Digital surfaces in DGtal
Topology module (since 0.5)

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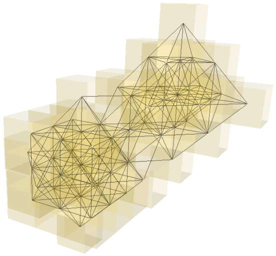
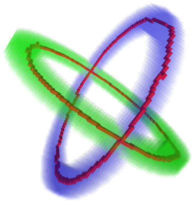
UMR 5127

Package Topology, available in DGtal 0.4

1. classical digital topology (*à la Rosenfeld*)

- Arbitrary adjacencies in $\mathbb{Z}^n$, but also in subdomains
- Digital topology = couple of adjacencies (Rosenfeld)
- Object = Topology + Set
- Operations : neighborhoods, border, connectedness and connected components, decomposition into digital layers, simple points


Adjacencies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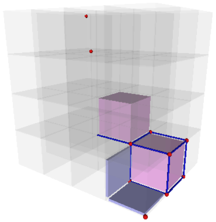
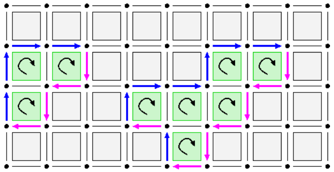
Package Topology, available in DGtal 0.4

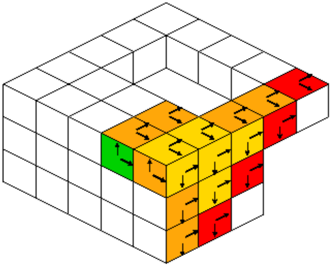
1. classical digital topology (*à la Rosenfeld*)
2. cubical cellular topology + algebraic topology
   - cells, adjacent and incident cells, faces and cofaces
   - signed cells, signed incidence, boundary operators

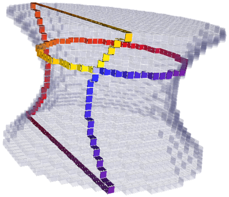




Package Topology, available in DGtal 0.4

1. classical digital topology (*à la Rosenfeld*)
2. cubical cellular topology + algebraic topology
3. digital surface topology (*à la Herman*)
   - surfels, surfel adjacency, surfel neighborhood
   - surface tracking (normal, fast), contour tracking in *n*D



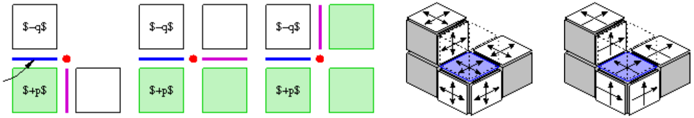


Package Topology, new in DGtal 0.5

Digital Surface

$$\left.\begin{array}{rl} & \text{surfels / signed } n-1\text{-cells} \\ + & \text{adjacencies between surfels} \end{array}\right\}$$

- kind of "dual" graph
- kind of manifold



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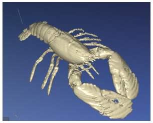
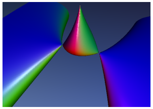
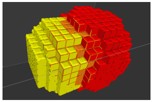
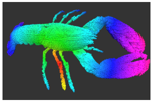
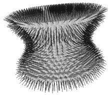
1. High-level `DigitalSurface` class for representing any kind of digital surface
2. Many container classes for digital surfaces
   - boundary of digital shape
   - boundary of implicitly defined shape
   - set of surfels
   - implicitly defined set of surfels
   - light containers
3. a `DigitalSurface` is a graph
4. a `DigitalSurface` is a combinatorial surface (with umbrellas)

Direct applications

- marching cubes algorithm
- tracking implicit polynomial surfaces
- representing boundary of regions and frontier between regions
- breadth-first visiting on surfaces
- estimating normals on surfaces

Necessary concepts and classes for digital surfaces

One must choose

- the representation of cellular grid space : model of `CCellularGridSpaceND`
  e.g. `KhalimskySpaceND`$< N, int >$, `Z2i::KSpace`, `Z3i::KSpace`
- the kind of adjacency between surfels, `SurfelAdjacency`$< N >$
- the kind of surface container : model of `CDigitalSurfaceContainer`

```
typedef Z3i::Point Point; // 3D digital point
typedef Z3i::Domain Domain;
typedef Z3i::DigitalSet DigitalSet; // a set of
    digital points
typedef Z3i::KSpace KSpace; // 3D cellular grid space
typedef SurfelAdjacency<3> SAdj; // surfel adjacency.
typedef DigitalSetBoundary<KSpace,DigitalSet>
    Container; // kind of surface container
typedef DigitalSurface<Container> MyDigSurf; //
    concrete digital surface
```

Concrete instanciations for digital surfaces

Then, the chosen types are instantiated. Here
digital surface = boundary of two intersecting balls

```
Point p1( -20, -20, -20 ), p2( 20, 20, 20 );
KSpace K; K.init( p1, p2, true ); // init space
DigitalSet someShape( Domain( p1, p2 ) );
Shapes<Domain>::addNorm2Ball( someShape, Point
    (-3,0,0), 4 );
Shapes<Domain>::addNorm2Ball( someShape, Point
    (3,0,0), 4 );
SAdj surfAdj( true );  // the adjacency
Container surfContainer( K, someShape, surfAdj );
MyDigSurf digSurf( surfContainer ); // digital
    surface
```

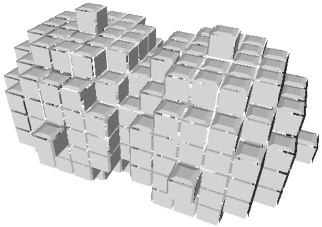
Using the digital surface (displays 518) :

```
cout << "-␣nb␣surfels/vertices␣=␣"
     << digSurf.size() << endl;
```

How to use digital surfaces (I)

Just enumerating its elements...

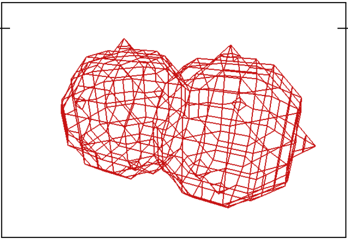
```
QApplication application( argc, argv );
Viewer3D viewer; // QGL viewer
viewer.show();
for( MyDigSurf::ConstIterator it = digSurf.begin(),
 itend = digSurf.end(); it != itend; ++it )
  viewer << *it;
viewer << Viewer3D::updateDisplay;
return application.exec();
```



How to use digital surfaces (II)

Getting the neighbors and drawing the graph...

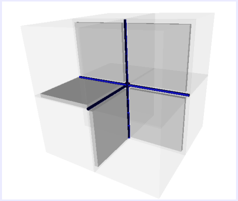
```cpp
typedef std::vector<Vertex> Neighborhood;
for ( ConstIterator it = digSurf.begin(),
        itend = digSurf.end(); it != itend; ++it )
  {
    Neighborhood N;
    back_insert_iterator<Neighborhood> itN = back_inserter( N );
    digSurf.writeNeighbors( itN , *it );
    Point p = K.sKCoords( *it );
    for ( unsigned int i = 0; i < N.size(); ++i )
      {
        Point q = K.sKCoords( N[ i ] );
        viewer.addLine ( p[0]/2.0, p[1]/2.0, p[2]/2.0,
                         q[0]/2.0, q[1]/2.0, q[2]/2.0,
                         DGtal::Color ( 200,20,20 ), 2.0 );
      }
  }
```



How to use digital surfaces (III)

Digital surfaces are combinatorial surfaces

- in n-D
- vertices $= n - 1$-cells
- edges $\approx n - 2$-cells
- faces $\approx n - 3$-cells



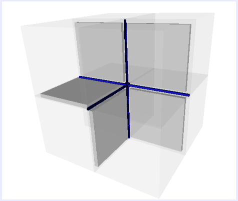
Inner types `Vertex`, `Arc`, `Face`, `xxxRange`, `xxxSet`

```
FaceRange   facesAroundVertex( const Vertex & v )
VertexRange verticesAroundFace( const Face & f )
FaceRange   facesAroundArc( const Arc & a )
FaceSet     allFaces()
FaceSet     allClosedFaces()
FaceSet     allOpenFaces() ...
```

How to use digital surfaces (III)

Digital surfaces are combinatorial surfaces

- in 3-D
- vertices = surfels
- edges ≈ linels
- faces = umbrellas



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FaceSet     allFaces()
FaceSet     allClosedFaces()
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```

How to use digital surfaces (IV)

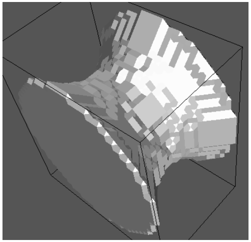
Getting the faces and outputing their vertices

```
typedef typename FaceSet::const_iterator FaceSetIter;
typedef typename VertexRange::const_iterator VertexRangeIter;
FaceSet faces = digSurf.allClosedFaces();
for ( FaceSetIter itf = faces.begin(),
        itf_end = faces.end(); itf != itf_end; ++itf )
  {
    Face face = *itf;
    out << face.nbVertices;
    VertexRange vtcs = digSurf.verticesAroundFace( face );
    for ( VertexRangeIter itv = vtcs.begin(),
            itv_end = vtcs.end(); itv != itv_end; ++itv )
      out << "␣" << index[ *itv ];
    out << std::endl;
  }
```

e.g. export in OFF format

```
void exportSurfaceAs3DOFF ( std::ostream
     & out )

template <typename CellEmbedder>
void exportEmbeddedSurfaceAs3DOFF
( std::ostream & out, const CellEmbedder
     & cembedder )
```

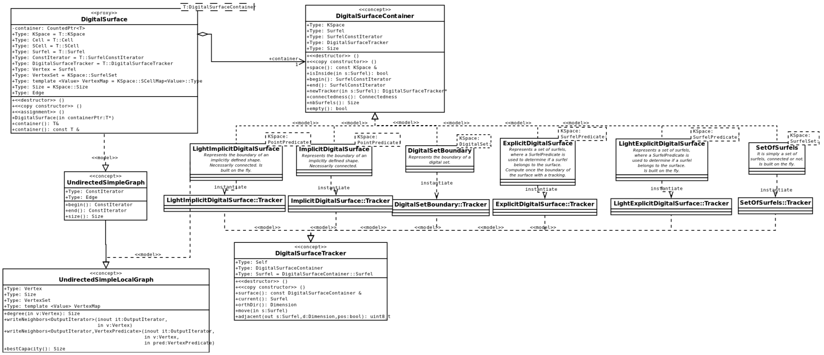


Diversity of digital surfaces

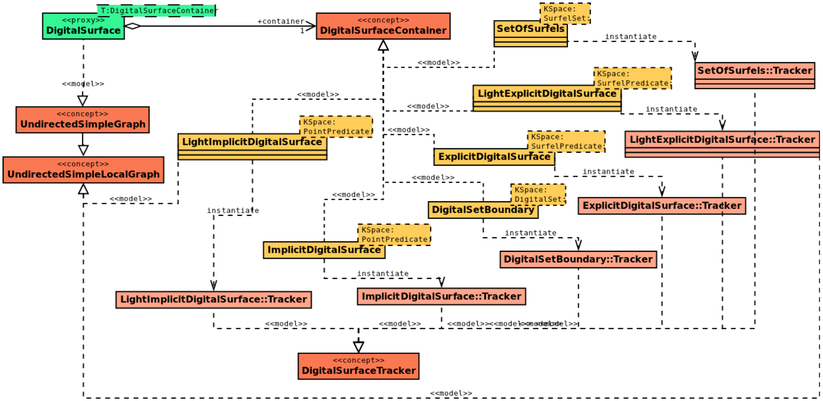
- may be open or closed
- may be connected or not
- may be defined explicitly with their surfels
- may be defined implicitly as the boundary of some shape
- the surfels may be listed or known only through a predicate
- the shape may be described by its points or known only through a predicate
- the surface may be big or infinite so that only lazy extraction is reasonnable

You wish to process them with the same object : `DigitalSurface<T>`
T is a model of `CDigitalSurfaceContainer`

Partial architecture


T:DigitalSurfaceContainer
<<proxy>>
DigitalSurface
-container: CountedPtr<T>
+Type: KSpace = T::KSpace
+Type: Cell = T::Cell
+Type: SCell = T::SCell
+Type: Surfel = T::Surfel
+Type: ConstIterator = T::SurfelConstIterator
+Type: DigitalSurfaceTracker = T::DigitalSurfaceTracker
+Type: Vertex = Surfel
+Type: VertexSet = KSpace::SurfelSet
+Type: template <Value> VertexMap = KSpace::SCellMap<Value>::Type
+Type: Size = KSpace::Size
+Type: Edge
+<<destructor>> ()
+<<copy constructor>> ()
+<<assignment>> ()
+DigitalSurface(in containerPtr:T*)
+container(): T&
+container(): const T &
+container
1
<<concept>>
DigitalSurfaceContainer
+Type: KSpace
+Type: Surfel
+Type: SurfelConstIterator
+Type: DigitalSurfaceTracker
+Type: Size
+<<destructor>> ()
+<<copy constructor>> ()
+space(): const KSpace &
+isInside(in s:Surfel): bool
+begin(): SurfelConstIterator
+end(): SurfelConstIterator
+newTracker(in s:Surfel): DigitalSurfaceTracker*
+connectedness(): Connectedness
+nbSurfels(): Size
+empty(): bool
<<model>>
<<model>>
<<model>>
<<model>>
<<model>>
<<model>>
KSpace: PointPredicate
LightImplicitDigitalSurface
Represents the boundary of an implicitly defined shape. Necessarily connected. Is built on the fly.
KSpace: PointPredicate
ImplicitDigitalSurface
Represents the boundary of an implicitly defined shape. Necessarily connected.
KSpace: DigitalSet
DigitalSetBoundary
Represents the boundary of a digital set.
KSpace: SurfelPredicate
ExplicitDigitalSurface
Represents a set of surfels, where a SurfelPredicate is used to determine if a surfel belongs to the surface. Compute once the boundary of the surface with a tracking.
KSpace: SurfelPredicate
LightExplicitDigitalSurface
Represents a set of surfels, where a SurfelPredicate is used to determine if a surfel belongs to the surface. Is built on the fly.
KSpace: SurfelSet
SetOfSurfels
It is simply a set of surfels, connected or not. Is built on the fly.
instantiate
instantiate
instantiate
instantiate
instantiate
instantiate
LightImplicitDigitalSurface::Tracker
ImplicitDigitalSurface::Tracker
DigitalSetBoundary::Tracker
ExplicitDigitalSurface::Tracker
LightExplicitDigitalSurface::Tracker
SetOfSurfels::Tracker
<<model>>
<<model>>
<<model>>
<<model>>
<<model>>
<<model>>
<<model>>
<<concept>>
UndirectedSimpleGraph
+Type: ConstIterator
+Type: Edge
+begin(): ConstIterator
+end(): ConstIterator
+size(): Size
<<model>>
<<concept>>
UndirectedSimpleLocalGraph
+Type: Vertex
+Type: Size
+Type: VertexSet
+Type: template <Value> VertexMap
+degree(in v:Vertex): Size
+writeNeighbors<OutputIterator>(inout it:OutputIterator, in v:Vertex)
+writeNeighbors<OutputIterator,VertexPredicate>(inout it:OutputIterator, in v:Vertex, in pred:VertexPredicate)
+bestCapacity(): Size
<<concept>>
DigitalSurfaceTracker
+Type: Self
+Type: DigitalSurfaceContainer
+Type: Surfel = DigitalSurfaceContainer::Surfel
+<<destructor>> ()
+<<copy constructor>> ()
+surface(): const DigitalSurfaceContainer &
+current(): Surfel
+orthDir(): Dimension
+move(in s:Surfel)
+adjacent(out s:Surfel,d:Dimension,pos:bool): uint8_t

Partial architecture



Digital surface containers

- `DigitalSetBoundary`<KSpace,DigitalSet> Represents the boundary of a digital set (a set of digital points, considered as the set of pixels/voxels/spels of the space).
  $\boxed{\Rightarrow \text{interpixel boundary of a digital shape}}$

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- `ImplicitDigitalSurface`<KSpace,PointPredicate> Represents the (connected) boundary of shape defined implicitly by a predicate. + `Light` version.
  $\boxed{\Rightarrow \text{ implicit surface computed once or on-the-fly}}$

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- `ExplicitDigitalSurface`<KSpace,SurfelPredicate> Represents a (connected) set of surfels defined implicitly by a predicate. + `Light` version.
  $\boxed{\Rightarrow \text{ frontier between regions in images, computed once or on-the-fly}}$

Package description

Should contain

- classical digital topology Ã *la* Rosenfeld
- cartesian cellular topology
- digital surface topology Ã *la* Herman
- must be the base block of geometric algorithms

Examples

- adjacencies, connected components, simple points, thinning
- cells, boundary operators, incidence, opening, closing
- contours, surfel adjacency, surface tracking
- topological invariants

Location

- `{DGtal}/src/DGtal/topology`
- `{DGtal}/src/DGtal/helpers`
- `{DGtal}/tests/topology`