24/07/24 UNIT-I Probability and Random Variable

LMIA → event

$$P(A) = \frac{n(A)}{n(S)} \rightarrow \text{cardinality}$$

$$0 \leq P(A) \leq 1$$

$$^nC_r = \frac{n!}{(n-r)!\,r!} \qquad {}^nC_1 = n$$

$$^nP_r = \qquad {}^nC_0 = 1$$

$$^nC_n = 1$$

$$^nC_{n-1} = n$$

$$^5C_2 = \frac{5!}{(5-2)!\,2!}$$

$$^5C_2 = \frac{5 \times 4}{2 \times 1} = \frac{5!}{3! \times 2!} = \frac{5 \times 4 \times \cancel{3} \times \cancel{2} \times 1}{\cancel{3} \times \cancel{2} \times 1 \times 2 \times 1} = 10$$

$$= 10$$

Properties:

Multiplication Property

$$P(A \cap B) = P(A) \times P(B)$$

A and B are independent

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1. A commity of 5 members drawing 8 boys 6 girls. Find the probability the committee will consist 2 boys and 3 girls.

$$\rightarrow P(A) = \frac{^8C_2 \times {}^6C_3}{^{14}C_5}$$

$$= \frac{28 \times 20}{2002} = \frac{560}{2002}$$

$$= 0.28.$$

2. In a city 60% people read news paper A. 40% people read B, 30% read C, 20% read A & B, 30% read A & C, 10% read B & C, 15% read A & B & C.

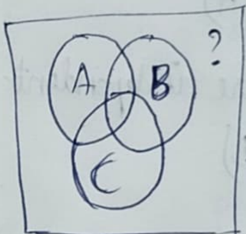
$\frac{60}{100}$

→ $P(A) = 0.6$ $\quad P(A \cap C) = 0.3$

$P(B) = 0.4$ $\quad P(B \cap C) = 0.1$

$P(C) = 0.3$ $\quad P(A \cap B \cap C) = 0.15$

$P(A \cap B) = 0.2$ $\quad P(\overline{A \cup B \cup C}) = ?$



$$P(\overline{A \cup B \cup C}) = 1 - P(A \cup B \cup C)$$

$$= 1 - [P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) + P(B \cap C) + P(A \cap B \cap C)]$$

$$= 1 - [0.6 + 0.4 + 0.3 - 0.2 - 0.3 - 0.1 + 0.15]$$

$$= 1 - [1.3 - 0.6 + 0.15]$$

$$= 1 - [0.7 + 0.15]$$

$$= 1 - [0.85]$$

$$= 0.15$$

Conditional Probability

$P(A/B)$ — condition that B has already occurred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

given

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A/B) \times P(B)$$
$$= P(B/A) \times P(A)$$

If events A & B are independent

$$P(A \cap B) = P(A) \times P(B)$$

1. In a shooting test prob of hitting target is $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(C) = \frac{3}{4}$.

Find the probability of
(i) none of them hits the target
(ii) atleast one of them hit
(iii) exactly one of them hit
(iv) all of them hit the target.

→

The events are independent

(i) $P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$

$P(\bar{A}) = 1 - P(A)$ $\quad P(\text{none}) = \frac{1}{24}$

and → $\times$
or → $+$

(ii) $P(\text{atleast one}) = P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C) + P(\bar{A} \wedge \bar{B} \wedge \bar{C}) + P(A \cap \bar{B} \wedge C) + P(\bar{A} \wedge B \wedge C) + P(A \cap B \wedge \bar{C})$

$= 1 - P(\bar{A} \wedge \bar{B} \wedge \bar{C})$

$= 1 - \frac{1}{24} = \frac{23}{24}$.

(iii) $P(A \wedge \bar{B} \wedge \bar{C}) + P(\bar{A} \wedge B \wedge \bar{C}) + P(\bar{A} \wedge \bar{B} \wedge C)$

$= P(A) \times P(\bar{B}) \times P(\bar{C}) + P(\bar{A}) \times P(B) \times P(\bar{C}) + P(\bar{A}) \times P(\bar{B}) \times P(C)$

$= \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right)$

$= \frac{1}{24} + \frac{2}{24} + \frac{3}{24}$

$= \frac{6}{24} = \frac{1}{4}$.

(iv) $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{6}{24} = \frac{1}{4}$.

2. A bag contains 15 items of which 4 are defective. Items are selected at random one by one examined. The ones examined are not put back. What is the chance that 10^{th} one examined is the last defective

→

$$P(A) = P\left(\begin{array}{c}\text{6 good and 3} \\ \text{defective in nine} \\ \text{items}\end{array}\right) = \frac{{}^{11}C_6 \times {}^4C_3}{{}^{15}C_9}$$

$= \dfrac{\dfrac{11\times10\times9\times8\times7\times6!}{6!}\times 4}{\dfrac{15\times14\times13\times12\times11\times10\times\cancel{9!}}{9!}}$

$= 24/65$

$= 0.3692$

$P(B) = P(10^{th} \text{ item is defective}) = \frac{1}{6}$.

$P(B/A) =$

$\cancel{P(B/A) = \frac{P(A\cap B)}{P(A)}}$.

$P(A\cap B) = P(A) \times P(B/A)$

$= \frac{24}{65} \times \frac{1}{6} = \frac{4}{64}$

3. In a box 4 bad and 6 good tubes. Two are drawn out at a time one is tested and it is good. What is the probability that the other one is also good.

→ $\underset{\text{both good}}{P(A\cap B)} = \frac{6C_2}{10C_2} = \frac{\cancel{6}^{3}\times\cancel{8}/2\times1}{\cancel{10}\times\cancel{9}_{3}/2\times1}$

$= \frac{1}{3}$

$= 0.333$

$P\left(\begin{array}{c}\text{getting one}\\ \text{tube is good}\end{array}\right) = \cancel{\frac{1}{6}}\ \frac{6C_1}{10C_1} = \frac{6}{10}$

$P\left(\begin{array}{c}\text{getting one tube}\\ \text{is good}\end{array}\right) = P(B/A)$

$= P(A\cap B)/P(A)$

$= \frac{1/3}{6/10} = \frac{5}{9}$.

LM11 Total Probability & Bayes Theorem

$\text{Total probability} = \sum_{i=1}^{n} P(x_i)\,P(x_i/D)$

$\text{Bayes theorem} = \dfrac{P(A)\,P(A/D)}{\sum_{i=1}^{n} P(x_i)\,P(x_i/D)}$

~~probability that the~~

Bag 1 contains 2 red and 1 black balls. Bag 2 contains 3 red and 2 black balls. What is the probability that a ball drawn from one of the bag is red.

→ Total probability $= P(A) \times P(red/A) + P(B) \cdot P(red/B)$

$= \frac{1}{2} \times \frac{2C_1}{3C_1} + \frac{1}{2} \times \frac{3C_1}{5C_1}$

$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{3}{5}$

$= \frac{1}{3} + \frac{3}{10}$

$= \frac{19}{30}$

2. A factory production line is manufacturing bolt using three machines. Machine A is responsible for 25%, machine B 35% and machine C is at rest. 5% output from A is defective, 4% from B is defective, 2% from 'C' is defective. A bolt is choosen

at random from the production line and found to be defective What is the probability that it came from A, B, C?

$\rightarrow$

$P(A) = \frac{25}{100} = 0.25$

$P(D/A) = 0.05$

$P(B) = \frac{35}{100} = 0.35$

$P(D/B) = 0.04$

$P(C) = 1 - \frac{25}{100} - \frac{35}{100} = 0.4$

$P(D/C) = 0.02$

i) $P(D) = \left(\frac{25}{100} \times \frac{5}{100}\right) + \left(\frac{35}{100} \times \frac{4}{100}\right) + \left(\frac{40}{100} \times \frac{2}{100}\right)$

$= \frac{5}{400} + \frac{7}{500} + \frac{4}{500}$

$= 0.0345$

(ii) $P(A/D) = \frac{P(A)\,P(D/A)}{0.0345} = \frac{0.0125}{0.0345}$

$= 0.3623$

(iii) $P(B/D) = \frac{P(B) \times P(D/B)}{\sum P(x_i)\,P(D/x_i)}$

$= \frac{0.35 \times 0.04}{0.035} =$

(iv) $P(C/D) = \frac{0.4 \times 0.02}{0.0345} = 0.232.$

3. Bucket A contains 5 red marbles 4 blue marbles

Bucket B contains 7 red marbles and 5 blue marbles

One bucket selected at random and a market B drawn from A. If the marble is red. Find the probability the marble is in A.

$\rightarrow$

$P(A) = \frac{1}{2}$; $P(B) = \frac{1}{2}$

$P(R/A) = \frac{^5C_1}{^9C_1} = \frac{5}{9}$;

$P(R/B) = \frac{^7C_1}{^{12}C_1} = \frac{7}{12}$

$P(\text{red}) = P(A) \times P(R/A) + P(B) \cdot P(R/B)$

$= \left(\frac{1}{2} \times \frac{5}{9}\right) + \left(\frac{1}{2} \times \frac{7}{12}\right)$

$= \frac{5}{18} + \frac{7}{24}$

$= 0.5694$

By Bayes theorem

$P(A/R) = \frac{5/18}{0.5694}$

$= 0.4878$

31/07

Random variable

Discrete — Continuous

LM!!! Discrete
↓
Finite
Countable

x	0	1	2	3 ... n
P(x)	P(0)	P(1)	P(2)	P(3) ...

Two coins toss.

X = prob of getting head

$S = \{HH, HT, TH, TT\}$

X	0	1	2
P(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$= 1$

$\text{Mean} = \sum_{i=1}^{n} x_i P(x_i) = \sum x p(x)$

↳ $E(x) \to$ Expectation

$\text{Variance} = E(x^2) - (E(x))^2$

↳ $E(x^2) = \sum x^2 p(x)$

Properties

1. $\sum P(x_i) = 1$
2. $P(x) \geq 0$
3. $P(x) \leq 1$
4. No negative values

1. A random variable X has the following probability function values

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i). Find the value of k

ii) Compute $P(X<6)$, $P(X \geq 6)$, $P(0<x<5)$

iii) If $P(x \leq \lambda) > \frac{1}{2}$ then find the minimum value

iv) Compute $P\left(\frac{1.5<x<4.5}{x>2}\right)$

v) Mean

vi) Variable

vii) Cdf (Cumulative distribution function)

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

⟶

By property,

$\sum p(x) = 1$

$10k^2 + 9k = 1$

$10k^2 + 9k - 1 = 0$

$k = \frac{-9 \pm \sqrt{81+40}}{20}$

$= \frac{-9 \pm 11}{2}$

$= \frac{-20}{20}, \frac{2}{20}$

$= -1 + \frac{1}{10}$

ii)

x	0	1	2	3	4	5	6	7
$P(x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{3}{100}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$
cdf	0	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{8}{10}$	$\frac{81}{100}$	$\frac{83}{100}$	1

(ii) $P(x<6) = 1 - P(x \geq 6)$

$= 1 - [P(x=6) + P(x=7)]$

$= 1 - \frac{19}{100}$

$= \frac{81}{100}$

$P(x \geq 6) = \frac{19}{100}$

$P(0 < x < 5) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} = \frac{8}{10}$

$= \frac{4}{5}$

(iii) $k = 4$

iv) $P(1.5 < x < 4.5 / x > 2)$

$= \frac{P(1.5 < x < 4.5 \cap x > 2)}{P(x > 2)}$

$= \frac{P(x=3) + P(x=4)}{1 - P(x \leq 1)}$

$= \frac{\frac{2}{10} + \frac{3}{10}}{1 - \frac{1}{10}}$

$= \frac{5/10}{9/10} = \frac{5}{9}$

v) Mean:

$E(x) = \sum x P(x)$

$= 0 \times 0 + 1 \times \frac{1}{10} + 2 \times \frac{2}{10} + 3 \times \frac{2}{10} + 4 \times \frac{3}{10} + 5 \times \frac{1}{100} + 6 \times \frac{2}{100} + 7 \times \frac{17}{100}$

$= \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{119}{100}$

$= \frac{10+40+60+120+5+12+119}{100}$

$= \frac{366}{100} = 3.66$

vi) Variance: $Var(x) = E(x^2) - (E(x))^2$

$E(x^2) = \sum x^2 p(x)$

$= 0 \times 0 + 1 \times \frac{1}{10} + 4 \times \frac{2}{10} + 9 \times \frac{2}{10} + 16 \times \frac{3}{10} + 25 \times \frac{1}{100} + 36 \times \frac{2}{100} + 49 \times \frac{17}{100}$

$Var(x) = 16.8 - (3.66)^2$

$= 16.8 - 13.4$

$= 3.4$

Note

Prob

"

"

"

R.V.

Conti

$\int_{-\infty}^{\infty}$

E

Var

Note:

Prob that atmost 6 = $P(X \leq 6)$

" atleast 6 = $P(X \geq 6)$

" greater than 6 = $P(X > 6)$

" less than 6 = $P(X < 6)$

RV → LM IV

continuous

↓

Probability density function

$$f(x) = \begin{cases} a & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$-\infty$ ———————— ∞

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$P(0 < x < 1) = \int_0^1 f(x)\,dx.$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$\text{Variance} = f(x) = E(x^2) - E(x)^2$$

1. A random variable defined as pdf

$$f(x) = \begin{cases} ax & 0 \leq x \leq 1 \\ a & 1 \leq x \leq 2 \\ 3a - ax & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{cdf} = \int_{-\infty}^{x} f(x)\,dx$$

i) Find the value of a

ii) Find cdf

iii) Compute $P(x < 1.5)$

⟹ Sol:

By property $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Here, $\cancel{\int_{-\infty}^{0}} + \int_0^1 + \int_1^2 + \int_2^3 + \int_3^{\infty} f(x)\,dx = 1$

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 3a - ax\,dx = 1$$

$$\therefore\ \frac{ax^2}{2}\Big|_0^1 + ax\Big|_1^2 + 3ax - \frac{ax^2}{2}\Big|_2^3 = 1$$

$$\left[\left\{\frac{a}{2}\right\} - \{0\}\right] + \left[\{2a\} - \{a\}\right] + \left[\left\{9a - \frac{9a}{2}\right\} - \left\{6a - \frac{4a}{2}\right\}\right] = 1$$

$$0.5a + a + 9a - 4.5a - 4a = 1$$

$$10a - 4a - 4a = 1$$

$$2a = 1$$

$$a = 1/2$$

ii) Cdf $F(x) = \int_{-\infty}^{x} f(x)\,dx$

cumulative distributive function

if x lies in b/w 0 to 1

$$F(x) = \cancel{\int_{-\infty}^{0}} + \int_0^x f(x)\,dx$$

$$= \int_0^x \frac{1}{2} x\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^x$$

$$= \frac{x^2}{4}.$$

if x lies in b/w 1 to 2

$$F(x) = \int_{-\infty}^{0} + \int_0^1 + \int_1^x f(x)\,dx$$

$$= 0 + \int_0^1 \frac{x}{2}\,dx + \int_1^x \frac{1}{2}\,dx$$

$$= \frac{x^2}{4}\Big|_0^1 + \frac{x}{2}\Big|_1^x$$

$$= \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$$

$$= \frac{x}{2} - \frac{1}{4}.$$

if x lies in b/w 2 to 3

$$F(x) = \int_{-\infty}^{0} + \int_0^1 + \int_1^2 + \int_2^x f(x)\,dx$$

$$= 0 + \frac{1}{4} + \int_1^2 \frac{1}{2}\,dx + \int_2^x \frac{3}{2} - \frac{x}{2}\,dx$$

$$= \frac{1}{4} + \frac{x}{2}\Big|_1^2 + \left(\frac{3x}{2} - \frac{x^2}{4}\right)\Big|_2^x$$

$$= \frac{1}{4} + 1 - \frac{1}{2} + \frac{3x}{2} - \frac{x^2}{4} - \left\{\frac{6}{2} - \ldots\right.$$

$$= \frac{1}{4} + 1 - \frac{1}{2} - \frac{3x}{2} - \frac{x^2}{4} - 2$$

$$= -\left(\frac{5}{4} + \frac{3x}{2} + \frac{x^2}{4}\right)$$

if x lies above 3

$$\text{cdf} = \begin{cases} \dfrac{x^2}{4} & 0 \le x \le 1 \\[4pt] \dfrac{x}{2} - \dfrac{1}{4} & 1 \le x \le 2 \\[4pt] -\left(\dfrac{5}{4} + \dfrac{3x}{2} + \dfrac{x^2}{4}\right) & 2 \le x \le 3 \\[4pt] 1 & 3 < x < \infty \end{cases}$$

LM:5 MGF

06/09

Discrete random variable = summation

Continuous random variable : integration

distance
time

$$P(a < x < b) = \int_a^b f(x)\,dx$$

Total probability = 1

1. The mileage x (in thousand of miles) which car owners get with a certain kind of tyre is a random variable having probability density

$$f(x) = \begin{cases} \frac{1}{20} e^{-x/20}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Find the probability that one of these tyres will last.

i) at most 10,000 miles

ii) anywhere from 16,000 to 24,000.

(iii) atleast 30,000 miles

Ans: $P(a<x<b) = \int_a^b f(x)\,dx$

i) $P(X \leq 10) = P(-\infty < x \leq 10)$

$$= \int_{-\infty}^{10} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_0^{10} f(x)\,dx$$

$$= 0 + \int_0^{10} \frac{1}{20} e^{-x/20}\,dx$$

$$\cancel{\frac{1}{20} e^{-10}}$$

$$= \frac{1}{20}\left(\frac{e^{-x/20}}{-1/20}\right)_0^{10}$$

$$= -\left(e^{-x/20}\right)_0^{10}$$

$$= -\left(e^{-1/2} - 1\right)$$

$$= 1 - e^{-1/2}$$

$$= 0.3935.$$

(ii) $P(16 < x < 24) = \int_{16}^{24} f(x)\,dx$

$$= \int_{16}^{24} \frac{1}{20} e^{-x/20}$$

$$= \frac{1}{20}\left[\frac{e^{-x/20}}{-1/20}\right]_{16}^{24}$$

$$= -\left[e^{-x/20}\right]_{16}^{24}$$

$$= -\left[e^{-24/20} - e^{-16/20}\right]$$

$$= -\left[0.3012 - 0.4493\right]$$

$$= 0.1481.$$

(iii) $P(x \geq 30) = \int_{30}^{\infty} f(x)\,dx$

$$= \int_{30}^{\infty} \frac{1}{20} e^{-x/20}\,dx$$

$e^{-\infty} = 0$

$e^{0} = 1$

$e^{\infty} = \infty$

$$= \frac{1}{20}\left(\frac{e^{-x/20}}{-1/20}\right)_{30}^{\infty}$$

$$= \cancel{-(e^{-\infty}} = -\left[e^{-x/20}\right]_{30}^{\infty}$$

$= -[e^{\infty} - e^{-3/2}]$

$= -[0 - 0.223]$

$= 0.223$

Bayes Theorem

In probability and statistics, it describes the probability of an event, based on prior knowledge of conditions related to event.

Formula:

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B)$$

$$= P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)$$

A company has two plants to manufacture scooters. Plant I manufactures 80% of the scooters and plant II manufactures 20%. At plant I; 85 out of 100 scooters are rated standard quality or better. At plant II only 65 out of 100 scooters are rated standard quality or better.

i). What is the probability that a scooter selected at random came from plant I if it is known that the scooter is of standard quality?

ii). What is the probability of that the scooter came from plant II if it is known that the scooter is of standard quality

$\rightarrow$

80% plant - I

$$P(A_1) = 80\% = \frac{80}{100} = 0.8$$

20% plant - II

$$P(A_2) = 20\% = \frac{20}{100} = 0.2$$

85 out of 100 from plant I is standard

$$P(B/A_1) = \frac{85}{100} = 0.85$$

65 out of 100 from plant - II is standard.

$$P(B/A_2) = \frac{65}{100} = 0.65$$

i) Probability of scooter from plant - I :

$$P(B) = P(A_1)\,P(B/A_1) = 0.8\,(0.85)$$

$$= 0.68$$

ii). Probability of scooter from plant - II :

$$P(B) = P(A_2)\, P(B/A_2)$$
$$= (0.2)\,(0.65)$$
$$= 0.13$$

Probability of an event is quantified as a number b/t 0 and 1

0 indicates impossibility
1 indicates certainty

Unbiased → Proper

$$P(A) = \frac{n(A)}{n(S)}$$

The conditional probability of A given B, is the probability of event A occurs, provided event B has already occured

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \text{ and } B)$$

Bayes theorem describes the probability of an event, based on conditions that might be related to an event

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$

→ deals with conditional probability

$P(A/B)$ is the conditional probability of event A occuring, given that B is true

$P(B/A)$ is the conditional probability of event B occuring given that A is true

$P(A)$ and $P(B)$ are the probabilities of A and B occuring independently

1. There is a cricket match tomorrow. In recent years, it has rained only 5 days each year. Unfortunately, the meteorologist has predicted the rain for tomorrow. When it rains, the meteorologist correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the match-day?

→

Sample space = Two mutually exclusive events - it rains or it does not rain.

Third event → Meteorologist predicts rain.

Event A_1 : It rains on the matchday
Event A_2 : It does not rain on match day
Event B : meteorologist predicts rain

$P(A_1) = 5/365 = 0.0136$ [Rains 5 days in a yr]
$= 0.014$

$P(A_2) = 360/365 = 0.986$ [No rain for 360 days]

$P(B/A_1) = 0.9$ [Meteorologist predicts rain 90% of the time when it rains]

$P(B/A_2) = 0.1$ [Meteorologist predicts rain 10% of the time when it does not rain]

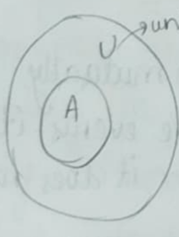
Probability it will rain on the matchday:

By Bayes theorem:

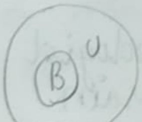
$$P(A1/B) = \frac{P(A_1) \cdot P(B/A_1)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)}$$

$$P\left(\frac{A1}{B}\right) = \frac{(0.014)(0.9)}{(0.014)(0.9) + (0.986)(0.1)}$$

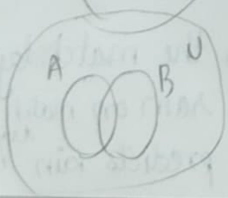
$= 0.111$



$P(A) = \frac{A}{U}$

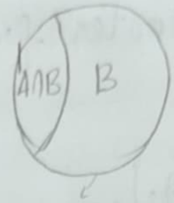


$P(B) = \frac{B}{U}$



$P(A \cap B) = \frac{A \cap B}{U}$

Conditional probability



B becomes universe

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$P(A \cap B) = P(A/B) * P(B)$
$P(A \cap B) = P(B/A) * P(A)$

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$ ← Bayes Theorem

2. Each user has a daily login probability of 1/4. If log in on a day and on the previous day, probability of a user spending more than 5 mins on a webpage is 4/5. If log in on a day and did not log in on the previous day, probability of a user spending more than 5 mins on a page is 1/3.

Given that a user spent more than 5 mins on the webpage today, what is the probability that user logged in both today and yesterday?

$$P(\text{login}) = \frac{1}{4}$$

$$P(>5\text{min} \mid \text{login yesterday and today}) = \frac{4}{5}$$

$$P(>5\text{min} \mid \text{login today, not yesterday}) = \frac{1}{3}$$

$P\left(\text{login yesterday \& today} / >5\text{min}\right) = ?$

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$

$$P\left(\text{login yest \& today} / >5\text{min}\right) = \frac{P(>5\text{min}/\text{login yesterday \& today}) * P(\text{login yesterday \& today})}{P(>5\text{min})}$$

$$= \frac{\frac{4}{5} \times \left(\frac{1}{4}\right)^2}{\frac{4}{5} * \left(\frac{1}{4}\right)^2 + \frac{1}{3} * \left(\frac{1}{4} * \frac{3}{4}\right)}$$

$$= \frac{4}{9}.$$

09/09

UNIT-II

Probability distribution

Discrete — Continuous

Discrete	Continuous
Binomial distribution	Uniform
Poisson "	Exponential (service)
Geometric "	Gamma
	Normal

Binomial distribution:

P = P(success) or P(failure)

$p+q=1$

$$P(X=x) = \sum_{n=1}^{n} nC_x \, p^x q^{n-x}$$

n - no.of trials

$P(X=x)$ = exactly equal to x

$P(X\leq x)$ = atmost x

$P(X\geq x)$ = atleast x

$P(\text{atmost } 2) = P(x=0) + P(x=1) + P(x=2)$

$$nC_x = \frac{n!}{(n-x)!\,x!} = \frac{n(n-1)\ldots\cancel{(n-x)!}}{\cancel{(n-x)!}\,x!}$$

Mean = $E(x) = np$

Variance = $E(x^2) - E(x)^2 = npq$

1. The probability of a man hitting a target is 1/4.

i) If he fires 7 times, what is the probability of hitting target is atleast twice?

ii) how many times must he fire so that the probability of hitting target ~~is~~ atleast once is greater than 2/3?

Sol:

Gn, $P(\text{success}) = P = \frac{1}{4}$

$q = 1-p = \frac{3}{4}$

i) $n=7$, $P(\text{target atleast twice}) = P(X\geq 2)$

$= 1 - P(X<2)$

$= 1 - [P(X=0) + P(X=1)]$

$$= 1 - \left[7C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^7 + 7C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^6\right]$$

$= 0.5551$

ii) $P(\text{at least once}) = P(X\geq 1)$

$= 1 - P(X<1)$

$= 1 - P(X=0)$

$$= 1 - {}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n$$

$$= 1 - 1 \times 1 \times \left(\frac{3}{4}\right)^n$$

$$= 1 - \left(\frac{3}{4}\right)^n$$

$$1 - \left(\frac{3}{4}\right)^n \geq \frac{2}{3}$$

$$1 - \frac{2}{3} \geq \left(\frac{3}{4}\right)^n \qquad \frac{3}{4} = \frac{3}{4}$$

$$\frac{1}{3} \geq \left(\frac{3}{4}\right)^n \qquad \left(\frac{3}{4}\right)^2 = \frac{9}{16} \quad \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$n = 4 \qquad \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

λ = no. of arrivals

Mean $= E(x) = \lambda$

Variance $= Var(x) = \lambda$

$$P(x=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$

$$P(x<2) = P(x=0) + P(x=1)$$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!}$$

1. The number of monthly breakdown of a computer is a random variable having poisson distribution with mean equal to 1.8. Find the probability that the computer will function for a month

i) without breakdown

ii) with only one breakdown

iii) with atleast one breakdown

Sol:

$\lambda = 1.8$

i) $P(\text{without breakdown}) = P(x=0)$

$$= \frac{e^{-1.8}(1.8)^0}{0!}$$

$$= e^{-1.8}$$

$$= 0.1653$$

ii) $P(\text{only one breakdown}) = P(x=1)$

$$= \frac{e^{-1.8}\, 1.8^1}{1!}$$

$$= e^{-1.8} \times 1.8$$

$$= 0.2975$$

(iii) $P(\text{atleast one breakdown}) = P(x \geq 1)$

$$= 1 - P(x<1)$$

$$= 1 - [P(x=0)]$$

$$= 1 - 0.1653$$

$$= 0.8347$$

2. If x is a poisson distribution
$P(X=2) = 9\,P(X=4) + 90\,P(X=6)$
Find the variance.

Sol:

Apply the poisson distribution formula.

$$P(X=2) = 9\,P(X=4) + 90\,P(X=6)$$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 9 \times \frac{e^{-\lambda}\lambda^4}{4!} + 90 \times \frac{e^{-\lambda}\lambda^6}{6!}$$

$\div\, e^{-\lambda}\lambda^2$

$$\frac{1}{2} = 9\frac{\lambda^2}{24} + 90\frac{\lambda^4}{720}$$

$\times 2$

$$1 = 9\frac{\lambda^2}{12} + 90\frac{\lambda^4}{360}$$

$$1 = \frac{3}{4}\lambda^2 + \frac{1}{4}\lambda^4$$

$$4 = 3\lambda^2 + \lambda^4$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

$$\boxed{\lambda = 1}$$

1 0 3 0 -4

(or)

Trial & Error.

$\therefore$ Variance = 1.

Negative binomial distribution

$$P(X=x) = {}^{x-1}C_{k-1}\, p^k q^{x-k}$$

$x \to$ no. of trials required to produced the k success.
$k \to$ no. of success.
$p \to$ probability of success
$q \to$ probability of failure

1. A football player, his success rate of goal is 70%. What is the probability that the player third goal on his fifth attempt?

Sol:

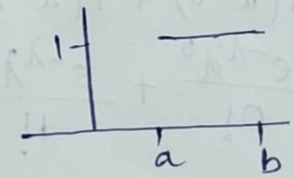
$x = 5 \quad k = 3 \quad p = 0.7 \quad q = 0.3$

$$P(X=5) = {}^4C_2 (0.7)^3 (0.3)^2 \qquad \frac{\overset{2}{\cancel{4}} \times 3}{\cancel{2} \times 1}$$

$$= 6\,(0.343)(0.09)$$

$$= 0.1852$$

Uniform distribution



$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{others} \end{cases}$$

$$\text{Mean} = \frac{b+a}{2}$$

$$\text{Variance} = \frac{(b-a)^2}{12}$$

1. The current measured in a piece wire is known to follow a uniform distribution over the interval $[0, 25]$. Find mean, variance and cdf

Sol:

$$\text{pdf } f(x) = \begin{cases} \frac{1}{25-0} & 0<x<25 \\ 0 & \text{others} \end{cases}$$

$$= \begin{cases} \frac{1}{25} & 0<x<25 \\ 0 & \text{others} \end{cases}$$

$$\text{Mean} = \frac{b+a}{2} = \frac{25+0}{2} = 12.5$$

$$\text{Variance} = \frac{(b-a)^2}{12} = \frac{25^2}{12} = \frac{625}{12}$$

$$= 52.08$$

$$\text{Cdf} = \begin{cases} 0 & x<0 \\ x/25 & 0<x<25 \\ 1 & x>25 \end{cases}$$

$\int_0^x f(x)\,dx$

2. Bus arrive at a specified stop at 15 min interval starting at 7 am. (i.e) they arrive at 7, 7.15, 7.30, 7.45 and 8.00 am. and so on. If a passenger arrives at the stop at a random time that is uniform distribution between 7 and 7.30 am. Find the probability he waits

a) less than 5 mins for a bus &

b) atleast 12 mins for a bus

Sol:

a) P(less than 5 mins for a bus)

$$= P(7.10 \text{ to } 7.15) + P(7.25 \text{ to } 7.30)$$

$$= P(10<x<15) + P(25<x<30)$$

$$= \int_{10}^{15} \frac{1}{30}\,dx + \int_{25}^{30} \frac{1}{30}\,dx$$

$$= \frac{1}{30} \int_{10}^{15} dx + \int_{25}^{30} dx$$

length b/w a and b

$$= \frac{10}{30}$$

$\iint dx\,dy = \text{area}$ $\quad = \frac{1}{3}$

$\int_a^b f(x)\,dx = \text{area}$

b) P(atleast 12 mins) =

$$= P(7.00 \text{ to } 7.03) + P(7.15 \text{ to } 7.18)$$

$$= P(0<x<3) + P(15<x<18)$$

$$= \int_0^3 \frac{1}{30}\,dx + \int_{15}^{18} \frac{1}{30}\,dx$$

$$= \frac{1}{30}(3+3) = \frac{6}{30} = \frac{1}{5}$$

Exponential distribution:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x>0, \lambda>0 \\ 0, & \text{otherwise} \end{cases}$$

Mean $E(x) = \frac{1}{\lambda}$

Variance $= \frac{1}{\lambda^2}$

Memoryless property:

$$P(x > s+t / x>s) = P(x>t)$$

1. The time required to repair a machine is exponential distribution with parameter $\lambda = \frac{1}{2}$.

i) What is the probability that the repair time exceeds 2 hours.

Sol:

$$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{1}{2}x}, & x>0 \\ 0, & \text{otherwise} \end{cases}$$

$P(x>2) = ?$

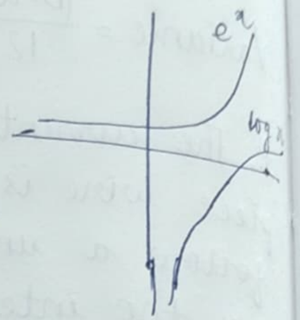
unknown time ∴

$$= \int_2^\infty \frac{1}{2} e^{-\frac{1}{2}x} dx = \frac{1}{2}\left[\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\right]_2^\infty$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$e^{-\infty} =$ $\quad e^{\infty} =$

$e^0 =$



$$= \frac{1}{2}\left[\{0\} - \{-2(e^{-1})\}\right]$$

$$= \frac{1}{2} \cdot 2 e^{-1}$$

$$= e^{-1} = 0.3479$$

ii) the conditional probability that a repair takes at least 10 hrs given that its duration exceeds 9 hrs.

$$P(x>10/x>9) =$$

$$\boxed{P(A/B) = \frac{P(A\cap B)}{P(B)}}$$

By memoryless property:

$$= P(x>9+1/x>9)$$

$$= P(x>1)$$

$$P(x>1) = \int_1^\infty \frac{1}{2} e^{-\frac{1}{2}x} dx$$

$$= \frac{1}{2}\left[\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\right]_1^\infty$$

$= \frac{1}{2} \left[\{0\} - \{-2e^{-1/2}\} \right]$

$= e^{-1/2} = 0.6065$