Homework 13

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1 8.138

1.1 a)

Specification: Two forces $\vec{F_1}$, and $\vec{F_2}$, act at the same point. Determine the direction and magnitude of the resultant force $\vec{F_R}$, and the angles that $\vec{F_1}$, and $\vec{F_2}$, make with $\vec{F_R}$ (forces in N).

$$\vec{F_1} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \vec{F_2} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}, \tag{1}$$

Exercise:

$$\vec{F_R} = \vec{F_1} + \vec{F_2} \tag{2}$$

$$\vec{F_R} = \begin{pmatrix} 14 \\ 5 \end{pmatrix} \tag{3}$$

$$|\vec{F_R}| = \sqrt{14^2 + 5^2} \tag{4}$$

$$|\vec{F_R}| = \sqrt{221} \tag{5}$$

$$\cos(\alpha) = \frac{\vec{F_1} \cdot \vec{F_R}}{|\vec{F_1}| * |\vec{F_R}|} \tag{6}$$

$$\cos(\alpha) = \frac{104}{107.200746266} \tag{7}$$

$$\alpha = \arccos(0.970142500146) \tag{8}$$

$$\alpha = 14.03624^\circ \tag{9}$$

$$\cos(\beta) = \frac{\vec{F_2} \cdot \vec{F_R}}{|\vec{F_2}| * |\vec{F_R}|} \tag{10}$$

$$\cos(\beta) = \frac{117}{119.854077945} \tag{11}$$

$$\beta = \arccos(0.976187060182) \tag{12}$$

$$\beta = 12.52881^\circ \tag{13}$$

Answer: The direction of $\vec{F_R}$ is $\begin{pmatrix} 14 \\ 5 \end{pmatrix}$ and its magnitude is $\sqrt{221}N$, the angle α between $\vec{F_1}$ and $\vec{F_R}$ is 14.03624° and the angle β between $\vec{F_2}$ and $\vec{F_R}$ is 12.52881°.

2 8.43

Specification: A sled is pulled along a distance s by a rope with a constant force $\vec{F}$. The sled is pulled by the rope. The rope forms an angle ϕ with the horizontal surface.

2.1 b)

Requirements: Calculate the direction of the force and the amount of work W done while pulling.

$$s = 2.3km; F = |\vec{F}| = 55N; \phi = 32^\circ \tag{14}$$

Exercise:

$$\vec{F_x} = 55\cos(\phi) \tag{15}$$
$$\vec{F_x} = 46.6426441414 \tag{16}$$
$$\vec{F_y} = 55\sin(\phi) \tag{17}$$
$$\vec{F_y} = 29.1455613688 \tag{18}$$
$$\vec{F} = \begin{pmatrix} 46.6426441414 \\ 29.1455613688 \end{pmatrix} \tag{19}$$

$$W = \vec{F_x} * s \tag{20}$$
$$W = 46.6426441414N * 2300m \tag{21}$$
$$W = 107278.081525Nm \tag{22}$$

Answer: The force $\vec{F}$ is $\begin{pmatrix} 46.6426441414 \\ 29.1455613688 \end{pmatrix}$ and the Work done while pulling the sled on the ground is $107278.081525Nm$.

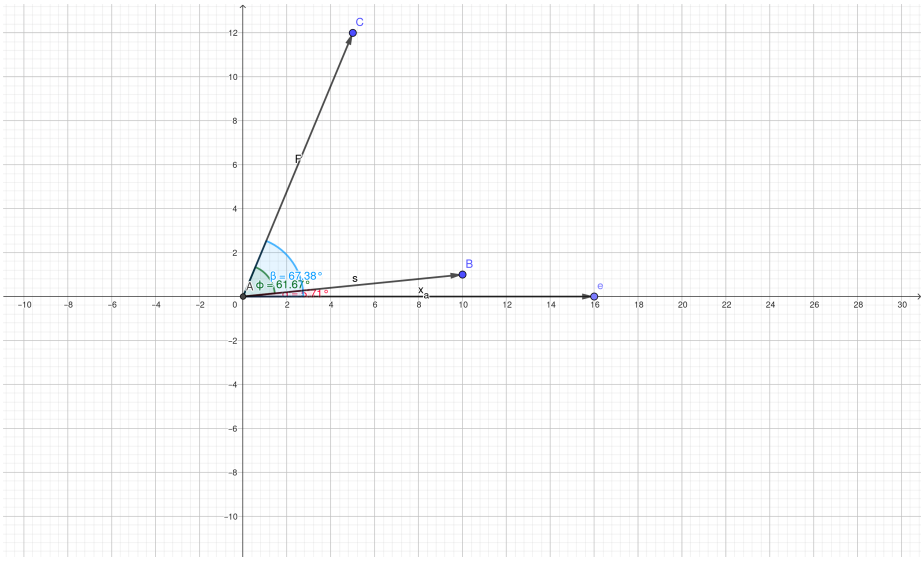
3 8.44

3.1 a)

Specification: A body is pulled in the direction of $\vec{r}$ on a distance s with the gradient k by a force $\vec{F}$.

Requirements: Calculate the work done in the process ($F = |\vec{F}|$).

$$s = 650m; k = 10\%; F = 65N; \vec{r} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{23}$$



Exercise:

$$\phi = \beta - \alpha \tag{24}$$
$$W = \cos(\phi) * F * s \tag{25}$$

$$\cos(\beta) = \frac{\vec{r_x}}{|\vec{r}|} \tag{26}$$
$$\beta = \arccos(\frac{5}{13}) \tag{27}$$
$$\beta = 67.38014° \tag{28}$$

$$\alpha = \arctan(\frac{1}{10}) \tag{29}$$
$$\alpha = 5.7105931375° \tag{30}$$

$$\phi = 67.38014^\circ - 5.7105931375^\circ \tag{31}$$
$$\phi = 61.6695468625^\circ \tag{32}$$

$$W = \cos(61.6695468625^\circ) * 65 * 650 \tag{33}$$
$$W = 20049.9828187 \tag{34}$$

Answer: $20049.9828187Nm$ are done in the process of pulling the body.