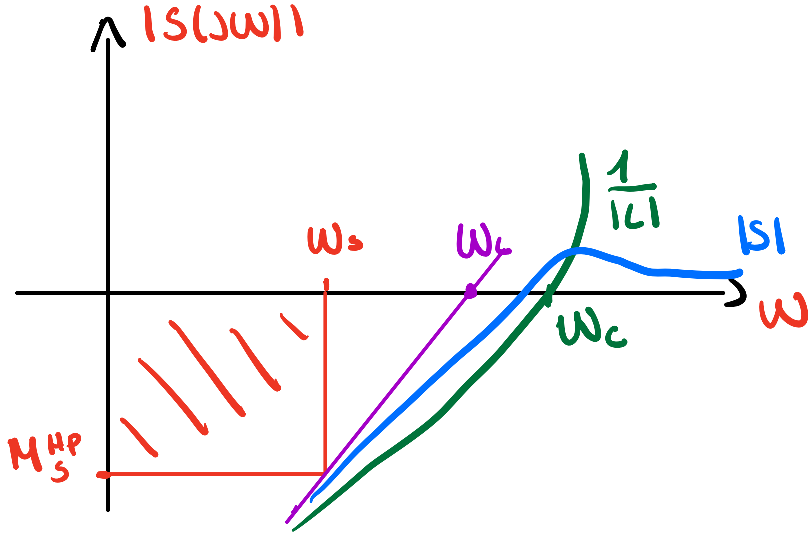
$$= \lim_{s \to 0} \frac{\frac{K_p}{s^p} D_a}{1 + \frac{K_p}{s^p} \frac{K_c}{s^\nu} G_a G_f G_s} = 0 \qquad \Rightarrow \nu = 1 \ \forall K_c$$

Requirement 4

$$|e_{dp}^{\infty}| \leq 2{,}10^{-3} \Rightarrow |a_p G_{dp}(j\omega_p) \sin(\omega_p t)| \leq |a_p G_{dp}(j\omega_p)|$$

$$\Rightarrow \left| a_p \frac{1}{1+L} \right| \leq 2 \cdot 10^{-3} \Rightarrow S = \frac{2 \cdot 10^{-3}}{a_p} = 0{,}0125$$

$$\Rightarrow M_S^{HF} = -38 \text{dB}$$



$$\omega_L = \omega_p \, 10^{-\frac{M_S^{HP}}{40}} = 0{,}2683$$

$$\omega_c \geq 2 \cdot \omega_L \Rightarrow \omega_c \geq 0{,}5367$$

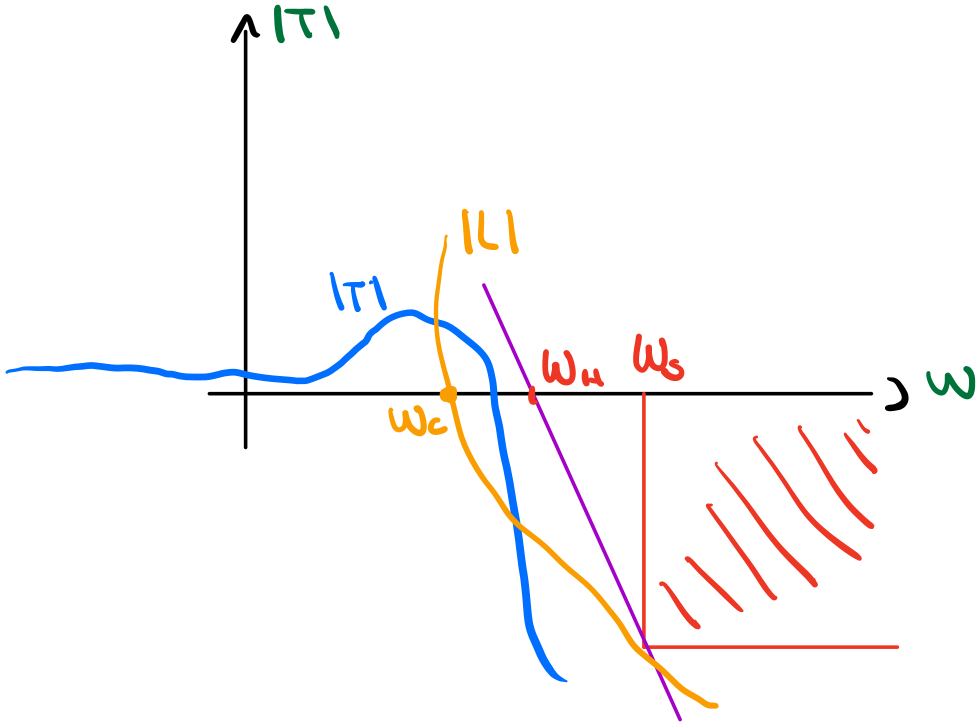
Requirement 5

$$|d_s^\infty| \leq 8 \cdot 10^{-4} \Rightarrow |a_s\, G_{ds}(\omega_s t) \sin(\omega_s t)| \leq |a_s\, G_{ds}(\omega_s t)|$$

$$\Rightarrow \left| a_s\, |T(j\omega_s)| \frac{1}{G_s} \right| \leq 8 \cdot 10^{-4}$$

$$|T(j\omega_s)| \leq \frac{8 \cdot 10^{-4} \cdot G_s}{a_s} = 0{,}0040$$

$$\Rightarrow M_T^{SP} = -48\,dB$$



$$W_H = W_s \cdot 10^{\frac{M_T^{SP}}{40}} = 3{,}7947 \text{ Rad/s} \Rightarrow W_c \le \frac{W_H}{2} = 1{,}89735$$

Requirement 6

$$zeta = \frac{|\ln(1-Rot)|}{\sqrt{\pi^2 + (\ln(1-Rot))^2}} = 0{,}5594$$

$$W_c \ge \frac{1}{t_r\sqrt{1-zeta^2}}\left(\pi - \arccos(zeta)\right) \cdot \sqrt{\sqrt{1+4\,zeta^4} - 2\,zeta^2}$$

Requirement 7

Summary Requirement

1. $\nu = 1$

   $G_f = 1$

2. $\nu = 1$

   $K_c \geq 21{,}43$

3. $\nu = 1$

   $\forall K_c$

4. @ $\omega_p = 0{,}03$ rad/s $\Rightarrow |S| \leq -38\,dB$

   $\omega_c \geq 0{,}5367$

5. @ $\omega_s = 60$ rad/s $\Rightarrow |T| \leq -48\,dB$

   $\omega_c \leq 1{,}8973$

6. $\omega_c \geq 0{,}9717$

7. $\omega_c \geq 0{,}498$

8. $T_p = 1{,}078$

   $S_p = 1{,}3939$

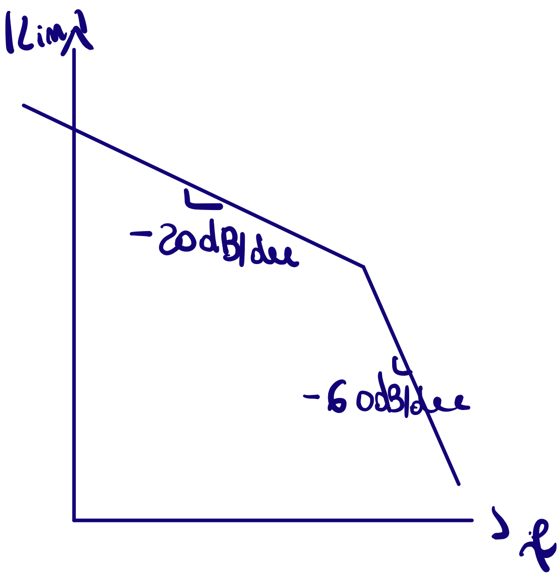
Now we have to chooce the right sign of K_c, let's assume that $K_c > 0$

$K_c = 21{,}43$

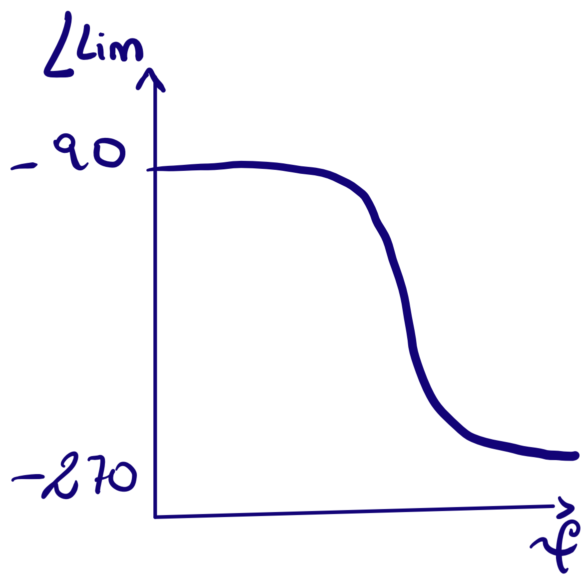
$$L_{im} = \frac{30}{s^3 + 5{,}5s + 4{,}5s} \Rightarrow pol: \emptyset$$

Bode

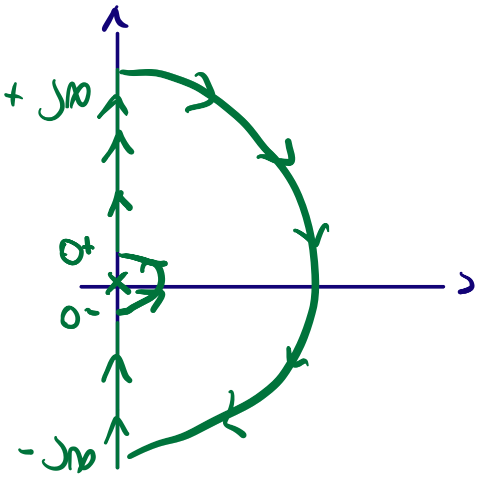
MAGNITUDE



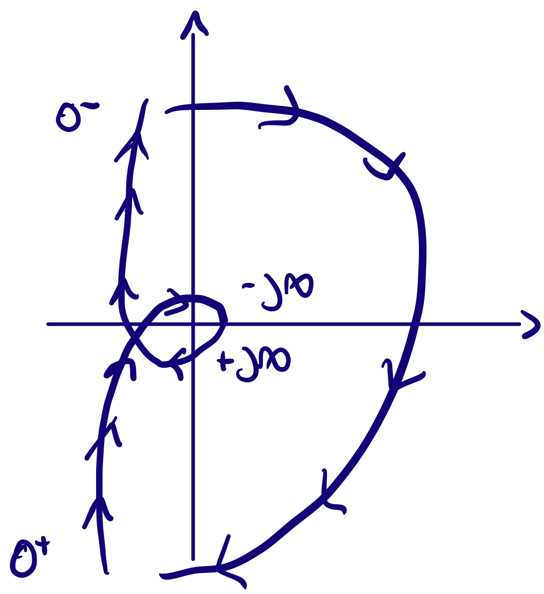
PHASE



Nyquist contour



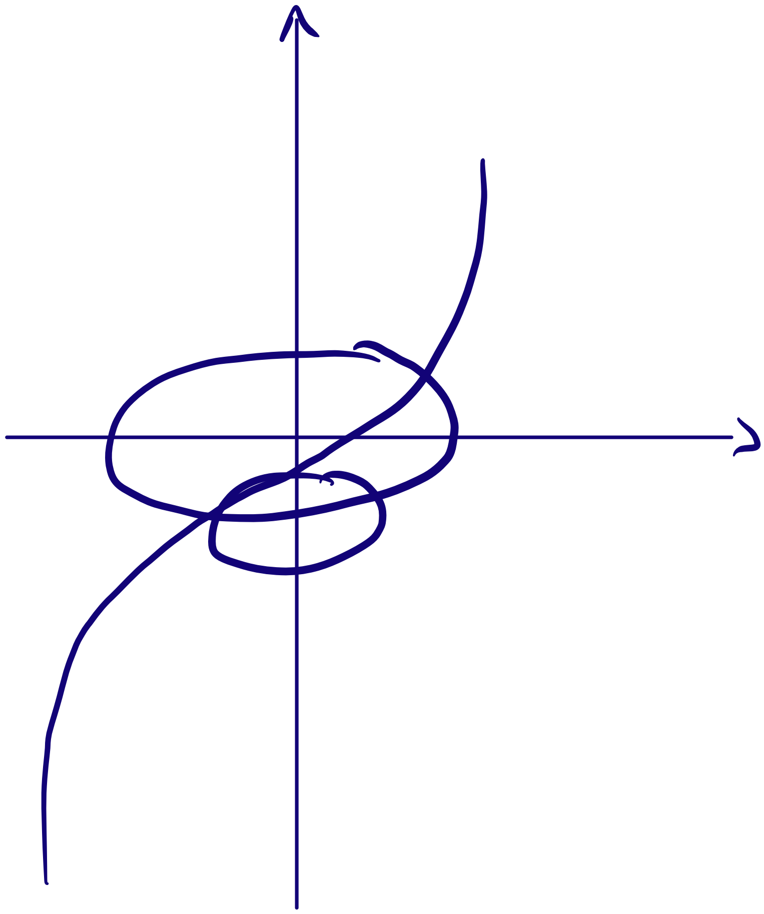
Nyquist diagram



So $N = 0$

$P_{cl} = N + P_{ol} = 0 \Rightarrow K_c > 0$

Now we plot the nychols diagram and we choose ω_c dis



$\omega_c \in [0{,}9717 ; 1{,}8973]$

$\omega_c = 1{,}60 \Rightarrow$ $|L| = 6{,}4\ dB \rightarrow 0\ dB$

$\angle L = -167\ deg \rightarrow -107\ deg$

$\Downarrow$

We need to gain a phase of 60°

So we use one zero and one lead network

zero => + 30 deg

$\Downarrow$

$|L| = 7,52$

$\angle L = -139$

lead => + 30 deg

$|L| = 8,93$ dB

$\angle L = -110$ deg

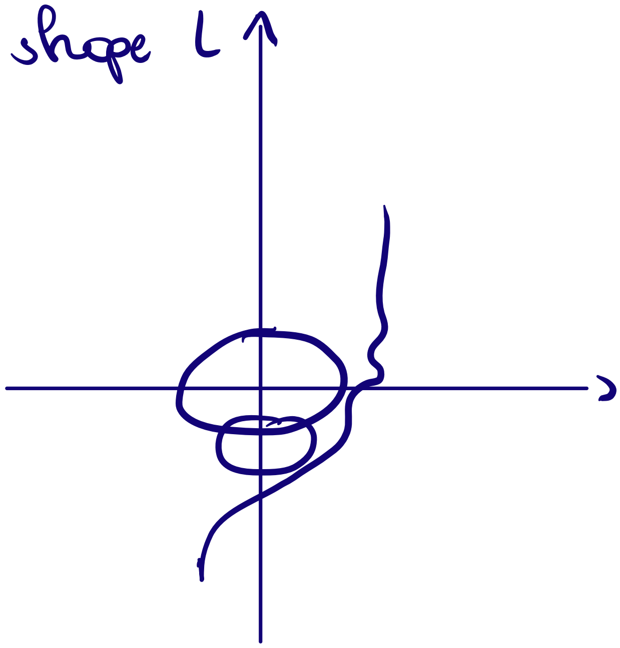
Now we decreas the magnitude

lag => − 8,93 dB

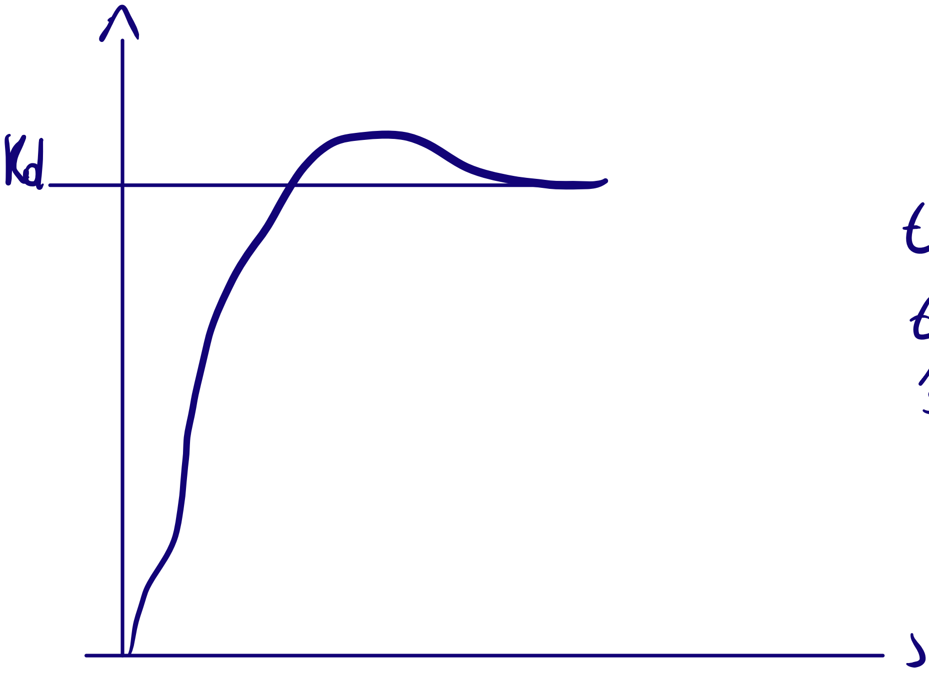
$|L| = 0$ dB

$\angle L = -110$ deg

Find shape L



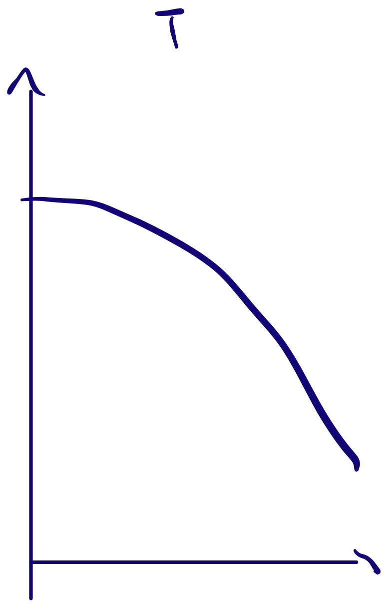
Step response



t_r ✓

t_s ✓

$\hat{s}$ ✓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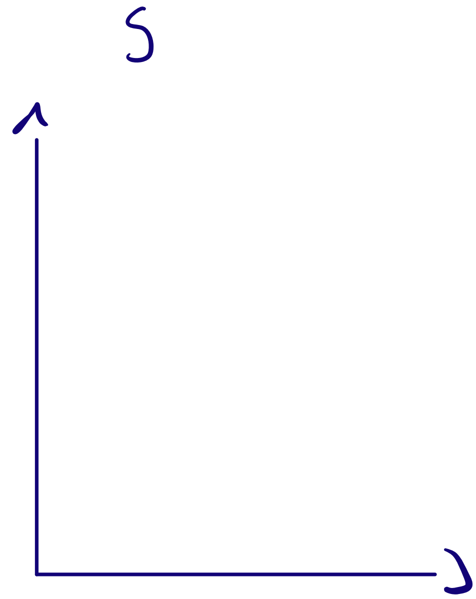
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