Applied Mathematics Exercises for 15. and 16. of February

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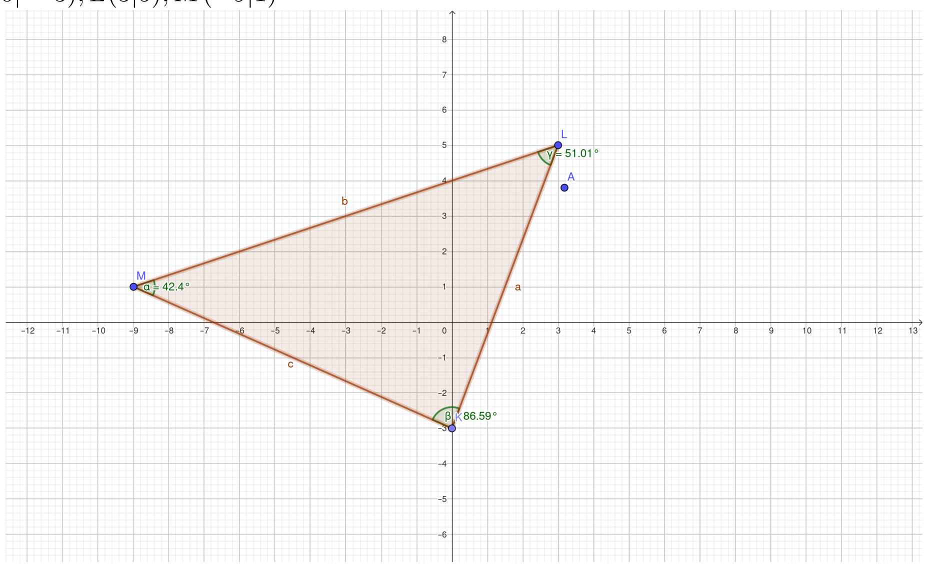
Contents

1 8.23

1.1 b)

Requirement: Calculate the size of the interior angles of the triangle with the given vertices.
$K(0|-3), L(3|5), M(-9|1)$



Exercises:

$$\vec{KL} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{1}$$

$$\vec{KL} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \tag{2}$$

$$\vec{LM} = \begin{pmatrix} -9 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{3}$$

$$\vec{LM} = \begin{pmatrix} -12 \\ -4 \end{pmatrix} \tag{4}$$

$$\vec{MK} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} - \begin{pmatrix} -9 \\ 1 \end{pmatrix} \tag{5}$$

$$\vec{MK} = \begin{pmatrix} 9 \\ -4 \end{pmatrix} \tag{6}$$

$$\cos(\alpha) = \frac{\vec{LM} \cdot (-\vec{MK})}{|\vec{LM}| * |\vec{MK}|} \tag{7}$$

$$\cos(\alpha) = \frac{\begin{pmatrix} -12 \\ -4 \end{pmatrix} \cdot (-\begin{pmatrix} 9 \\ -4 \end{pmatrix})}{\sqrt{12^2 + 4^2 * \sqrt{9^2 + 4^2}}} \tag{8}$$

$$\cos(\alpha) = \frac{92}{124.579292019} \tag{9}$$

$$\alpha = \arccos(0.738485493929) \tag{10}$$

$$\alpha = 42.39744^\circ \tag{11}$$

$$\cos(\beta) = \frac{\vec{KL} \cdot (-\vec{MK})}{|\vec{KL}| * |\vec{MK}|} \tag{12}$$

$$\cos(\beta) = \frac{\begin{pmatrix} 3 \\ 8 \end{pmatrix} \cdot (-\begin{pmatrix} 9 \\ -4 \end{pmatrix})}{\sqrt{3^2 + 8^2 * \sqrt{9^2 + 4^2}}} \tag{13}$$

$$\cos(\beta) = \frac{5}{84.1486779456} \tag{14}$$

$$\beta = \arccos(0.0594186399842) \tag{15}$$

$$\beta = 86.59356^\circ \tag{16}$$

$$\gamma = 180^\circ - (\alpha + \beta) \tag{17}$$

$$\gamma = 180^\circ - (42.39744^\circ + 86.59356^\circ) \tag{18}$$

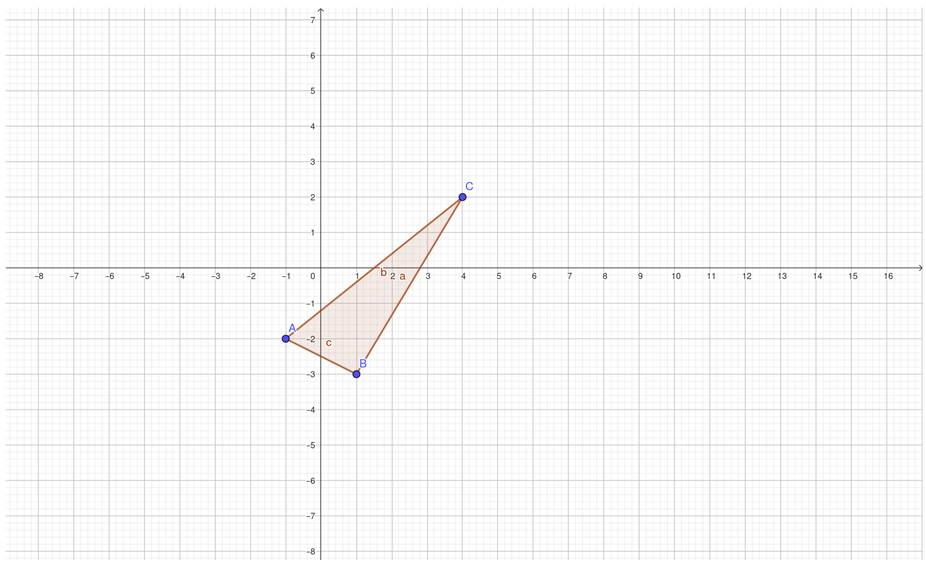
$$\gamma = 51.00901^\circ \tag{19}$$

2 8.26

2.1 b)

Requirements: Check if these points form a right angled triangle:

$$A(-1|-2) \qquad B(1|-3) \qquad C(4|2)$$



Exercises:

$$\vec{BC} \cdot \vec{CA} = BC_x * CA_x + BC_y * CA_y \tag{20}$$
$$\vec{BC} \cdot \vec{CA} = -1 * 1 + -2 * -3 \tag{21}$$
$$\vec{BC} \cdot \vec{CA} = -1 + 6 \tag{22}$$
$$\vec{BC} \cdot \vec{CA} = 5 \tag{23}$$

$$\vec{BC} \cdot \vec{AB} = BC_x * AB_x + BC_y * AB_y \tag{24}$$
$$\vec{BC} \cdot \vec{AB} = -1 * 4 + -2 * 2 \tag{25}$$
$$\vec{BC} \cdot \vec{AB} = -4 + -4 \tag{26}$$
$$\vec{BC} \cdot \vec{AB} = -8 \tag{27}$$

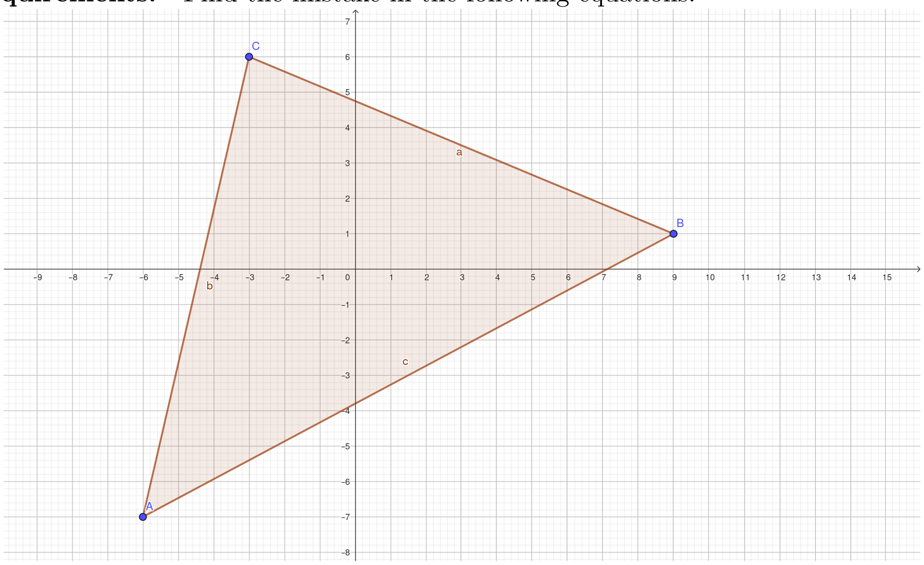
$$\vec{CA} \cdot \vec{AB} = CA_x * AB_x + CA_y * AB_y \tag{28}$$
$$\vec{CA} \cdot \vec{AB} = 1 * 4 + -3 * 2 \tag{29}$$
$$\vec{CA} \cdot \vec{AB} = 4 + -6 \tag{30}$$
$$\vec{CA} \cdot \vec{AB} = -2 \tag{31}$$

Answer: The triangle is not a right angle as none of the angles α, β, γ are $90°$. This is shown be the fact that none of the dot products of the triangles sides are 0.

3 8.29

3.1 1)

Requirements: Find the mistake in the following equations:



$$\vec{c} = \vec{AB} \tag{32}$$

$$= \begin{pmatrix} 15 \\ 8 \end{pmatrix} \tag{33}$$

$$c = |\vec{c}| \tag{34}$$

$$= 17cm \tag{35}$$

$$\vec{a} = \vec{BC} \tag{36}$$

$$= \begin{pmatrix} -12 \\ 5 \end{pmatrix} \tag{37}$$

$$a = |\vec{a}| \tag{38}$$

$$= 13cm \tag{39}$$

$$\tag{40}$$

$$\begin{aligned}
\cos(\beta) &= \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| * |\vec{b}|} && (41)\\
&= \frac{\begin{pmatrix} 15 \\ 8 \end{pmatrix} * \begin{pmatrix} -12 \\ 5 \end{pmatrix}}{13 * 17} && (42)\\
&= -\frac{140}{221} && (43)\\
&= -0.633... && (44)
\end{aligned}$$

$$\begin{aligned}
\beta &= \arccos(-0.633...) && (45)\\
&\approx 129.31^\circ && (46)
\end{aligned}$$

$$\begin{aligned}
A &= \frac{a * c * \sin(\beta)}{2} && (47)\\
&= \frac{13 * 17 * \sin(129.31^\circ)}{2} && (48)\\
&\approx 85.5cm^2 && (49)
\end{aligned}$$

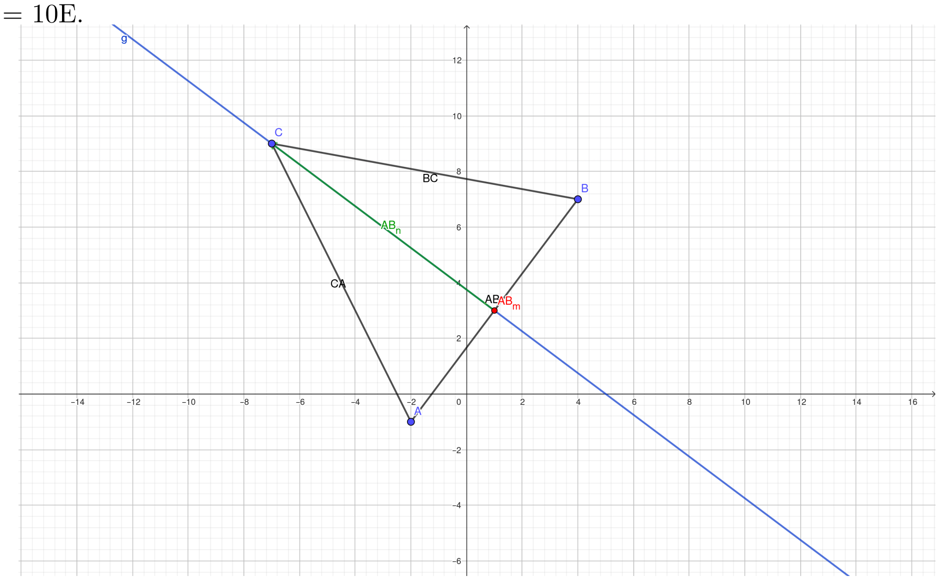
Answer: The obtuse value of β (129.31°) was used instead of the acute value of β (50.69°). This didn't affect the result because $\sin(\beta)$ returns the same value for the obtuse and the acute value of β

$$\sin(\alpha) = \sin(180^\circ - \alpha) \tag{50}$$

$$\sin(129.31^\circ) = 0.77368 \tag{51}$$
$$\sin(50.69^\circ) = 0.77368 \tag{52}$$

4 8.31

Specification: An isosceles triangle ABC, which is named in the mathematically positive direction, has the base AB with $A(-2|-1)$, $B(4|7)$ and the height h = 10E.



Requirements: Calculate the missing point C, the angle ϕ which is enclosed by $\vec{AB}$ and $\vec{AC}$, and the area A of the triangle.

Definitions:

$$h = 10E \tag{53}$$

$$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{54}$$

$$B = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{55}$$

$$\vec{c} = \vec{AB} \tag{56}$$

$$\vec{c_m} = \vec{OA} + \frac{1}{2}\vec{c} \tag{57}$$

Exercise:

Point C**:**

$$\vec{c} = \vec{OB} - \vec{OA} \tag{58}$$

$$\vec{c} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{59}$$

$$\vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{60}$$

$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{61}$$

$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{62}$$

$$\vec{c_m} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{63}$$

$$\vec{c_n} = \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{64}$$

$$\vec{c_{n0}} = \frac{1}{|\vec{c_n}|}\vec{c_n} \tag{65}$$

$$\vec{c_{n0}} = \frac{1}{\sqrt{(-8)^2 + 6^2}} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{66}$$

$$\vec{c_{n0}} = \frac{1}{10} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{67}$$

$$\vec{c_{n0}} = \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \tag{68}$$

$$\vec{OC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 10 \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \tag{69}$$

$$\vec{OC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix} \tag{70}$$

Angle ϕ:

$$\vec{AC} = \vec{OC} - \vec{OA} \tag{71}$$
$$\vec{AC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{72}$$
$$\vec{AC} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} \tag{73}$$

$$\cos(\phi) = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| * |\vec{AC}|} \tag{74}$$
$$\cos(\phi) = \frac{\begin{pmatrix} 6 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 10 \end{pmatrix}}{\sqrt{6^2 + 8^2} * \sqrt{(-5)^2 + 10^2}} \tag{75}$$
$$\cos(\phi) = \frac{50}{111.803398875} \tag{76}$$
$$\phi = \arccos(0.4472135955) \tag{77}$$
$$\phi = 63.43495^\circ \tag{78}$$

Area:

$$A = \frac{1}{2} c h_c \tag{79}$$
$$A = 5c \tag{80}$$

$$c = |\vec{c}| \tag{81}$$
$$c = \sqrt{6^2 + 8^2} \tag{82}$$
$$c = 10 \tag{83}$$

$$A = 50E^2 \tag{84}$$

Answer: The point C is at the location $(-7|9)$, the angle between $\vec{AB}$ and $\vec{AC}$ is 63.43495°, and the area of the triangle is $50E^2$.

5 8.33

Specification: In a rowing competition on a lake, two buoys at positions $B_1(150|140)$ and $B_2(500|250)$ must be passed. Then the team has to return to the starting point. A rowing team starts at point $A(120|10)$.
Note: Units in meters

5.1 1)

Requirements Calculate the minimum angle to the horizontal at which the team should approach the first buoy B_1.

Exercises:

$$\cos(\alpha) = \frac{\vec{i} \cdot \vec{AB_1}}{|\vec{i}| * |\vec{AB_1}|} \tag{85}$$

$$\cos(\alpha) = \frac{\begin{pmatrix}1\\0\end{pmatrix} \cdot \begin{pmatrix}30\\130\end{pmatrix}}{\sqrt{30^2 + 130^2}} \tag{86}$$

$$\cos(\alpha) = \frac{30}{133.416640641} \tag{87}$$

$$\alpha = \arccos(0.224859506699) \tag{88}$$

$$\alpha = 77.00538° \tag{89}$$

Answer: The minimum angle that the team has to approach the buoy at is 77.00538°.

5.2 2)

Requirements Determine the angle between $\vec{B_1A}$ and $\vec{B_1B_2}$.

Exercise:

$$\cos(\beta) = \frac{\vec{AB_1} \cdot (-\vec{B_1B_2})}{|\vec{AB_1}| * |\vec{B_1B_2}|} \tag{90}$$

$$\cos(\beta) = \frac{\begin{pmatrix}30\\130\end{pmatrix} \cdot (-\begin{pmatrix}350\\110\end{pmatrix})}{\sqrt{30^2 + 130^2} * \sqrt{350^2 + 110^2}} \tag{91}$$

$$\cos(\beta) = \frac{-24800}{48947.72722} \tag{92}$$

$$\beta = \arccos(-0.506662952675) \tag{93}$$

$$\beta = 120.4418° \tag{94}$$

Answer: The angle between $\vec{B_1A}$ and $\vec{B_1B_2}$ is 120.4418°.

5.3 3)

Requirements Calculate the water area enclosed by the course AB_1B_2.

Exercises:

$$A = \frac{1}{2}|(AB_1)_x(B_1B_2)_y - (AB_1)_y(B_1B_2)_x| \tag{95}$$

$$A = \frac{1}{2}|30 * 110 - 130 * 350| \tag{96}$$

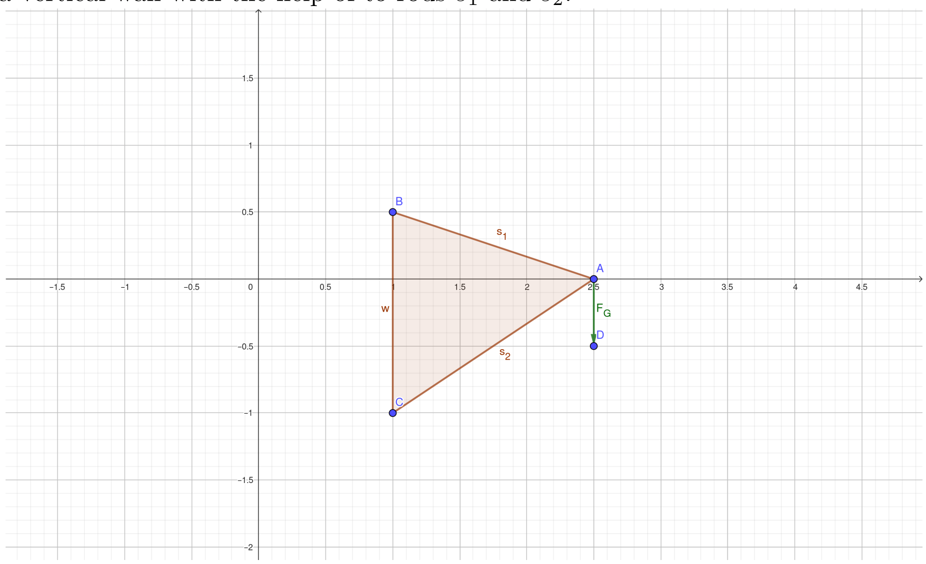
$$A = \frac{1}{2}| - 42200| \tag{97}$$

$$A = 21100 \tag{98}$$

Answer: The water area enclosed by the course AB_1B_2 is $21100m^2$.

6 8.37

Specification A Lamp with the weight $F_G = |\vec{F_G}| = 45N$ should be mounted to a vertical wall with the help of to rods s_1 and s_2.



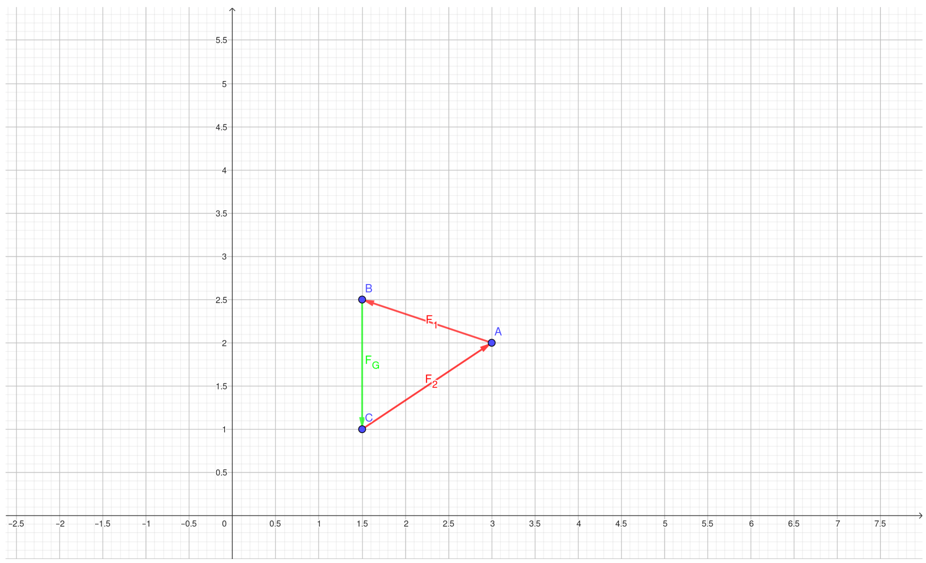
6.1 1)

Requirements: Sketch the force triangle of the forces acting on the point A where the lamp is mounted. Denote the directions of the forces.

Answer: The direction of the force $\vec{F_1}$ is taken from the direction of $s_1 = \begin{pmatrix} -1.5 \\ 0.5 \end{pmatrix}$.

The same goes for the force $\vec{F_2}$: $s_2 = \begin{pmatrix} 1.5 \\ 1 \end{pmatrix}$.

The weight force points vertically down with $45N$: $\vec{F_G} = \begin{pmatrix} 0 \\ -45 \end{pmatrix}$.



6.2 2)

Requirements: Calculate the magnitudes of the forces in the rods s_1 and s_2. Note: The sum of the forces acting on A has to be the zero vector.

$$\vec{F_G} + \vec{F_1} + \vec{F_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{99}$$

$$\vec{F_1} + \vec{F_2} = -\vec{F_G} \tag{100}$$

$$r * \begin{pmatrix} -1.5 \\ -.5 \end{pmatrix} + t * \begin{pmatrix} 1.5 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 45 \end{pmatrix} \tag{101}$$

I. $-1.5r + 1.5t = 0 \Rightarrow t = r$
II. $0.5r + t = 45$
I. in II. $1.5r = 45 \Rightarrow r = 30, t = 30$

$$\vec{F_1} = r + s_1 = \begin{pmatrix} -45 \\ 15 \end{pmatrix} \tag{102}$$

$$|\vec{F_1}| = 47.343...N \approx 47N \tag{103}$$

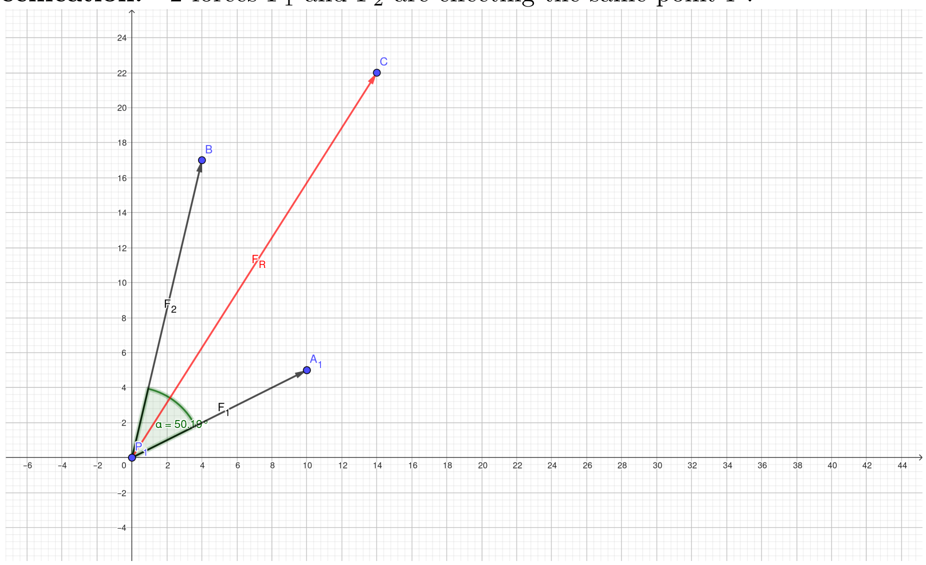
$$\vec{F_2} = r + s_2 = \begin{pmatrix} 45 \\ 30 \end{pmatrix} \tag{104}$$

$$|\vec{F_2}| = 54.083...N \approx 54N \tag{105}$$

7 8.38

7.1 a)

Specification: 2 forces $\vec{F_1}$ and $\vec{F_2}$ are effecting the same point P.



Requirements: Calculate the resulting force $\vec{F_R}$ and the angle ϕ between $\vec{F_1}$ and $\vec{F_2}$ (unit = Newton).

Definitions:

$$\vec{F_1} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \tag{106}$$

$$\vec{F_2} = \begin{pmatrix} 4 \\ 17 \end{pmatrix} \tag{107}$$

Exercises:

$$\vec{F_R} = \vec{F_1} + \vec{F_2} \tag{108}$$

$$\vec{F_R} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 17 \end{pmatrix} \tag{109}$$

$$\vec{F_R} = \begin{pmatrix} 14 \\ 22 \end{pmatrix} \tag{110}$$

$$|\vec{F_R}| = \sqrt{14^2 + 22^2} \tag{111}$$

$$|\vec{F_R}| = 26.0768096208 \tag{112}$$

$$\cos(\phi) = \frac{\vec{F_1} \cdot \vec{F_2}}{|\vec{F_1}| * |\vec{F_2}|} \tag{113}$$

$$\cos(\phi) = \frac{\begin{pmatrix} 10 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 17 \end{pmatrix}}{\sqrt{125} * \sqrt{305}} \tag{114}$$

$$\cos(\phi) = \frac{125}{195.256241898} \tag{115}$$

$$\phi = \arccos(0.640184399663) \tag{116}$$

$$\phi = 50.19443^\circ \tag{117}$$

Answer: The resulting force F_R has a magnitude of 26.0768096208 Newton and the angle ϕ between the forces $\vec{F_1}$ and $\vec{F_2}$ is 50.19443°.

8 8.39

8.1 b)

Specification: The forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ form a force triangle.

Requirements: The forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ form a force triangle. Find the missing force graphically and through calculation, such that the force triangle is balanced. (units = kN)

Definitions:

$$\vec{F_1} = \begin{pmatrix} 20 \\ 30 \end{pmatrix} \tag{118}$$

$$\vec{F_2} = \begin{pmatrix} -10 \\ 15 \end{pmatrix} \tag{119}$$

$$\vec{F_3} = ? \tag{120}$$

Exercise:

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{121}$$

$$\vec{F_1} + \vec{F_2} = \begin{pmatrix} 20 \\ 30 \end{pmatrix} + \begin{pmatrix} -10 \\ 15 \end{pmatrix} \tag{122}$$

$$\vec{F_1} + \vec{F_2} = \begin{pmatrix} 10 \\ 45 \end{pmatrix} \tag{123}$$

$$\begin{pmatrix} 10 \\ 45 \end{pmatrix} + \vec{F_3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{124}$$

$$\vec{F_3} = \begin{pmatrix} -10 \\ -45 \end{pmatrix} \tag{125}$$

Answer: The force F_3 has the value $\begin{pmatrix} -10 \\ -45 \end{pmatrix}$.