Interim Report

PHY2071-Celestial Mechanics

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Introduction:

This project's aim is to simulate the solar system's constituents' positions and velocities over a period of time, this is done by solving the n-body problem. Solving the n-body problem requires taking the instantaneous positions and velocities of the bodies concerned and using this to predict their future positions and motions by considering the accelerations due to gravity on each body from every other body (Trenti & Hut 2008). To do this Newton's law of gravity and Euler's method of numerical integration is employed. Predicting the positions and velocities of planets and larger bodies has many applications in astronomy and the discrepancies in such simulations have resulted in the theory of dark matter (Koupelis, 2014).

Summary of Progress Made:

The initial positions and velocity of the planets have been inputted into the code in cartesian heliocentric coordinates and were taken from the NASA's Horizons system, at 01/01/2000 00:00 (Chamberlin & Park, 2019). The code to calculate the acceleration in 3 dimensions caused by the gravitational influence by all other bodies, for each planet, was implemented using the following series (Heggie, 2005),

$$\frac{d\mathbf{v_i}}{dt} = -G\sum_{j=1,j\neq i}^{j=N}\frac{m_j(\mathbf{r_i}-\mathbf{r_j})}{|\mathbf{r_i}-\mathbf{r_j}|^3}(1)$$

The Euler method was then implemented to integrate this ODE over a chosen period and step, dt (Barker, 2017). The step, dt is the time that the bodies are assumed to travel at one speed; by increasing dt the results then become less accurate, but the computation time is decreased and vice versa. The next step was to calculate each bodies' new position and velocity for each step which was done successfully using the following equation (Heggie, 2005),

$$\mathbf{r}_{i,2} = \boldsymbol{v}_{i,1}\times dt + \mathbf{r}_{i,1}\ (2); \mathbf{v}_{i,2} = \frac{d\mathbf{v}_{i,1}}{dt}\times dt + \mathbf{v}_{i,1}\ (3)$$

The positions of the bodies were printed to a file to allow for plotting. The X position of the Earth after 1 year of simulation time was then compared to the value given by NASA's Horizon system and from this the ability to calculate the percentage error in the simulations value was added to the program, along with code to calculate the computation time of the simulation (Chamberlin & Park, 2019).

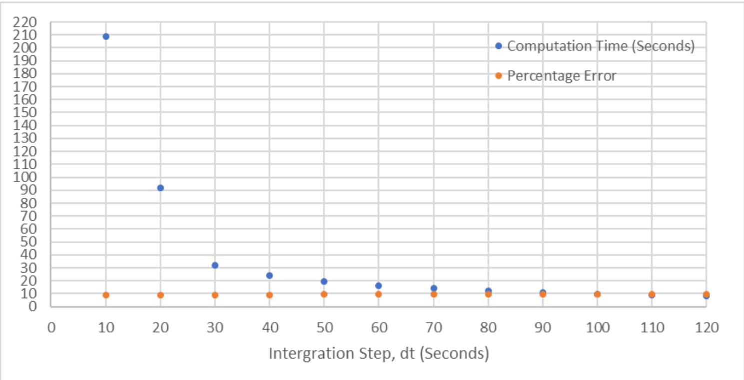

Figure 1: Graph of percentage error in Earth's X-position and computation time against time step for 1-year simulation time (Chamberlin & Park, 2019).

The program was run for 13 different step times from 1 to 120 seconds and the computation time and percentage error was recorded. A computation time of 1808.25±0.05 seconds and a percentage error of 9.023% was found for a step of one second, this value is not displayed in figure 1 so as not to obscure the relationship for lower computation times. This test indicated that the best trade-off between computation time and error was found at a step of 40 seconds, with a computation time of 24.18±0.05 seconds and an error of 9.262%. The ability to calculate the average distance of each body from the sun and the percentage error in these values was added to the simulation. Code to calculate the orbit time of the planets has been partially implemented, but successful results have not been obtained. This is due to difficulties caused by planets doing multiple orbits over simulation times required to see one orbit in Neptune resulting in orbit times being given in unknown multiples of the single orbit time. Orbit times were further obscured by the errors in the coordinates.

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Summary of Preliminary Results:

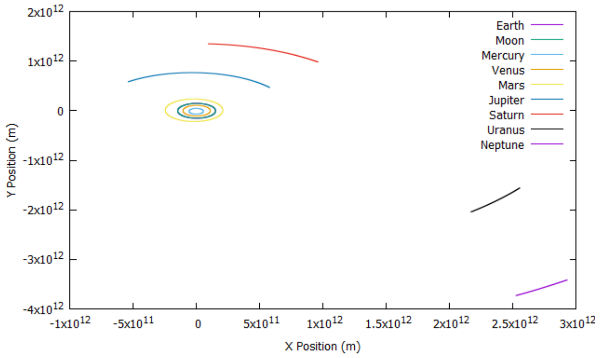

Figure 2: Graph of X and Y position of all planets for a 3-year simulation time and a step of 150 seconds

The results of a 3-year simulation are shown in figure 2, from this we can see that the orbits are ellipses with the orbits of the outer planets only being partially completed in 3 years as is expected. A larger step time had to be used so that Gnuplot could plot the results without running out of memory. Different simulation times were tested to manually check if the planets would complete their orbits in their expected orbit time, a time of 88 days was tested for Mercury, 365 days for the Earth and 1.88 years for Mars. The planets were found to complete their orbit in these times with little or no overlap in the graph, indicating the program was successful in this area (Freedman & Kaufmann, 2005).

Body	Average Distance(AU)	Percentage Error
Mercury	0.402	3.77
Venus	0.725	0.26
Earth	1.001	0.11
Mars	1.532	0.55
Juipiter	5.232	0.56
Saturn	9.509	0.66
Uranus	20.018	0.09
Neptune	30.002	0.21

Figure 3: Table displaying the average distance from the sun for all planets and the percentage error in these values for a 180-year simulation and a 40 second step time (Freedman & Kaufmann, 2005)

The average distances between the planets and the sun are shown in the table given in figure 3 along with their percentage errors calculated by comparison to the values given in Freedman's and Kaufmann's book, Universe (Freedman & Kaufmann, 2005). The values calculated for average distances show a good accuracy in the simulation with the largest error in Mercury. This can be explained in part by the fact that Mercury is in very close proximity to the sun and as such accelerates incredibly quickly; the assumption made by the Euler model that the planets will have a constant acceleration for the 40 second step time used subsequently causes a larger error in Mercury than in any other planet, especially considering the 745 orbits it completes in the 180-year simulation time.

Outstanding Things to be Done:

The results obtained so far produced expected average distances to the sun, but a 9% error in Earth's X coordinate after one year means that the program is not good enough for precise predictions. To reduce this error, different numerical methods to solve ODEs, such as the modified Euler method, will be implemented and their error and computation time will be noted, the most suitable method will then be chosen. It may take up to 7 hours to understand and implement the different methods and then test their errors. Code to calculate the orbit time of each of the bodies will be implemented, along with code to calculate the eccentricity of the orbits. This may take up to 5 hours to work out the least resource intensive method of doing this and analyse the errors produced. The initial conditions will also be read from a file to tidy up the code, a simple process that shouldn't take any longer than an hour to correctly format the data in the file and test it. Wider applications of the code will be tested, such as what it predicts for the planets orbiting around a binary star system or a black hole, and then compared to current simulations for comparison. The main challenge of this will be to come up with suitable initial conditions that will result in sensible orbits.

References:

Trenti, M. & Hut, P. 2008. N-body simulations (gravitational). [Online]. [17 March 2019]. Available from: http://www.scholarpedia.org/article/N-body_simulations_(gravitational)

Koupelis, T (2014). In Quest of The Universe. (7th ed.). Burlington, MA: Jones & Bartlett Learning.

Chamberlin, A.B. & Park, R.S. (2019). Horizons Web Interface. [Online]. [1st March 2019]. Available from: https://ssd.jpl.nasa.gov/?horizons

Heggie, D.C (2005). The Classical Gravitational N-Body Problem. Edinburgh: University of Edinburgh.

Barker, C.A. 2017. Euler's Method Theoretical Introduction. [Online]. [17 March 2019]. Available from: Numerical Methods for Solving Differential Equations Euler's Method

Freedman, R.A & Kaufmann, W.J (2005). Universe. (7th ed.). New York: W H Freeman and Company.