

Sheet1

<b>ASCENDING</b>	Insertion	Bubble	Selection
N=50000			
Trial 1	0.000893	0.000216	2.81378
Trial 2	0.000808	0.000386	2.55934
Trial 3	0.000807	0.000562	2.63204
Average	0.000836	0.000388	2.668387
N=100000			
Trial 1	0.001696	0.000701	10.2581
Trial 2	0.00161	0.001005	10.1433
Trial 3	0.001619	0.00112	10.2172
Average	0.001642	0.000942	10.2062
N=200000			
Trial 1	0.001964	0.00194	40.9844
Trial 2	0.002084	0.002289	40.9052
Trial 3	0.002796	0.001996	40.6289
Average	0.002281	0.002075	40.8395
N=300000			
Trial 1	0.003521	0.002193	91.3997
Trial 2	0.00432	0.003277	91.7247
Trial 3	0.00286	0.001765	91.5082
Average	0.003567	0.002412	91.5442
N=400000			
Trial 1	0.005322	0.003928	163.275
Trial 2	0.008047	0.003019	162.81
Trial 3	0.003592	0.002568	162.738
Average	0.005654	0.003172	162.941

<b>DESCENDING</b>	Insertion	Bubble	Selection
N=50000			
Trial 1	3.63327	12.5636	2.73254
Trial 2	3.59967	12.6725	2.69035
Trial 3	3.57632	12.5421	2.71377
Average	3.603087	12.59273	2.71222
N=75000			
Trial 1	8.12066	28.2384	6.05225
Trial 2	8.05223	28.1151	6.01711
Trial 3	8.20243	28.1068	6.12157
Average	8.125107	28.15343	6.063643
N=100000			
Trial 1	14.4003	50.0188	10.7616
Trial 2	14.3893	55.7995	10.7599
Trial 3	14.449	50.0044	10.6929
Average	14.41287	51.9409	10.73813
N=125000			
Trial 1	22.3221	82.46	16.7379
Trial 2	22.3549	78.3356	16.8473
Trial 3	22.4042	78.1765	16.7597
Average	22.3604	79.65737	16.78163
N=150000			
Trial 1	32.6394	112.964	24.1541
Trial 2	32.3632	113.13	24.2058
Trial 3	32.2676	113.423	24.1297

	Quick	Merge
N=500000		
Trial 1	0.032088	0.081636
Trial 2	0.044354	0.086208
Trial 3	0.045204	0.093128
Average	0.040549	0.086991
N=1000000		
Trial 1	0.098173	0.124276
Trial 2	0.071426	0.155342
Trial 3	0.094972	0.134638
Average	0.08819	0.138085
N=2000000		
Trial 1	0.134492	0.205823
Trial 2	0.151569	0.205635
Trial 3	0.141811	0.206165
Average	0.142624	0.205874
N=4000000		
Trial 1	0.177861	0.468442
Trial 2	0.221913	0.443342
Trial 3	0.209582	0.428878
Average	0.203119	0.446887
N=8000000		
Trial 1	0.375416	0.893485
Trial 2	0.387257	0.891984
Trial 3	0.425358	0.894704
Average	0.39601	0.893391

	Quick	Merge
N=500000		
Trial 1	0.102932	0.099919
Trial 2	0.043863	0.080435
Trial 3	0.072941	0.086499
Average	0.073245	0.088951
N=1000000		
Trial 1	0.14953	0.162726
Trial 2	0.129775	0.174267
Trial 3	0.109296	0.09885
Average	0.129534	0.145281
N=2000000		
Trial 1	0.211382	0.242709
Trial 2	0.191602	0.205352
Trial 3	0.20075	0.206515
Average	0.201245	0.218192
N=4000000		
Trial 1	0.363897	0.4309
Trial 2	0.399091	0.446067
Trial 3	0.375588	0.427509
Average	0.379525	0.434825
N=8000000		
Trial 1	0.771117	0.917299
Trial 2	0.755377	0.889698
Trial 3	0.767636	0.892223

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Average 32.4234 113.1723 24.1632

**RANDOM**

N=50000

	Insertion	Bubble	Selection
Trial 1	1.84076	12.0542	2.85693
Trial 2	1.84207	12.1433	2.82463
Trial 3	1.84156	11.7733	2.83329

Average 1.841463 11.99027 2.838283

N=75000

Trial 1	4.0471	26.5201	6.33825
Trial 2	4.10224	26.5403	6.33871
Trial 3	4.10646	26.5469	6.33456

Average 4.085267 26.53577 6.337173

N=100000

Trial 1	7.2261	47.2062	11.3481
Trial 2	7.29068	46.9308	11.2488
Trial 3	7.37978	47.1627	11.3049

Average 7.298853 47.0999 11.3006

N=125000

Trial 1	11.7165	74.2431	17.6128
Trial 2	11.4327	73.8016	17.628
Trial 3	11.5872	73.5611	17.7921

Average 11.5788 73.8686 17.67763

N=150000

Trial 1	16.5181	106.097	25.5351
Trial 2	16.4305	105.706	25.39
Trial 3	16.7354	106.954	25.4319

Average 16.56133 106.2523 25.45233

Average 0.76471 0.89974

Quick Merge

N=500000

Trial 1	0.153533	0.144595
Trial 2	0.092047	0.129192
Trial 3	0.147916	0.138377

Average 0.131165 0.137388

N=1000000

Trial 1	0.212111	0.261591
Trial 2	0.221816	0.203607
Trial 3	0.169798	0.21392

Average 0.201242 0.226373

N=2000000

Trial 1	0.348157	0.420192
Trial 2	0.336985	0.434883
Trial 3	0.334415	0.418753

Average 0.339852 0.424609

N=4000000

Trial 1	0.70215	0.847799
Trial 2	0.705368	0.84962
Trial 3	0.701429	0.846143

Average 0.702982 0.847854

N=8000000

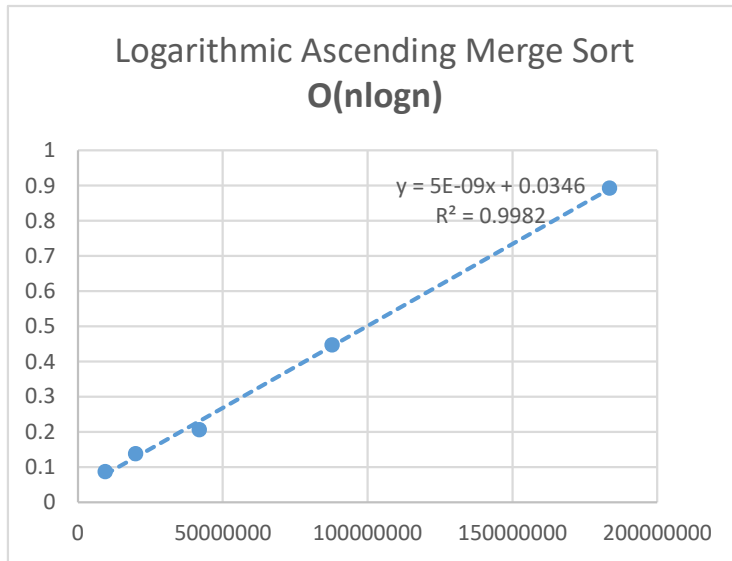
Trial 1	1.45214	1.78434
Trial 2	1.44642	1.76428
Trial 3	1.44973	1.78997

Average 1.44943 1.77953

**Merge Sort**

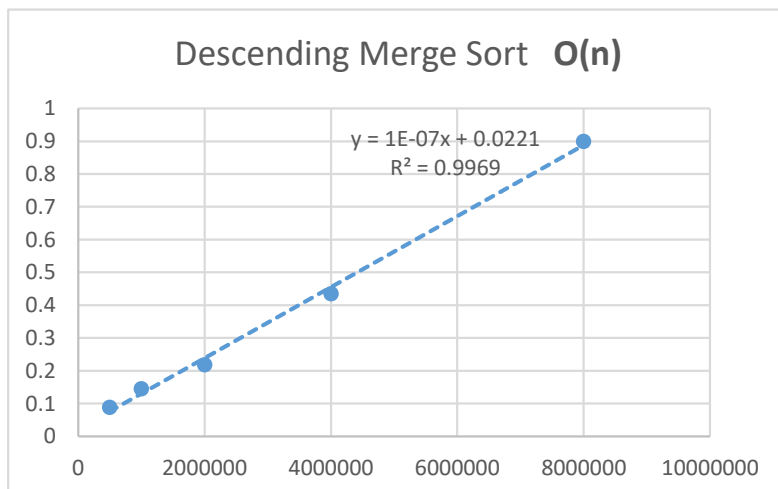
Ascending

500000	9465784.28	0.08699067
1000000	19931568.6	0.13808533
2000000	41863137.1	0.20587433
4000000	87726274.3	0.44688733
8000000	183452549	0.893391



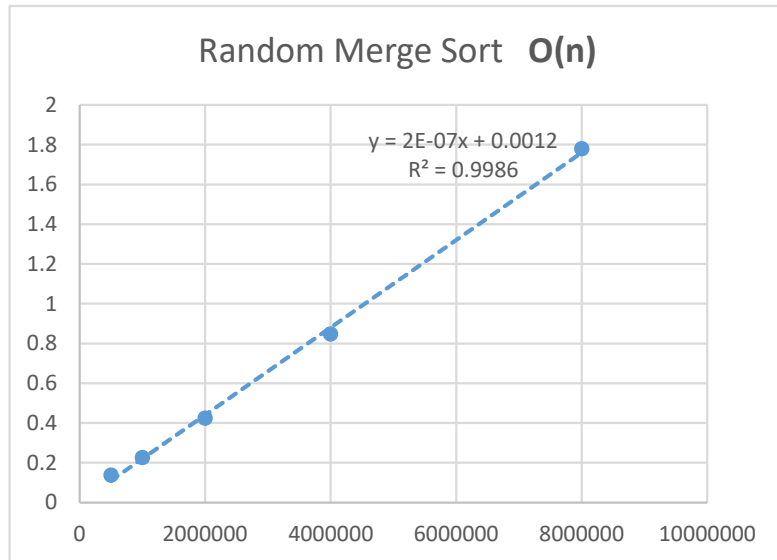
Descending

500000	0.088951
1000000	0.145281
2000000	0.218192
4000000	0.43482533
8000000	0.89974



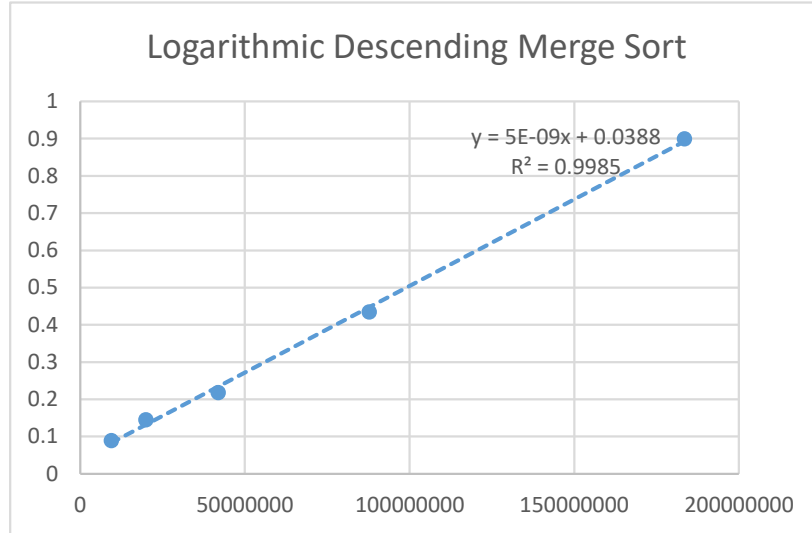
Random

500000	0.137388
1000000	0.22637267
2000000	0.42460933
4000000	0.847854
8000000	1.77953



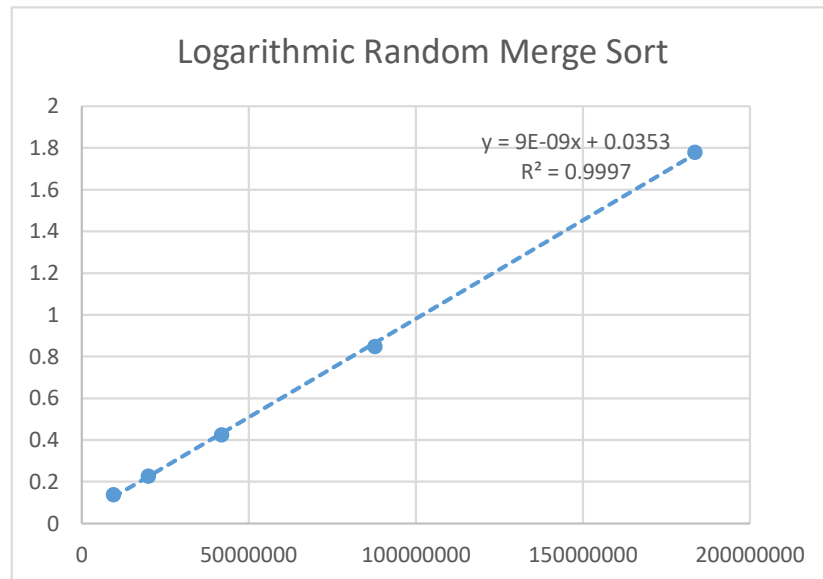
9465784.28	0.088951
19931568.6	0.145281
41863137.1	0.218192
87726274.3	0.43482533
183452549	0.89974

This column is  
N\*Log<sub>2</sub>(N)



9465784.28	0.137388
19931568.6	0.22637267
41863137.1	0.42460933
87726274.3	0.847854
183452549	1.77953

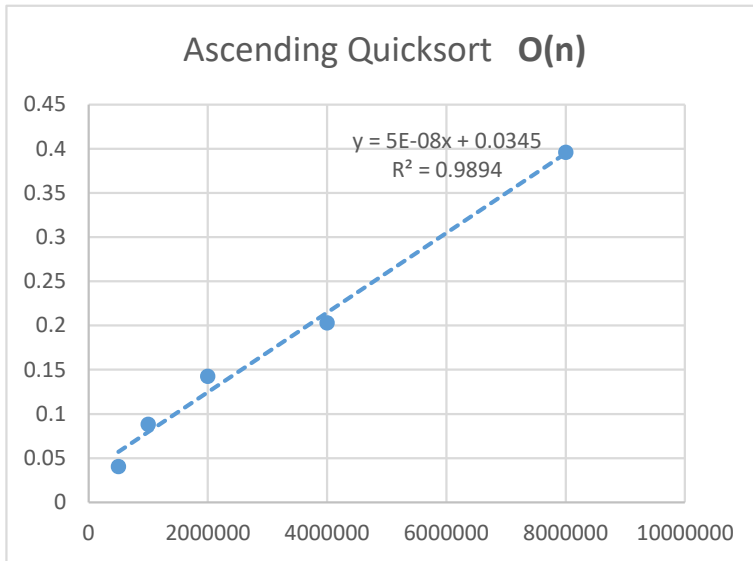
This column is  
N\*Log<sub>2</sub>(N)



**Quick Sort**

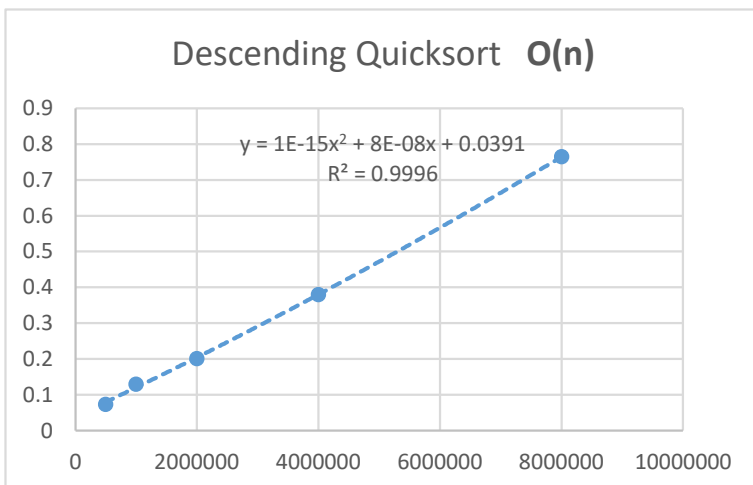
Ascending

500000	0.04054867
1000000	0.08819033
2000000	0.142624
4000000	0.20311867
8000000	0.39601033



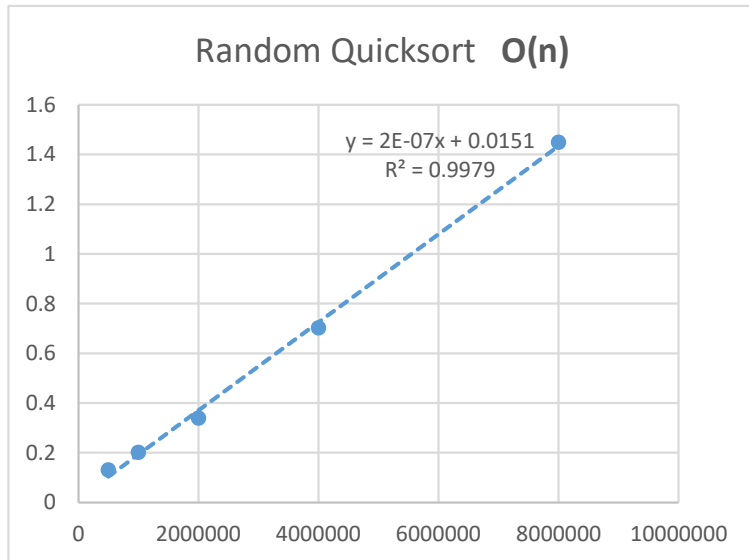
Descending

500000	0.07324533
1000000	0.12953367
2000000	0.20124467
4000000	0.37952533
8000000	0.76471



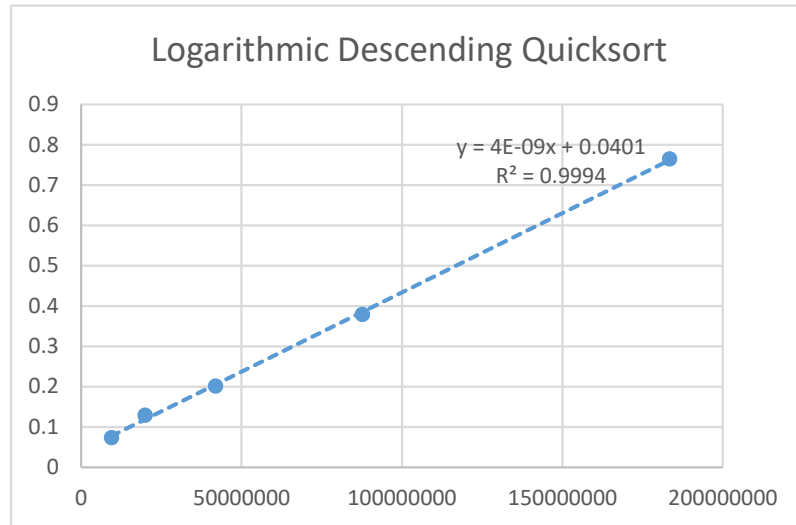
Random

500000	0.13116533
1000000	0.20124167
2000000	0.33985233
4000000	0.70298233
8000000	1.44943



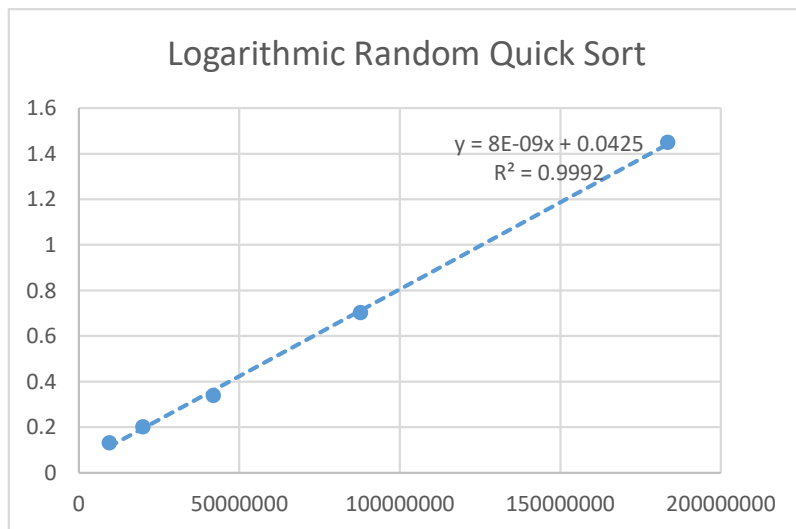
9465784.28	0.07324533
19931568.6	0.12953367
41863137.1	0.20124467
87726274.3	0.37952533
183452549	0.76471

This column is  
N\*Log<sub>2</sub>(N)



9465784.28	0.13116533
19931568.6	0.20124167
41863137.1	0.33985233
87726274.3	0.70298233
183452549	1.44943

This column is  
N\*Log<sub>2</sub>(N)

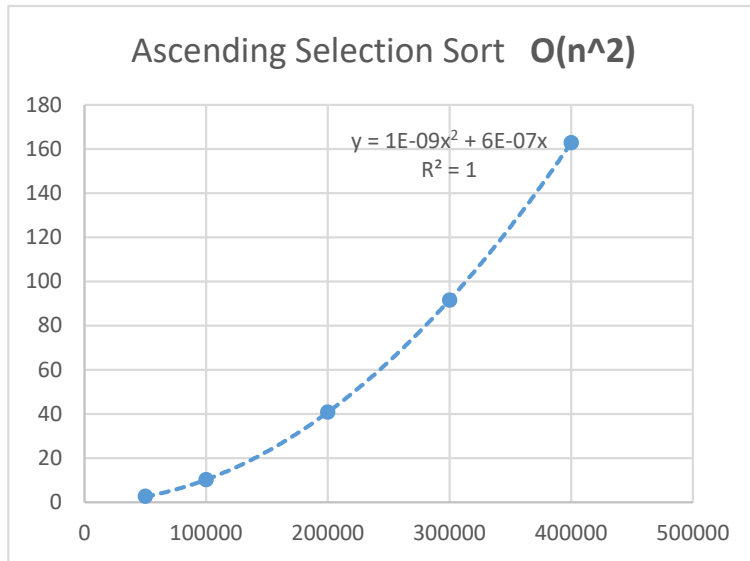




## Selection Sort

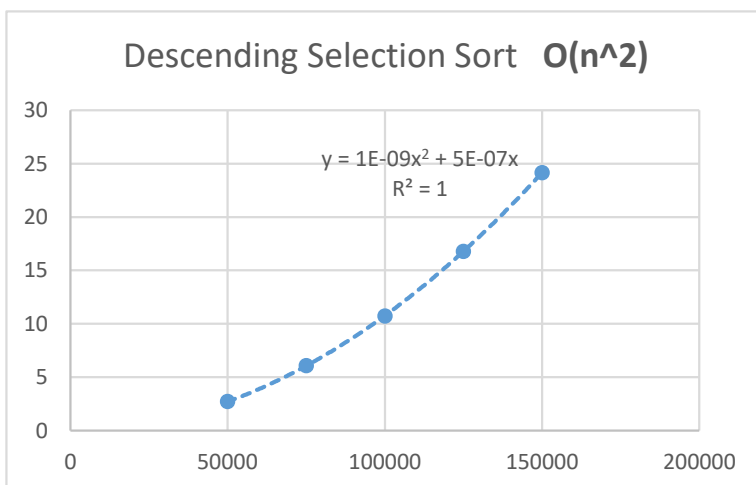
Ascending

50000	2.66838667
100000	10.2062
200000	40.8395
300000	91.5442
400000	162.941



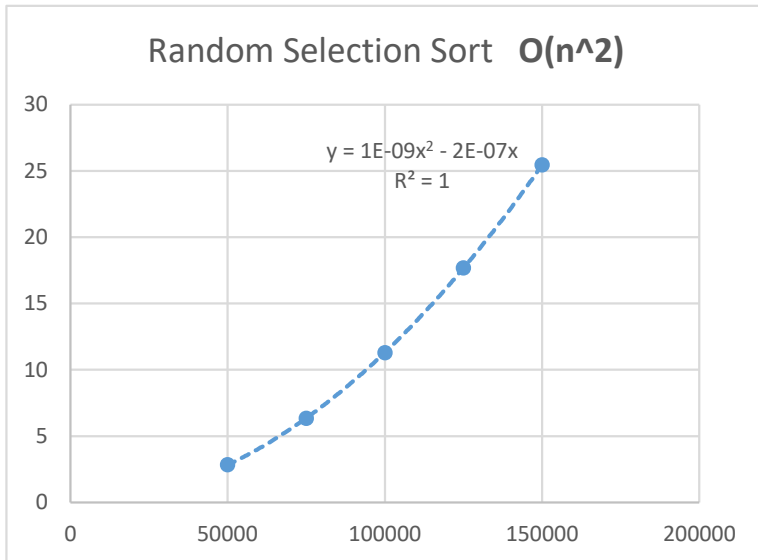
Descending

50000	2.71222
75000	6.06364333
100000	10.7381333
125000	16.7816333
150000	24.1632



Random

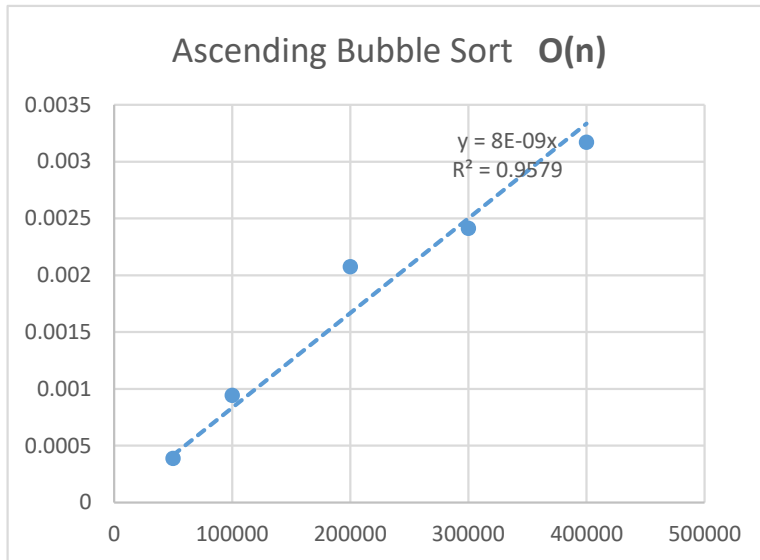
50000	2.83828333
75000	6.33717333
100000	11.3006
125000	17.6776333
150000	25.4523333



**Bubble Sort**

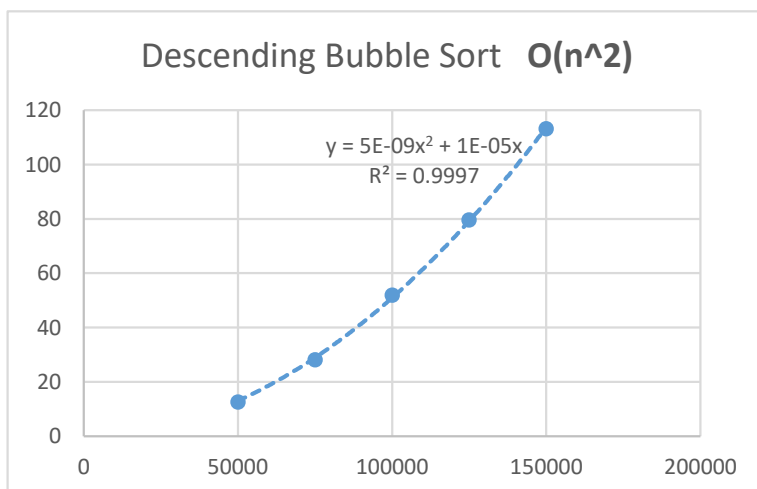
Ascending

50000	0.000388
100000	0.000942
200000	0.002075
300000	0.00241167
400000	0.00317167



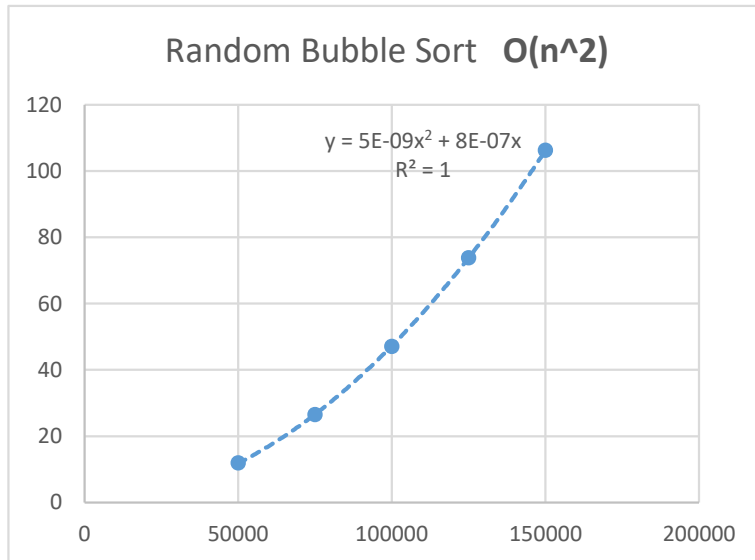
Descending

50000	12.5927333
75000	28.1534333
100000	51.9409
125000	79.6573667
150000	113.172333



Random

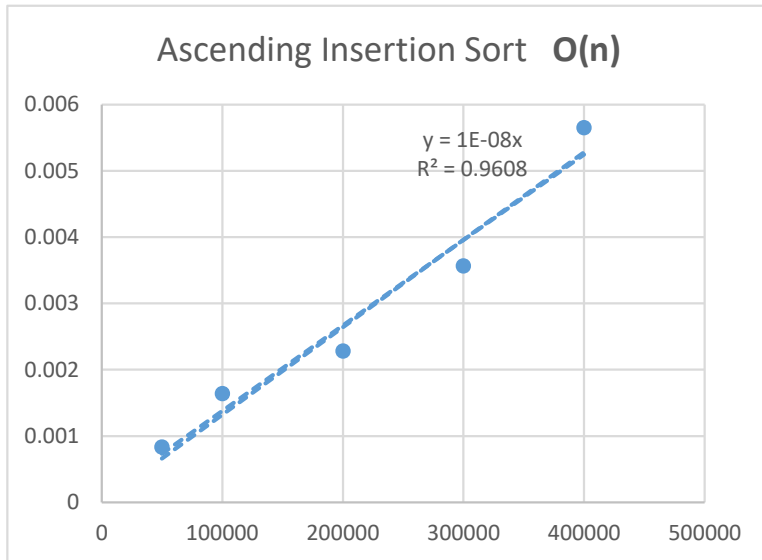
50000	11.9902667
75000	26.5357667
100000	47.0999
125000	73.8686
150000	106.252333



## Insertion Sort

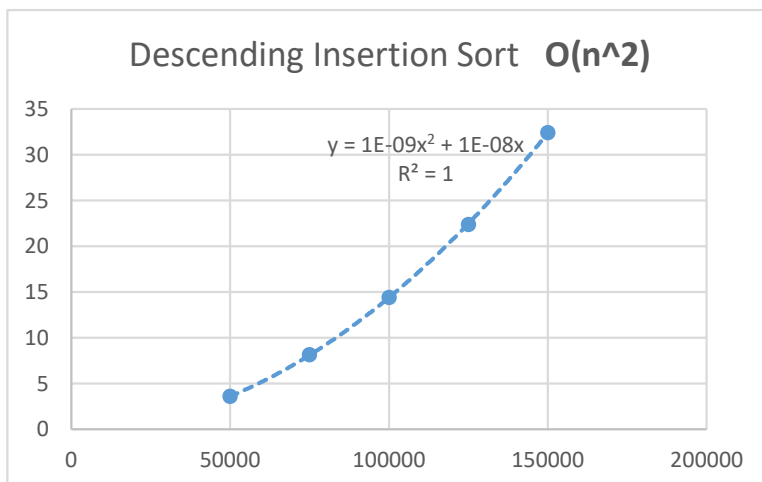
Ascending

50000	0.000836
100000	0.00164167
200000	0.00228133
300000	0.003567
400000	0.00565367



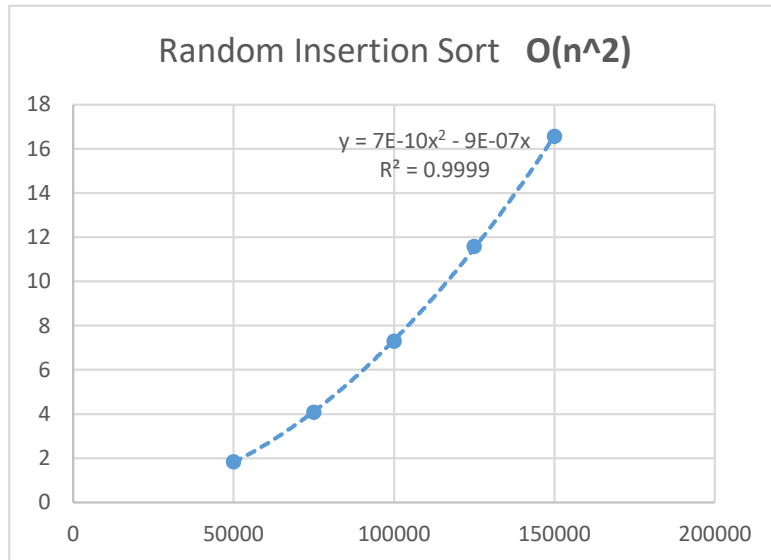
Descending

50000	3.60308667
75000	8.12510667
100000	14.4128667
125000	22.3604
150000	32.4234



Random

50000	1.84146333
75000	4.08526667
100000	7.29885333
125000	11.5788
150000	16.5613333



## Sheet1

Results      Note: All trials was run and tested on the same lab machine.

Insertion Sort should always have a growth rate of  $O(N^2)$ . The graphs show this to be true with almost 100% certainty. Similarly, the "slow" sorts should also be  $O(N^2)$ . They almost fit perfectly, with the worst having an  $R^2$  of 0.9998. That's very good. I can say with an extremely high degree of confidence that all of the  $O(N^2)$  ones are correctly marked. That is, all of the Selection sorts, and the descending and randomly distributed insertion and bubble sorts. In the case of the already sorted Insertion and Bubble sorts, all the algorithm would need to do is go through the array once and see that everything's already sorted. One pass through the array is  $O(N)$ , so we should expect those to be  $O(N)$ . Although the coefficient of determination ( $R^2$ ) is only about 0.96 for both, it's still a good estimate, and considering the times have higher variance as a result of the sorts taking very little time, it's a very reasonable result. If hundreds of trials were run on the ascending bubble/insertion sort, I would expect the variance to decrease, making our line of best fit more accurate.

Side note: Since all algorithms should take virtually zero seconds to sort an array of size zero, I forced the y-intercept to be 0 for all graphs. In most cases, this causes an almost negligible change in the line of best fit and coefficient of determination.

For the slow algorithms:

	Ascending	Descending	Random
Insertion	$O(N)$	$O(N^2)$	$O(N^2)$
Bubble	$O(N)$	$O(N^2)$	$O(N^2)$
Selection	$O(N^2)$	$O(N^2)$	$O(N^2)$

Predicted time to sort an array of size 10,000,000 (in seconds):

	Ascending	Descending	Random
Insertion	0.1	100000	69991
Bubble	0.08	500100	500008
Selection	100006	100005	99998

For the faster sorts, we should expect the ascending one to be linear, and the random ones to be  $O(N \log N)$ . The descending ones are the worst-case scenario, so the worst case for quicksort is  $O(N^2)$  and the worst case for merge sort is still  $O(N \log N)$ . In theory, it's easy to show those. However, since merge sort and quicksort are very efficient, even very large sizes,  $N$  in the millions get sorted in just a few seconds. This means every graph can be approximated relatively well by almost any type of function. So in order to find out which one it resembles most, we should look at the confidence coefficients. Every single one of them will be very close to 1, so it's mostly just a matter of trying them all and seeing which one is the best. And it does turn out that all of the functions most resemble their proven growth rate, even if the confidence coefficient only differs by a small amount. Even the  $O(N \log N)$  functions fit on a logarithmic graph better than a linear graph. Since  $N \log N$  is not a commonly used function, to find its coefficient of determination I plotted  $N \log N$  against time, and found the best linear fit for that graph. In every case, even the descending quicksort, the logarithmic graph fit better. So for the random and descending merge sorts, and the random quicksort, this was their best fit. The descending quicksort had  $R^2 = 0.9994$ , but a quadratic had  $R^2 = 0.9996$ , which, although a minor difference, means that at even larger values of  $N$ , it will behave more like  $O(N^2)$ . It's also important to note that because the merge sort and quicksort do have more initial startup time compared to the slower sorts, I didn't force the graph to start at  $t=0$  for  $n=0$ . Overall, my data supports each algorithm's growth rate, although the differences between calling some  $O(n)$  instead of  $O(n \log n)$  or  $O(n^2)$  can be very small, even at large values of  $n$ .

Side Note: The ascending merge sort modeled  $O(n \log n)$  better, making it have the largest  $O$

## Sheet1

For the fast algorithms:

	Ascending	Descending	Random
Merge	$O(N \log N)$	$O(N \log N)$	$O(N \log N)$
Quicksort	$O(N)$	$O(N^2)$	$O(N \log N)$

Predicted time to sort an array of size 10,000,000 (in seconds)

	Ascending	Descending	Random
Merge	0.13426	1.0221	2.12811
Quicksort	0.5345	0.97024	1.9028