

8.26

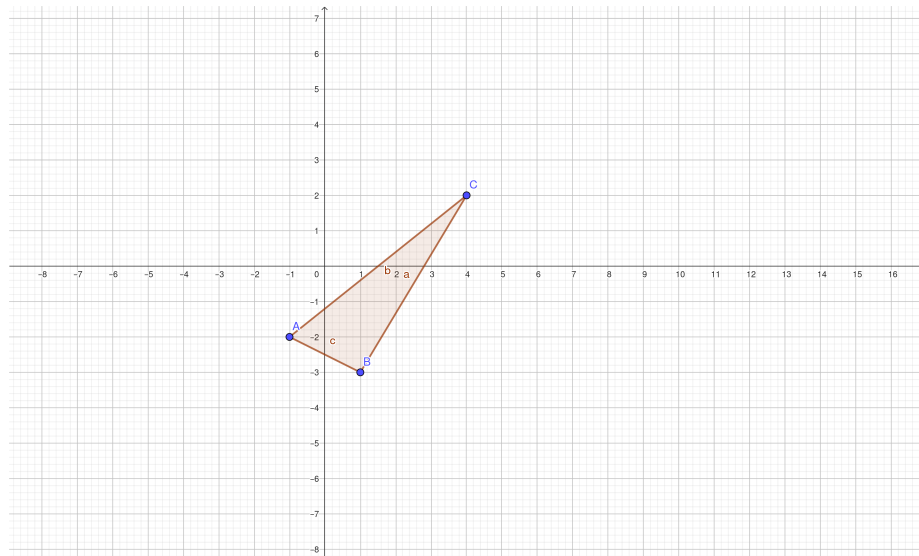
b)

Specification: Check if these points form a right angled triangle:

$$A(-1|-2)$$

$$B(1|-3)$$

$$C(4|2)$$



Definitions:

$$\vec{a} = \vec{BC}$$

$$\vec{b} = \vec{CA}$$

$$\vec{c} = \vec{AB}$$

Exercises:

$$\vec{a} \cdot \vec{b} = a_x * b_x + a_y * b_y \quad (1)$$

$$\vec{a} \cdot \vec{b} = -1 * 1 + -2 * -3 \quad (2)$$

$$\vec{a} \cdot \vec{b} = -1 + 6 \quad (3)$$

$$\vec{a} \cdot \vec{b} = 5 \quad (4)$$

$$\vec{a} \cdot \vec{c} = a_x * c_x + a_y * c_y \quad (5)$$

$$\vec{a} \cdot \vec{c} = -1 * 4 + -2 * 2 \quad (6)$$

$$\vec{a} \cdot \vec{c} = -4 + -4 \quad (7)$$

$$\vec{a} \cdot \vec{c} = -8 \quad (8)$$

$$\vec{a} \cdot \vec{c} = b_x * c_x + b_y * c_y \quad (9)$$

$$\vec{a} \cdot \vec{c} = 1 * 4 + -3 * 2 \quad (10)$$

$$\vec{a} \cdot \vec{c} = 4 + -6 \quad (11)$$

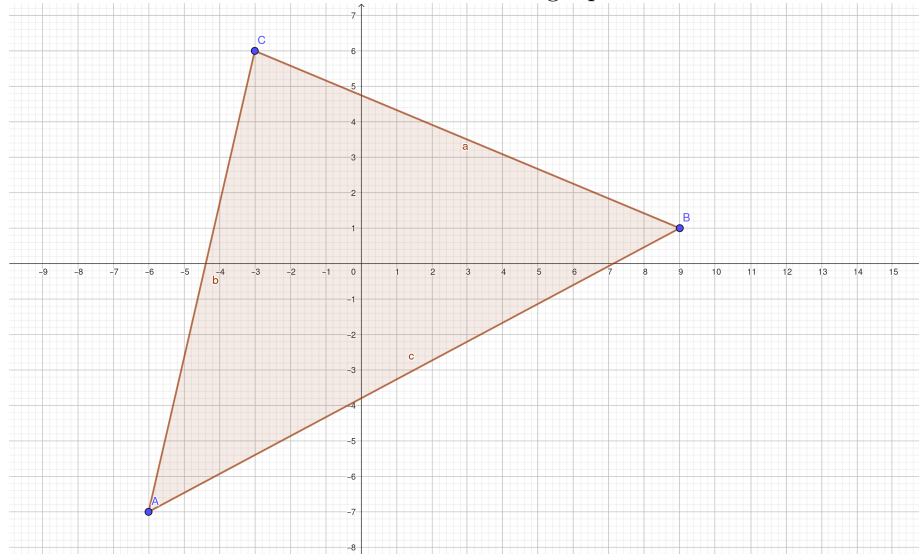
$$\vec{a} \cdot \vec{c} = -2 \quad (12)$$

Answer: The triangle is not a right angle as none of the angles α , β , γ are 90° . This is shown by the fact that none of the dot products of the triangles sides are 0.

8.29

1)

Specification: Find the mistake in the following equations:



$$\begin{aligned}\vec{c} &= \vec{AB} \\ &= \begin{pmatrix} 15 \\ 8 \end{pmatrix} \\ c &= |\vec{c}| \\ &= 17cm\end{aligned}$$

$$\begin{aligned}\vec{a} &= \vec{BC} \\ &= \begin{pmatrix} -12 \\ 5 \end{pmatrix} \\ a &= |\vec{a}| \\ &= 13cm\end{aligned}$$

$$\begin{aligned}
\cos(\beta) &= \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| * |\vec{b}|} \\
&= \frac{\begin{pmatrix} 15 \\ 8 \end{pmatrix} * \begin{pmatrix} -12 \\ 5 \end{pmatrix}}{13 * 17} \\
&= -\frac{140}{221} \\
&= -0.633...
\end{aligned}$$

$$\begin{aligned}
\beta &= \arccos(-0.633...) \\
&\approx 129.31^\circ
\end{aligned}$$

$$\begin{aligned}
A &= \frac{a * c * \sin(\beta)}{2} \\
&= \frac{13 * 17 * \sin(129.31^\circ)}{2} \\
&\approx 85.5 cm^2
\end{aligned}$$

Answer: The obtuse value of β (129.31°) was used instead of the acute value of β (50.69°). This didn't affect the result because $\sin(\beta)$ returns the same value for the obtuse and the acute value of β

$$\sin(\alpha) = \sin(180^\circ - \alpha)$$

$$\sin(129.31^\circ) = 0.77368$$

$$\sin(50.69^\circ) = 0.77368$$

8.31

Specification: An isosceles triangle ABC, which is named in the mathematically positive direction, has the base AB with $A(-2|-1)$, $B(4|7)$ and the height $h = 10E$. Calculate the missing point C, the angle ϕ which is enclosed by \vec{AB} and \vec{AC} , and the area A_{rea} of the triangle.

Definitions:

$$h = 10E$$

$$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

$$\vec{c} = \vec{AB}$$

$$\vec{c}_m = \vec{OA} + \frac{1}{2}\vec{c}$$

Exercise:

$$\vec{c} = \vec{B} - \vec{A}$$

$$\vec{c} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$\vec{c}_m = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$\vec{c}_m = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\vec{c}_m = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{c}_n = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$

$$\vec{c}_{n0} = \frac{1}{|\vec{c}_n|} \vec{c}_n$$

$$\vec{c}_{n0} = \frac{1}{\sqrt{(-8)^2 + 6^2}} \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$

$$\vec{c}_{n0} = \frac{1}{10} \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$

$$\vec{c}_{n0} = \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix}$$

$$\vec{OC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 10 \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix}$$

$$\vec{OC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}$$