Applied Mathematics Exercises for 15. and 16. of February

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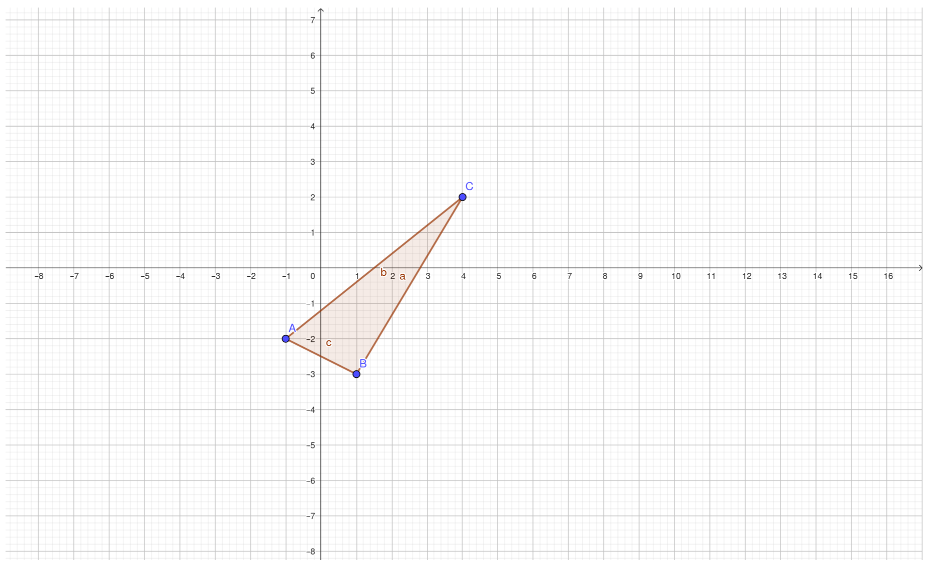
Contents

1 8.26

1.1 b)

Specification: Check if these points form a right angled triangle:

$$A(-1|-2) \qquad B(1|-3) \qquad C(4|2)$$



Definitions:

$$\vec{a} = \vec{BC}$$
$$\vec{b} = \vec{CA}$$
$$\vec{c} = \vec{AB}$$

Exercises:

$$\vec{a} \cdot \vec{b} = a_x * b_x + a_y * b_y \tag{1}$$
$$\vec{a} \cdot \vec{b} = -1 * 1 + -2 * -3 \tag{2}$$
$$\vec{a} \cdot \vec{b} = -1 + 6 \tag{3}$$
$$\vec{a} \cdot \vec{b} = 5 \tag{4}$$

$$\vec{a} \cdot \vec{c} = a_x * c_x + a_y * c_y \tag{5}$$
$$\vec{a} \cdot \vec{c} = -1 * 4 + -2 * 2 \tag{6}$$
$$\vec{a} \cdot \vec{c} = -4 + -4 \tag{7}$$
$$\vec{a} \cdot \vec{c} = -8 \tag{8}$$

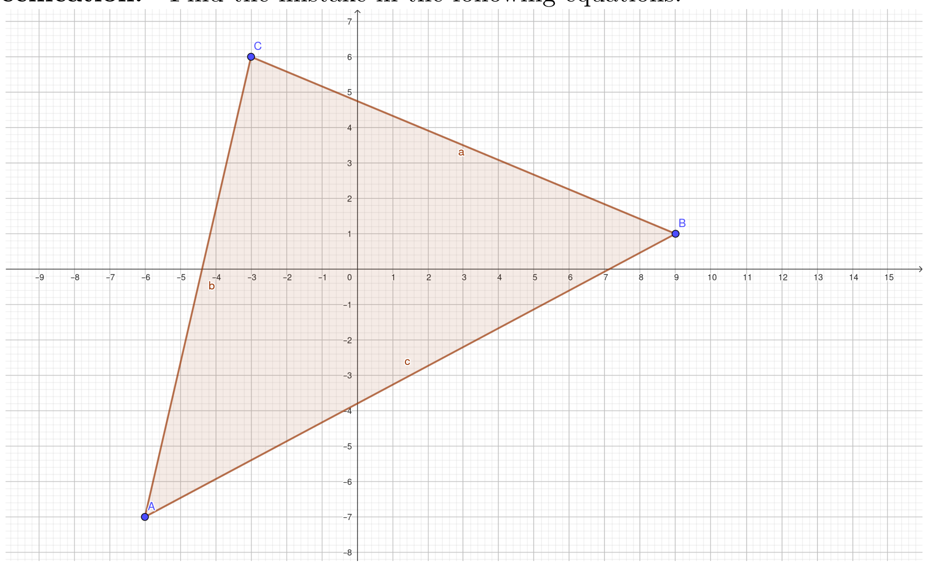
$$\vec{a} \cdot \vec{c} = b_x * c_x + b_y * c_y \tag{9}$$
$$\vec{a} \cdot \vec{c} = 1 * 4 + -3 * 2 \tag{10}$$
$$\vec{a} \cdot \vec{c} = 4 + -6 \tag{11}$$
$$\vec{a} \cdot \vec{c} = -2 \tag{12}$$

Answer: The triangle is not a right angle as none of the angles α, β, γ are $90°$. This is shown be the fact that none of the dot products of the triangles sides are 0.

2 8.29

2.1 1)

Specification: Find the mistake in the following equations:



$$\vec{c} = \vec{AB} \tag{13}$$
$$= \begin{pmatrix} 15 \\ 8 \end{pmatrix} \tag{14}$$
$$c = |\vec{c}| \tag{15}$$
$$= 17cm \tag{16}$$

$$\vec{a} = \vec{BC} \tag{17}$$
$$= \begin{pmatrix} -12 \\ 5 \end{pmatrix} \tag{18}$$
$$a = |\vec{a}| \tag{19}$$
$$= 13cm \tag{20}$$

$$\tag{21}$$

$$\cos(\beta) = \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| * |\vec{b}|} \tag{22}$$

$$= \frac{\begin{pmatrix} 15 \\ 8 \end{pmatrix} * \begin{pmatrix} -12 \\ 5 \end{pmatrix}}{13 * 17} \tag{23}$$

$$= -\frac{140}{221} \tag{24}$$

$$= -0.633... \tag{25}$$

$$\beta = \arccos(-0.633...) \tag{26}$$

$$\approx 129.31^\circ \tag{27}$$

$$A = \frac{a * c * \sin(\beta)}{2} \tag{28}$$

$$= \frac{13 * 17 * \sin(129.31^\circ)}{2} \tag{29}$$

$$\approx 85.5cm^2 \tag{30}$$

Answer: The obtuse value of β ($129.31°$) was used instead of the acute value of β ($50.69°$). This didn't affect the result because $\sin(\beta)$ returns the same value for the obtuse and the acute value of β

$$\sin(\alpha) = \sin(180° - \alpha) \tag{31}$$

$$\sin(129.31°) = 0.77368 \tag{32}$$

$$\sin(50.69°) = 0.77368 \tag{33}$$

3 8.31

Specification: An isosceles triangle ABC, which is named in the mathematically positive direction, has the base AB with $A(-2|-1)$, $B(4|7)$ and the height h = 10E. Calculate the missing point C, the angle ϕ which is enclosed by $\vec{AB}$ and $\vec{AC}$, and the area A_{rea} of the triangle.

Definitions:

$$h = 10E \tag{34}$$

$$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{35}$$

$$B = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{36}$$

$$\vec{c} = \vec{AB} \tag{37}$$

$$\vec{c_m} = \vec{OA} + \frac{1}{2}\vec{c} \tag{38}$$

Exercise:

Point C:

$$\vec{c} = \vec{OB} - \vec{OA} \tag{39}$$

$$\vec{c} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{40}$$

$$\vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{41}$$

$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{42}$$

$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{43}$$

$$\vec{c_m} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{44}$$

$$\vec{c_n} = \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{45}$$

$$\vec{c_{n0}} = \frac{1}{|\vec{c_n}|} \vec{c_n} \tag{46}$$

$$\vec{c_{n0}} = \frac{1}{\sqrt{(-8)^2 + 6^2}} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{47}$$

$$\vec{c_{n0}} = \frac{1}{10} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{48}$$

$$\vec{c_{n0}} = \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \tag{49}$$

$$\vec{OC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 10 \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \tag{50}$$

$$\vec{OC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix} \tag{51}$$

Angle ϕ:

$$\vec{AC} = \vec{OC} - \vec{OA} \tag{52}$$
$$\vec{AC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{53}$$
$$\vec{AC} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} \tag{54}$$

$$\cos(\phi) = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| * |\vec{AC}|} \tag{55}$$
$$\cos(\phi) = \frac{\begin{pmatrix} 6 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 10 \end{pmatrix}}{\sqrt{6^2 + 8^2} * \sqrt{(-5)^2 + 10^2}} \tag{56}$$
$$\cos(\phi) = \frac{50}{111.803398875} \tag{57}$$
$$\phi = \arccos(0.4472135955) \tag{58}$$
$$\phi = 63.43495^\circ \tag{59}$$

Area:

$$A = \frac{1}{2} c h_c \tag{60}$$
$$A = 5c \tag{61}$$

$$c = |\vec{c}| \tag{62}$$
$$c = \sqrt{6^2 + 8^2} \tag{63}$$
$$c = 10 \tag{64}$$

$$A = 50E^2 \tag{65}$$

Answer: The point C is at the location $(-7|9)$, the angle between $\vec{AB}$ and $\vec{AC}$ is 63.43495°, and the area of the triangle is $50E^2$.

4 8.33

4.1 1)

Specification: Given are the points $pA(120|10)$, $pB_1(150|140)$, and $pB_2(500|250)$. Calculate the angle β that encloses the horizontal and the vector $pA\vec{p}B_1$.

Definitions: Let $\vec{i}$ be the horizontal enclosing β.

Exercise:

$$\vec{AB} = \begin{pmatrix}150\\140\end{pmatrix} - \begin{pmatrix}120\\10\end{pmatrix} \tag{66}$$

$$\vec{AB} = \begin{pmatrix}30\\130\end{pmatrix} \tag{67}$$

$$\vec{H} = \vec{A} + \vec{i} \tag{68}$$

$$\vec{H} = \begin{pmatrix}120\\10\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix} \tag{69}$$

$$\vec{H} = \begin{pmatrix}121\\10\end{pmatrix} \tag{70}$$

$$\cos(\beta) = \frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|} \tag{71}$$

$$\beta = \arccos(\frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|}) \tag{72}$$

$$\beta = \arccos(\frac{\begin{pmatrix}30\\130\end{pmatrix} \cdot \begin{pmatrix}121\\10\end{pmatrix}}{\sqrt{30^2 + 130^2} * \sqrt{121^2 + 10^2}}) \tag{73}$$

$$\beta = \arccos(\frac{30 * 121 + 130 * 10}{133.416640641 * 121.412519947}) \tag{74}$$

$$\beta = 72.2809315108° \tag{75}$$

Answer: The angle β that is enclosed by the horizontal and the vector $\vec{AB}$ has a value of 72.2809315108°.

4.2 2)

Specification: Calculate the angle between the vectors $B_1\vec{A}$ and $B_1\vec{B}_2$.

Exercise:

$$\vec{B_1A} = \begin{pmatrix}120\\10\end{pmatrix} - \begin{pmatrix}150\\140\end{pmatrix} \tag{76}$$
$$\vec{B_1A} = \begin{pmatrix}-30\\110\end{pmatrix} \tag{77}$$

$$\vec{B_1B_2} = -1.0 \tag{78}$$
$$\vec{B_1B_2} = \begin{pmatrix}350\\110\end{pmatrix} \tag{79}$$

$$\cos(\alpha) = \frac{\vec{B_1A} \cdot \vec{B_1B_2}}{|\vec{B_1A}| * |\vec{B_1B_2}|} \tag{80}$$
$$\alpha = \arccos(\frac{\begin{pmatrix}-30\\110\end{pmatrix} \cdot \begin{pmatrix}350\\110\end{pmatrix}}{|\begin{pmatrix}-30\\110\end{pmatrix}| * |\begin{pmatrix}350\\110\end{pmatrix}|}) \tag{81}$$
$$\alpha = \arccos(\frac{1600}{41830.6108012}) \tag{82}$$
$$\alpha = 87.80793028^\circ \tag{83}$$

TODO: Area, correction

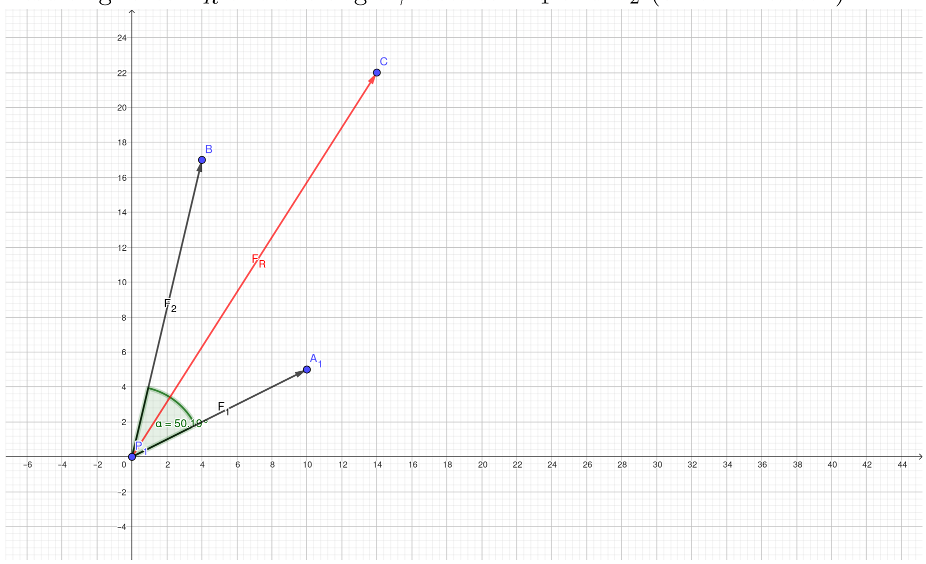
Answer: The angle between the vectors $\vec{B_1A}$ and $\vec{B_1B_2}$ is 87.80793028°.

5 8.37

6 8.38

6.1 a)

Specification: 2 forces $\vec{F_1}$ and $\vec{F_2}$ are effecting the same point P. Calculate the resulting force $\vec{F_R}$ and the angle ϕ between $\vec{F_1}$ and $\vec{F_2}$ (unit = Newton).



Definitions:

$$\vec{F_1} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \tag{84}$$

$$\vec{F_2} = \begin{pmatrix} 4 \\ 17 \end{pmatrix} \tag{85}$$

Exercises:

$$\vec{F_R} = \vec{F_1} + \vec{F_2} \tag{86}$$

$$\vec{F_R} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 17 \end{pmatrix} \tag{87}$$

$$\vec{F_R} = \begin{pmatrix} 14 \\ 22 \end{pmatrix} \tag{88}$$

$$|\vec{F_R}| = \sqrt{14^2 + 22^2} \tag{89}$$

$$|\vec{F_R}| = 26.0768096208 \tag{90}$$

$$\cos(\phi) = \frac{\vec{F_1} \cdot \vec{F_2}}{|\vec{F_1}| * |\vec{F_2}|} \tag{91}$$

$$\cos(\phi) = \frac{\begin{pmatrix} 10 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 17 \end{pmatrix}}{\sqrt{125} * \sqrt{305}} \tag{92}$$

$$\cos(\phi) = \frac{125}{195.256241898} \tag{93}$$

$$\phi = \arccos(0.640184399663) \tag{94}$$

$$\phi = 50.19443^\circ \tag{95}$$

Answer: The resulting force F_R has a magnitude of 26.0768096208 Newton and the angle ϕ between the forces $\vec{F_1}$ and $\vec{F_2}$ is 50.19443°.

7 8.39

7.1 b)

Specification: The forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ form a force triangle. Find the missing force graphically and through calculation, such that the force triangle is balanced. (units = kN)

Definitions:

$$\vec{F_1} = \begin{pmatrix} 20 \\ 30 \end{pmatrix} \tag{96}$$

$$\vec{F_2} = \begin{pmatrix} -10 \\ 15 \end{pmatrix} \tag{97}$$

$$\vec{F_3} = ? \tag{98}$$

Exercise:

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{99}$$

$$\vec{F_1} + \vec{F_2} = \begin{pmatrix} 20 \\ 30 \end{pmatrix} + \begin{pmatrix} -10 \\ 15 \end{pmatrix} \tag{100}$$

$$\vec{F_1} + \vec{F_2} = \begin{pmatrix} 10 \\ 45 \end{pmatrix} \tag{101}$$

$$\begin{pmatrix} 10 \\ 45 \end{pmatrix} + \vec{F_3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{102}$$

$$\vec{F_3} = \begin{pmatrix} -10 \\ -45 \end{pmatrix} \tag{103}$$

Answer: The force F_3 has the value $\begin{pmatrix} -10 \\ -45 \end{pmatrix}$.