

pPCA

Model:

$$X = W \cdot Z + \epsilon$$

- X : Data in High dimension
- Z : X projected to Low dimension
- W : Linear Transformation Matrix
- ϵ : noise

Constrain:

- X : gaussian distribution
- Z : gaussian distribution with $mean(Z) = 0$ and $cov(Z) = I$
- ϵ : gaussian distribution with $mean(\epsilon) = 0$ and $cov(\epsilon) = I \cdot s_0$

Goal: find Linear Transformation Matrix W

Create Data

```
clc; clear
[u,~,~] = svd(rand(3,4), 'econ');
S = diag(rand(3,1)*10)+eye(3)*0.5;
X = u*S*randn(3,1000);
X = X + randn(size(X))/100;
```

pPCA

```
W0 = rand(3,2);
s0 = 0.1;
[W, mu, para] = ppca_em(X, 2, 1e-6, 1000, W0, s0);
```

Project to 2D

```
Z_2d = W\ (X-mean(X,2));
```

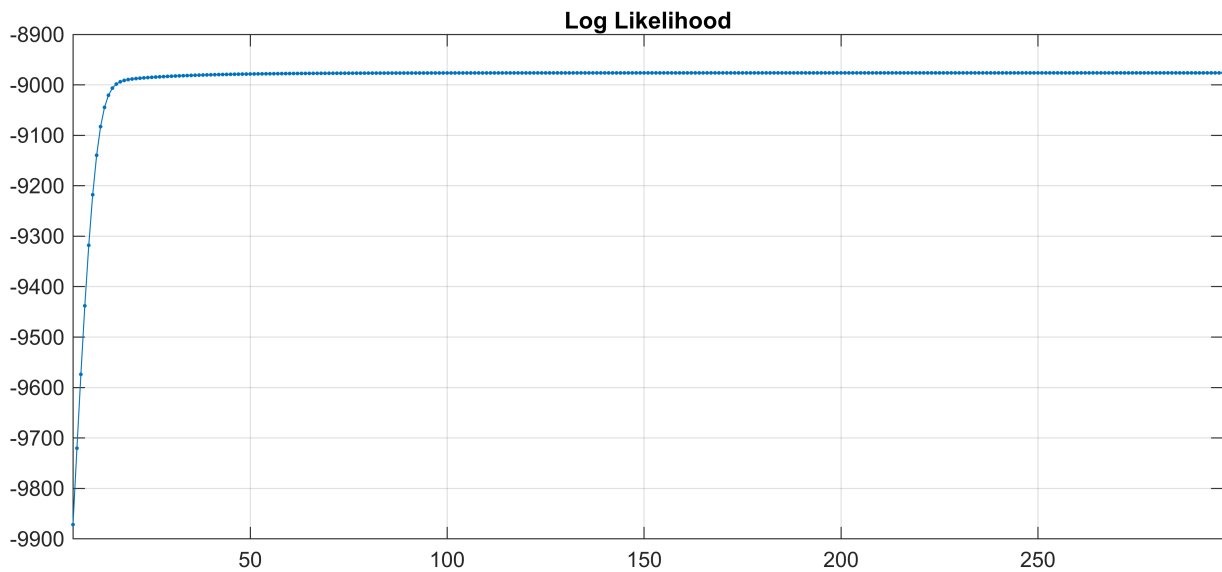
Check Projection Cov Matrix (expect close to I)

```
sig = Z_2d*Z_2d'/size(Z_2d,2)
```

```
sig = 2x2
    1.0135   -0.0023
   -0.0023    1.0146
```

Plot Log Likelihood

```
f = figure;
plot(5:para.iter,para.loglk_all(5:end),'.-');
grid on; title('Log Likelihood')
xlim([5,para.iter])
set(f,'Units','normalized','Position',[0,0,0.6,0.4])
```



Plot pPCA Axis

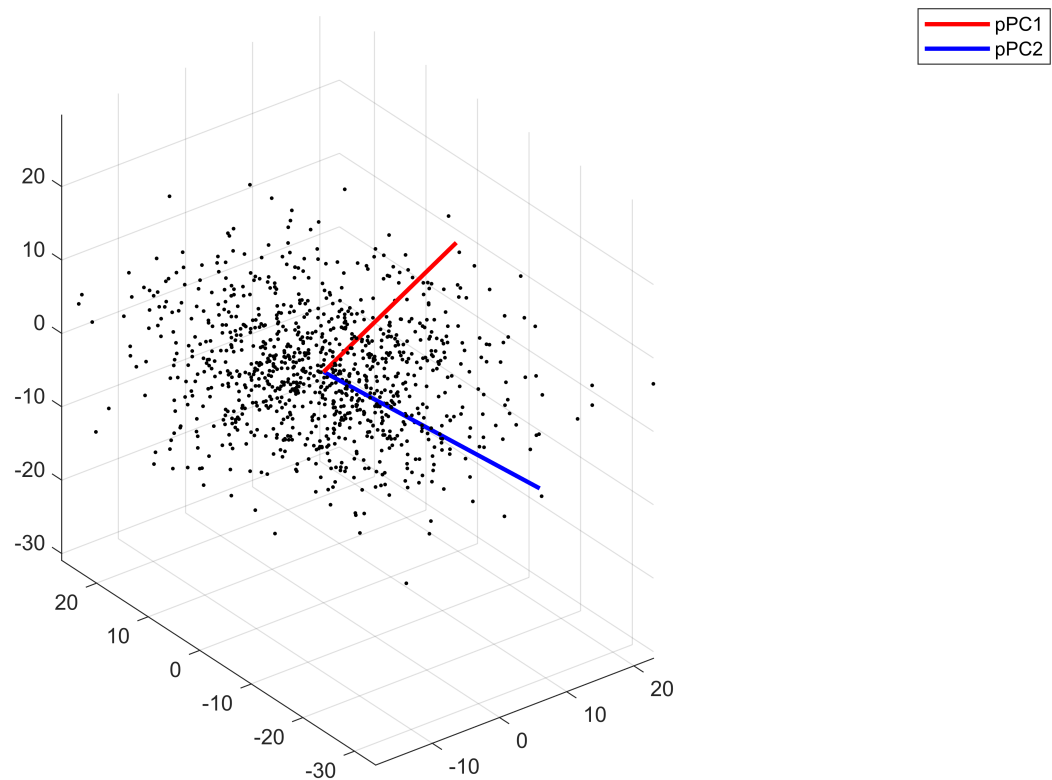
```
ppc1 = [zeros(3,1),W(:,1)]*3 + mu;
ppc2 = [zeros(3,2),W(:,2)]*3 + mu;

f = figure;
plot3(X(1,:),X(2,:),X(3,:),'.k'); hold on
p2 = plot3(ppc1(1,:),ppc1(2,:),ppc1(3,:),'-r','LineWidth',2); hold on
p3 = plot3(ppc2(1,:),ppc2(2,:),ppc2(3,:),'-b','LineWidth',2); hold on
grid on; axis equal
```

```

legend([p2,p3],{'pPC1','pPC2'})
set(f,'Units','normalized','Position',[0,0,0.6,0.6])

```



Plot 2D Projection

We expect Z is zero mean and Cov=1

```

elip = cov2ellip(sig,3,mean(Z_2d,2),50); % 3 sigma circle, I expect it close to r = 3
elip_ref = cov2ellip(eye(2),3,[0;0],50);

f = figure;
plot(Z_2d(1, :), Z_2d(2, :), '.k'); hold on
p2 = plot(elip(1,:),elip(2,:), '-','LineWidth',1); hold on
p3 = plot(elip_ref(1,:),elip_ref(2,:), ':','LineWidth',1); hold on
grid on; axis equal; title('2D Projection')
legend([p2,p3],{'3\sigma circle','ref: r = 3'})
set(f,'Units','normalized','Position',[0,0,0.6,0.6])

```

