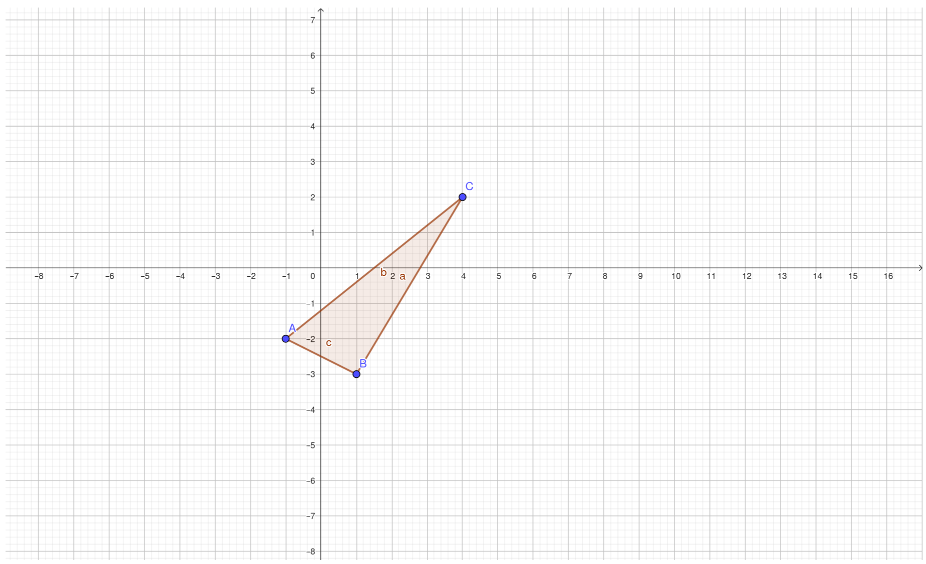
8.26

b)

Specification: Check if these points form a right angled triangle:

$$A(-1|-2) \qquad B(1|-3) \qquad C(4|2)$$



Definitions:

$$\vec{a} = \vec{BC}$$
$$\vec{b} = \vec{CA}$$
$$\vec{c} = \vec{AB}$$

Exercises:

$$\vec{a} \cdot \vec{b} = a_x * b_x + a_y * b_y \tag{1}$$
$$\vec{a} \cdot \vec{b} = -1 * 1 + -2 * -3 \tag{2}$$
$$\vec{a} \cdot \vec{b} = -1 + 6 \tag{3}$$
$$\vec{a} \cdot \vec{b} = 5 \tag{4}$$

$$\vec{a} \cdot \vec{c} = a_x * c_x + a_y * c_y \tag{5}$$
$$\vec{a} \cdot \vec{c} = -1 * 4 + -2 * 2 \tag{6}$$
$$\vec{a} \cdot \vec{c} = -4 + -4 \tag{7}$$
$$\vec{a} \cdot \vec{c} = -8 \tag{8}$$

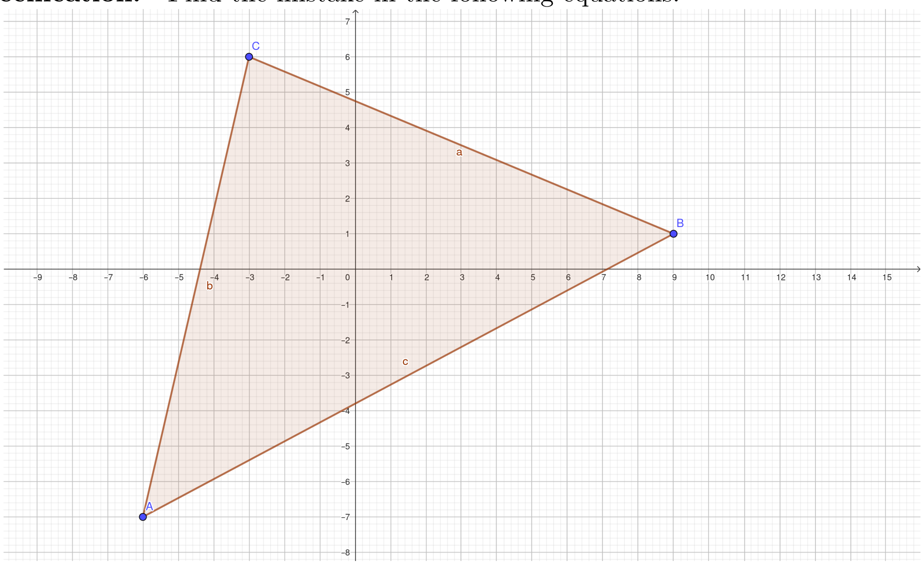
$$\vec{a} \cdot \vec{c} = b_x * c_x + b_y * c_y \tag{9}$$
$$\vec{a} \cdot \vec{c} = 1 * 4 + -3 * 2 \tag{10}$$
$$\vec{a} \cdot \vec{c} = 4 + -6 \tag{11}$$
$$\vec{a} \cdot \vec{c} = -2 \tag{12}$$

Answer: The triangle is not a right angle as none of the angles α, β, γ are $90°$. This is shown be the fact that none of the dot products of the triangles sides are 0.

8.29

1)

Specification: Find the mistake in the following equations:



$$\begin{aligned}
\vec{c} &= \vec{AB} \\
&= \begin{pmatrix} 15 \\ 8 \end{pmatrix} \\
c &= |\vec{c}| \\
&= 17cm
\end{aligned}$$

$$\begin{aligned}
\vec{a} &= \vec{BC} \\
&= \begin{pmatrix} -12 \\ 5 \end{pmatrix} \\
a &= |\vec{a}| \\
&= 13cm
\end{aligned}$$

$$
\begin{aligned}
\cos(\beta) &= \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| * |\vec{b}|} \\
&= \frac{\begin{pmatrix} 15 \\ 8 \end{pmatrix} * \begin{pmatrix} -12 \\ 5 \end{pmatrix}}{13 * 17} \\
&= -\frac{140}{221} \\
&= -0.633...
\end{aligned}
$$

$$
\begin{aligned}
\beta &= \arccos(-0.633...) \\
&\approx 129.31^\circ
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{a * c * \sin(\beta)}{2} \\
&= \frac{13 * 17 * \sin(129.31^\circ)}{2} \\
&\approx 85.5 cm^2
\end{aligned}
$$

Answer: The obtuse value of β (129.31°) was used instead of the acute value of β (50.69°). This didn't affect the result because $\sin(\beta)$ returns the same value for the obtuse and the acute value of β

$$\sin(\alpha) = \sin(180° - \alpha)$$

$$\begin{aligned} \sin(129.31°) &= 0.77368 \\ \sin(50.69°) &= 0.77368 \end{aligned}$$

8.31

Specification: An isosceles triangle ABC, which is named in the mathematically positive direction, has the base AB with $A(-2|-1)$, $B(4|7)$ and the height h = 10E. Calculate the missing point C, the angle ϕ which is enclosed by $\vec{AB}$ and $\vec{AC}$, and the area A_{rea} of the triangle.

Definitions:

$$h = 10E$$

$$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

$$\vec{c} = \vec{AB}$$

$$\vec{c_m} = \vec{OA} + \frac{1}{2}\vec{c}$$

Exercise:

$$\vec{c} = \vec{B} - \vec{A}$$
$$\vec{c} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$
$$\vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ 8 \end{pmatrix}$$
$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$
$$\vec{c_m} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{c_n} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$
$$\vec{c_{n0}} = \frac{1}{|\vec{c_n}|}\vec{c_n}$$
$$\vec{c_{n0}} = \frac{1}{\sqrt{(-8)^2 + 6^2}}\begin{pmatrix} -8 \\ 6 \end{pmatrix}$$
$$\vec{c_{n0}} = \frac{1}{10}\begin{pmatrix} -8 \\ 6 \end{pmatrix}$$
$$\vec{c_{n0}} = \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix}$$

$$\vec{OC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 10\begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix}$$
$$\vec{OC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix}$$

8.33

1)

Specification: Given are the points $pA(120|10)$, $pB_1(150|140)$, and $pB_2(500|250)$. Calculate the angle β that encloses the horizontal and the vector $p\vec{A}pB_1$.

Definitions: Let $\vec{i}$ be the horizontal enclosing β.

Exercise:

$$\vec{AB} = \begin{pmatrix}150\\140\end{pmatrix} - \begin{pmatrix}120\\10\end{pmatrix}$$
$$\vec{AB} = \begin{pmatrix}30.0\\130.0\end{pmatrix}$$

$$\vec{H} = \vec{A} + \vec{i}$$
$$\vec{H} = \begin{pmatrix}120\\10\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}$$
$$\vec{H} = \begin{pmatrix}121\\10\end{pmatrix}$$
$$\cos(\beta) = \frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|}$$
$$\beta = \arccos(\frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|})$$
$$\beta = \arccos(\frac{\begin{pmatrix}30.0\\130.0\end{pmatrix} \cdot \begin{pmatrix}121\\10\end{pmatrix}}{\sqrt{30.0^2 + 130.0^2} * \sqrt{121^2 + 10^2}})$$
$$\beta = \arccos(\frac{30 * 121 + 130 * 10}{133.416640641 * 121.412519947})$$
$$\beta = 72.2809315108°$$

Answer: The angle β that is enclosed by the horizontal and the vector $\vec{AB}$ has a value of 72.2809315108°.

2)

Specification: Calculate the angle between the vectors $\vec{B_1A}$ and $\vec{B_1B_2}$.

Exercise:

$$\vec{B_1A} = \begin{pmatrix}120\\10\end{pmatrix} - \begin{pmatrix}150\\140\end{pmatrix}$$
$$\vec{B_1A} = \begin{pmatrix}-30.0\\-130.0\end{pmatrix}$$

$$\vec{B_1B_2} = -1.0$$
$$\vec{B_1B_2} = \begin{pmatrix}350.0\\110.0\end{pmatrix}$$

$$\cos(\alpha) = \frac{\vec{B_1A} \cdot \vec{B_1B_2}}{|\vec{B_1A}| * |\vec{B_1B_2}|}$$