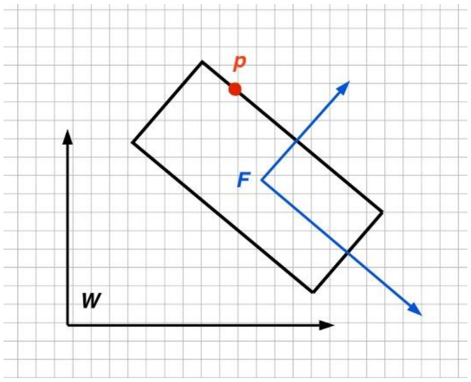
Equivalent Force on Object Frame



F is the body frame of the Object; W is the global frame.

p is a point on the Object located at [x,y] ref to the Body frame.

Frame F (or the transformation of Frame F ref to W) can be written as:

$$F\left(\theta,t_x,t_y\right) = {}^{W}T_F = \left[R(\theta),t\left(t_x,t_y\right)\right]$$

Then coordinate of point p ref to global frame is:

$${}^{W}p = {}^{W}T_F \cdot {}^{F}p = [R,t]\cdot\begin{bmatrix} x \\ y \end{bmatrix} = R\cdot\begin{bmatrix} x \\ y \end{bmatrix} + t$$

$${}^{W}p = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}\cdot\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{pmatrix} t_x + x\cos(\theta) - y\sin(\theta) \\ t_y + y\cos(\theta) + x\sin(\theta) \end{pmatrix}$$

Computes the Jacobian matrix of ${}^{W}p$ with respect to θ, t_x, t_y

$$J = \begin{bmatrix} \dfrac{\partial\, {}^{W}p_x}{\partial t_x} & \dfrac{\partial\, {}^{W}p_x}{\partial t_y} & \dfrac{\partial\, {}^{W}p_x}{\partial \theta} \\ \dfrac{\partial\, {}^{W}p_y}{\partial t_x} & \dfrac{\partial\, {}^{W}p_y}{\partial t_y} & \dfrac{\partial\, {}^{W}p_y}{\partial \theta} \end{bmatrix} = \begin{pmatrix} 1 & 0 & -y\cos(\theta) - x\sin(\theta) \\ 0 & 1 & x\cos(\theta) - y\sin(\theta) \end{pmatrix}$$

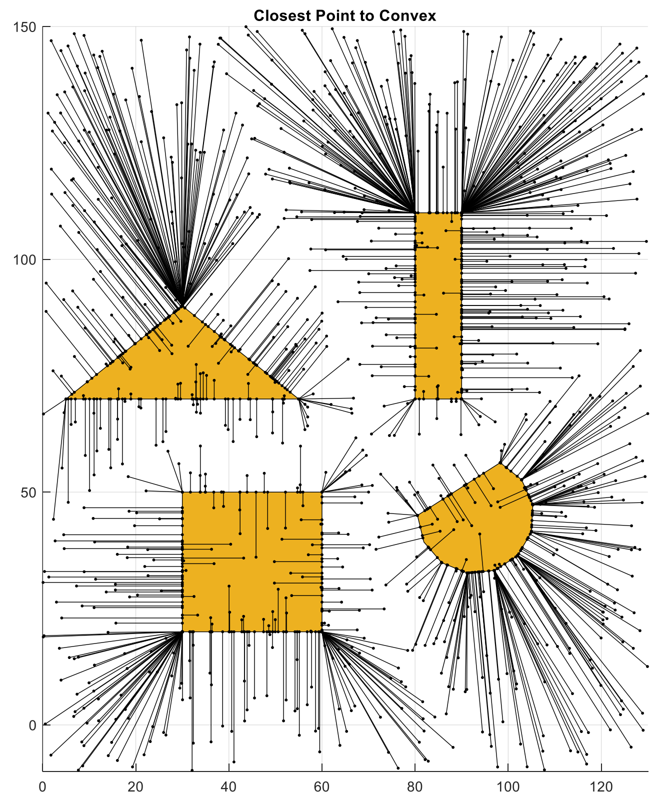
Equivalent Force on the Frame F

If external force applied on point p ref to global frame is: $f = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$

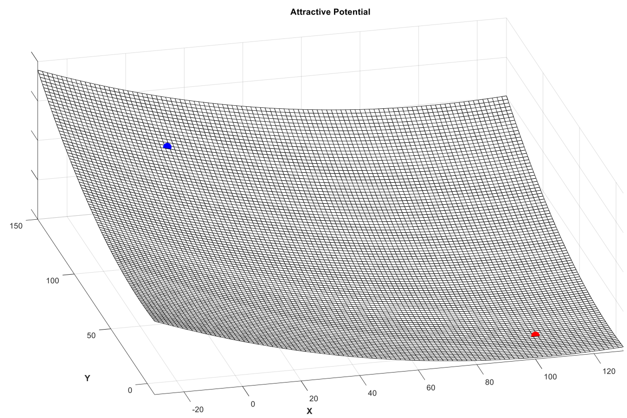
Then the equivalent force on Frame F is:

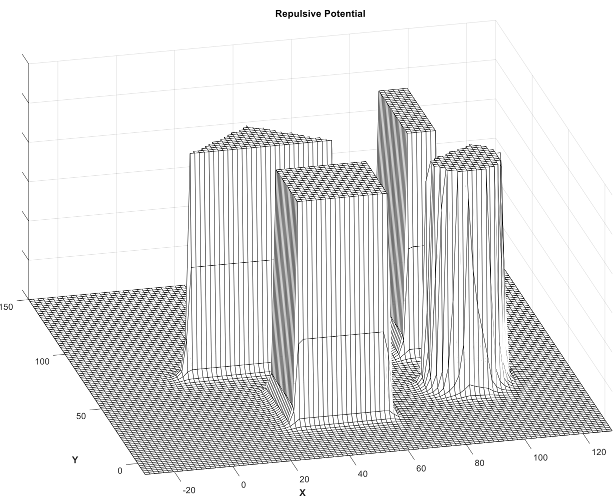
$$\begin{bmatrix} \text{Force}_x \\ \text{Force}_y \\ \text{Torque}_z \end{bmatrix} = J^T \cdot \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{pmatrix} f_x \\ f_y \\ f_y\left(x\cos(\theta) - y\sin(\theta)\right) - f_x\left(y\cos(\theta) + x\sin(\theta)\right) \end{pmatrix}$$

Find Closest Point on Obstacles to Control Points:

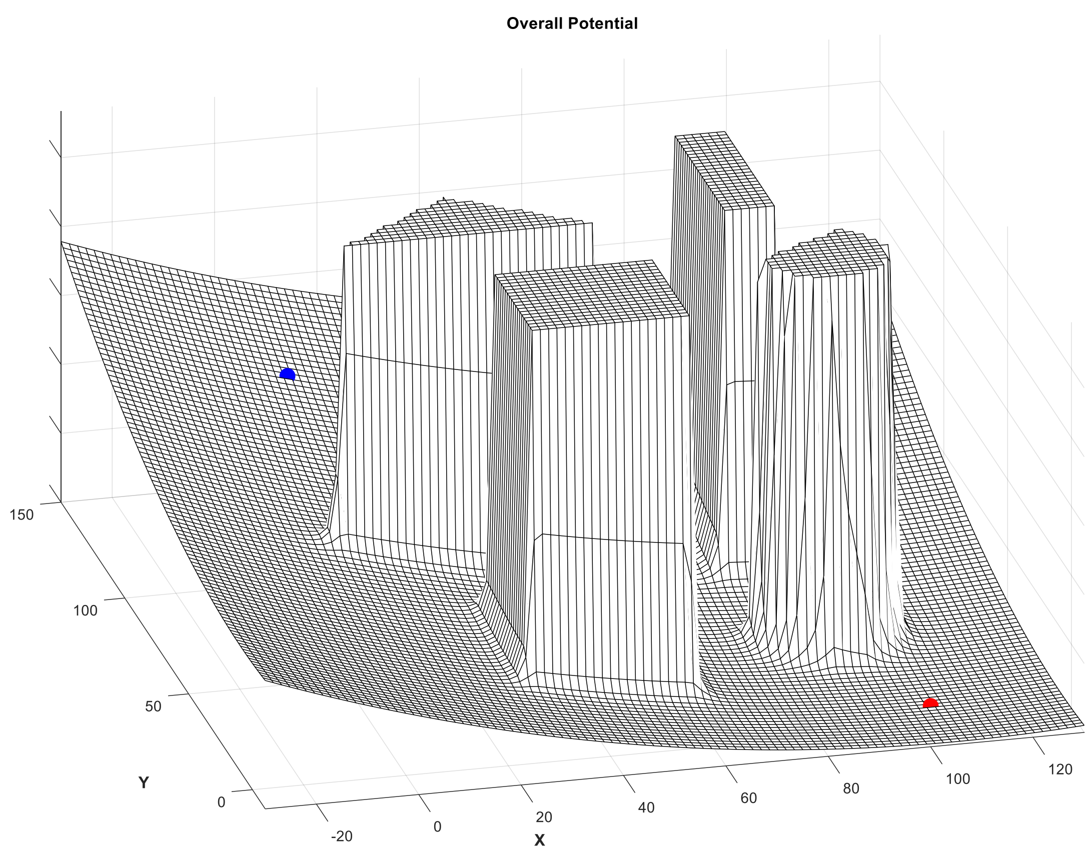


Attractive / Repulsive Potential

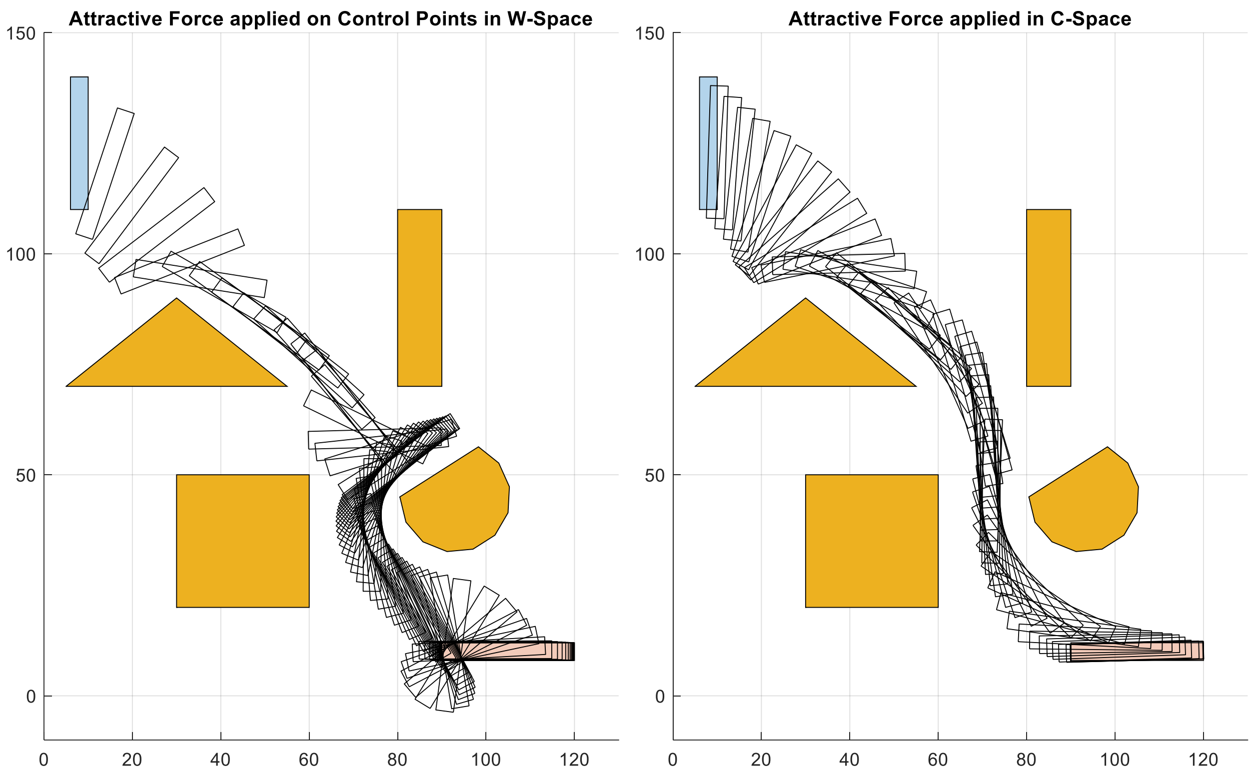




Overall Potential



Attractive Potential can be applied on both work space and configuration space:



Tilted Attractive Potential

Define the Folding Axis with 2 point, one of those is the goal point:

$$\text{axis} = \begin{bmatrix} q_{\text{tilt}} & q_G \end{bmatrix}$$

Distance from another point to the axis:

$$d = (q - q_{\text{tilt}}) \times \frac{q - q_{\text{tilt}}}{\| q - q_{\text{tilt}} \|}$$

d can be positive or negative, determined in which half plane.

Potential for the Half Plane:

$$U_{\text{att}} = \frac{1}{2} \cdot \zeta \cdot \| q_G - q \|^2 \cdot \left[c_2 \cdot \frac{d}{c_3} + 1 \right]$$

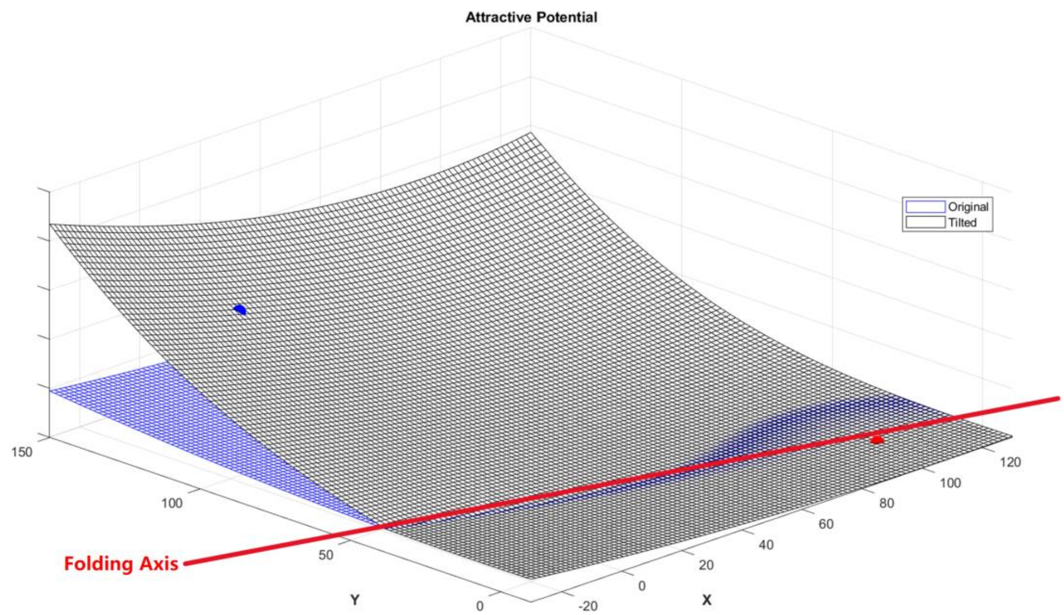
In order to keep the potential field continuous, using 2nd order instead 1st order:

$$U_{\text{att}} = \frac{1}{2} \cdot \zeta \cdot \| q_G - q \|^2 \cdot \left[c_2 \cdot \left(\frac{d}{c_3} \right)^2 + 1 \right]$$

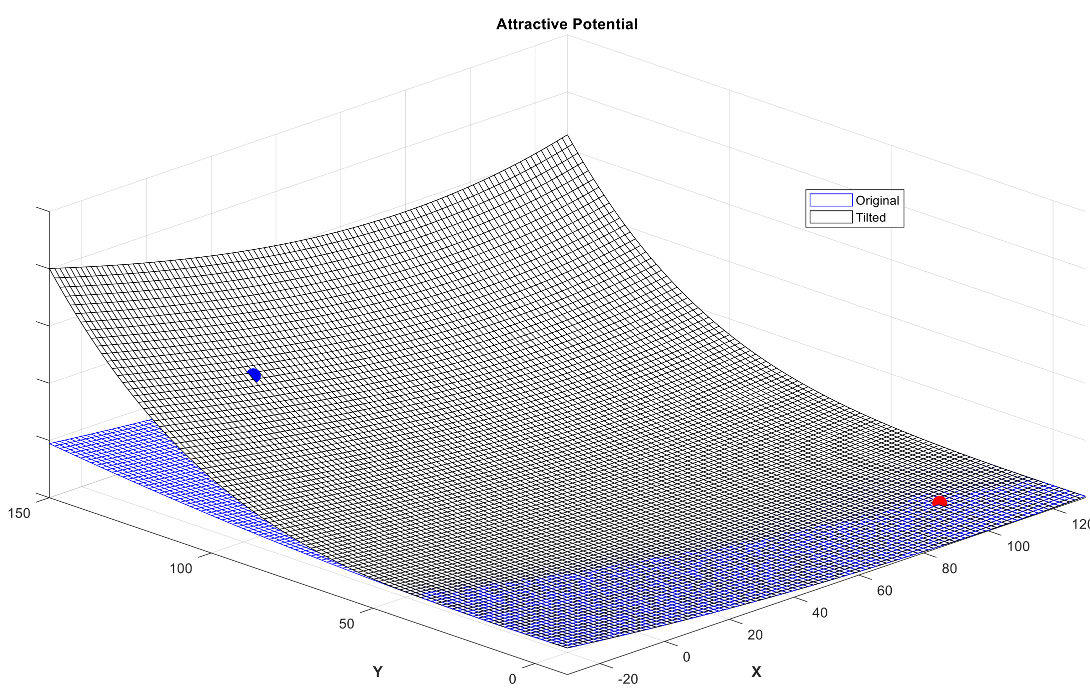
Where c_3 is a reference distance, when the distance from point to axis equal to c_3, the 2nd order term $\left(\frac{d}{c_3} \right)^2$ equal to 1.

c_2 is the magnitude of the "folding angle".

Tilted Attrctive Force (1st Order)



Tilted Attrctive Force (2nd Order)



Path using the Tilted Attractive Potential