0.1 Full FDTD for the 1D Case

- starting from six equations from maxwell from other note on plane waves

$$\partial_y E_z - \partial_z E_y = -\mu \partial_t H_x \tag{1}$$
$$\partial_z E_x - \partial_x E_z = -\mu \partial_t H_y \tag{2}$$
$$\partial_x E_y - \partial_y E_x = -\mu \partial_t H_z \tag{3}$$
$$\partial_y H_z - \partial_z H_y = \varepsilon \partial_t E_x + \sigma E_x \tag{4}$$
$$\partial_z H_x - \partial_x H_z = \varepsilon \partial_t E_y + \sigma E_y \tag{5}$$
$$\partial_x H_y - \partial_y H_x = \varepsilon \partial_t E_z + \sigma E_z \tag{6}$$

- partial x refers to partial derivation

- only consider discretization along one axis, here z-axis so any derivative along another axis can be ignored

- this gives the following six equations

$$\partial_z E_y = \mu \partial_t H_x \tag{7}$$
$$\partial_z E_x = -\mu \partial_t H_y \tag{8}$$
$$0 = -\mu \partial_t H_z \tag{9}$$
$$-\partial_z H_y = \varepsilon \partial_t E_x + \sigma E_x \tag{10}$$
$$\partial_z H_x = \varepsilon \partial_t E_y + \sigma E_y \tag{11}$$
$$0 = \varepsilon \partial_t E_z + \sigma E_z \tag{12}$$

- again discretize spatially and temporally in a staggered manner (Yee-grid)
- we can drop eqn 3. and 6, as they just state that there is no change

$$\frac{E_y^{n+1/2}(k+1) - E_y^{n+1/2}(k)}{\Delta z} = \mu \frac{H_x^{n+1}(k+1/2) - H_x^{n}(k+1/2)}{\Delta t} \tag{13}$$
$$\frac{E_x^{n+1/2}(k+1) - E_x^{n+1/2}(k)}{\Delta z} = -\mu \frac{H_y^{n+1}(k+1/2) - H_y^{n}(k+1/2)}{\Delta t} \tag{14}$$
$$-\frac{H_y^{n}(k+1/2) - H_y^{n}(k-1/2)}{\Delta z} = \varepsilon \frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t} + \sigma E_x^n(k) \tag{15}$$
$$\frac{H_x^{n}(k+1/2) - H_x^{n}(k-1/2)}{\Delta z} = \varepsilon \frac{E_y^{n+1/2}(k) - E_y^{n-1/2}(k)}{\Delta t} + \sigma E_y^n(k) \tag{16}$$

- new unknown variable, we can get that as simple average between two time steps, e.g.

$$E_x^n(k) = \frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{2} \tag{17}$$

- no we get recurrence relations for the algorithm by multiplying out and then rearranging

$$H_x^{n+1}(k+1/2) = H_x^n(k+1/2) + \frac{\Delta t}{\mu \Delta z}\left(E_y^{n+1/2}(k+1) - E_y^{n+1/2}(k)\right) \tag{18}$$

$$H_y^{n+1}(k+1/2) = H_y^n(k+1/2) - \frac{\Delta t}{\mu \Delta z}\left(E_x^{n+1/2}(k+1) - E_x^{n+1/2}(k)\right) \tag{19}$$

$$E_x^{n+1/2}(k) = E_x^{n-1/2}(k) - \frac{\Delta t}{\varepsilon \Delta z + \frac{1}{2}\sigma \Delta z \Delta t}\left(H_y^n(k+1/2) - H_y^n(k-1/2)\right) \tag{20}$$

$$E_y^{n+1/2}(k) = E_y^{n-1/2}(k) + \frac{\Delta t}{\varepsilon \Delta z + \frac{1}{2}\sigma \Delta z \Delta t}\left(H_x^n(k+1/2) - H_x^n(k-1/2)\right) \tag{21}$$

- in algorithm, first calculate 20/21, then 18/19
  - again CFL condition for the time step

$$\Delta t \leq \frac{\Delta z}{c} \tag{22}$$

  - for 18/19 term, we get

$$\frac{\Delta t}{\mu \Delta z} = \frac{1}{Z_0 \mu_r} \tag{23}$$

$$\frac{\Delta t}{\varepsilon \Delta z + \frac{1}{2}\sigma \Delta z \Delta t} = \frac{\frac{\Delta z}{c}}{\varepsilon \Delta z + \frac{1}{2}\sigma \Delta z \frac{\Delta z}{c}} = \frac{Z_0}{\varepsilon_r + \frac{1}{2} Z_0 \sigma \Delta z} \tag{24}$$

0.2 Simulations