

8.26

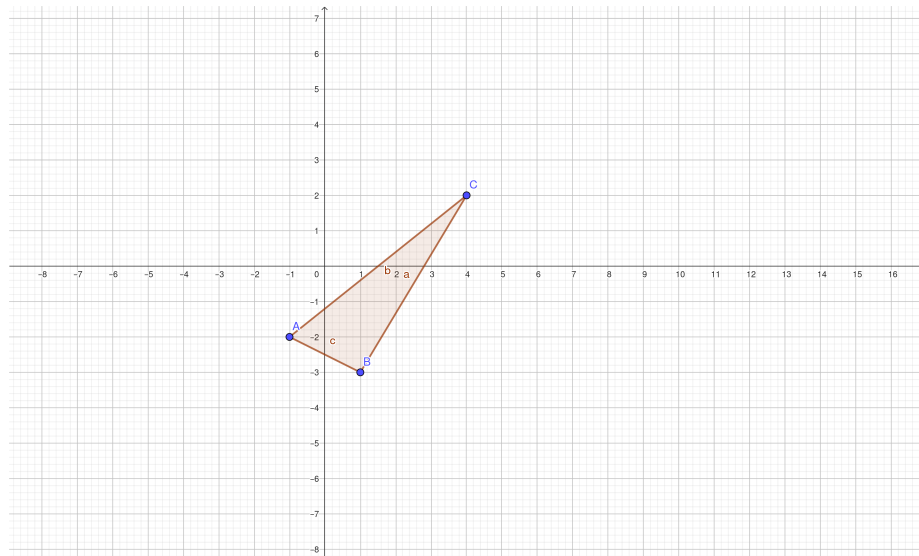
b)

**Specification:** Check if these points form a right angled triangle:

$$A(-1|-2)$$

$$B(1|-3)$$

$$C(4|2)$$



**Definitions:**

$$\vec{a} = \vec{BC}$$

$$\vec{b} = \vec{CA}$$

$$\vec{c} = \vec{AB}$$

**Exercises:**

$$\vec{a} \cdot \vec{b} = a_x * b_x + a_y * b_y \quad (1)$$

$$\vec{a} \cdot \vec{b} = -1 * 1 + -2 * -3 \quad (2)$$

$$\vec{a} \cdot \vec{b} = -1 + 6 \quad (3)$$

$$\vec{a} \cdot \vec{b} = 5 \quad (4)$$

$$\vec{a} \cdot \vec{c} = a_x * c_x + a_y * c_y \quad (5)$$

$$\vec{a} \cdot \vec{c} = -1 * 4 + -2 * 2 \quad (6)$$

$$\vec{a} \cdot \vec{c} = -4 + -4 \quad (7)$$

$$\vec{a} \cdot \vec{c} = -8 \quad (8)$$

$$\vec{a} \cdot \vec{c} = b_x * c_x + b_y * c_y \quad (9)$$

$$\vec{a} \cdot \vec{c} = 1 * 4 + -3 * 2 \quad (10)$$

$$\vec{a} \cdot \vec{c} = 4 + -6 \quad (11)$$

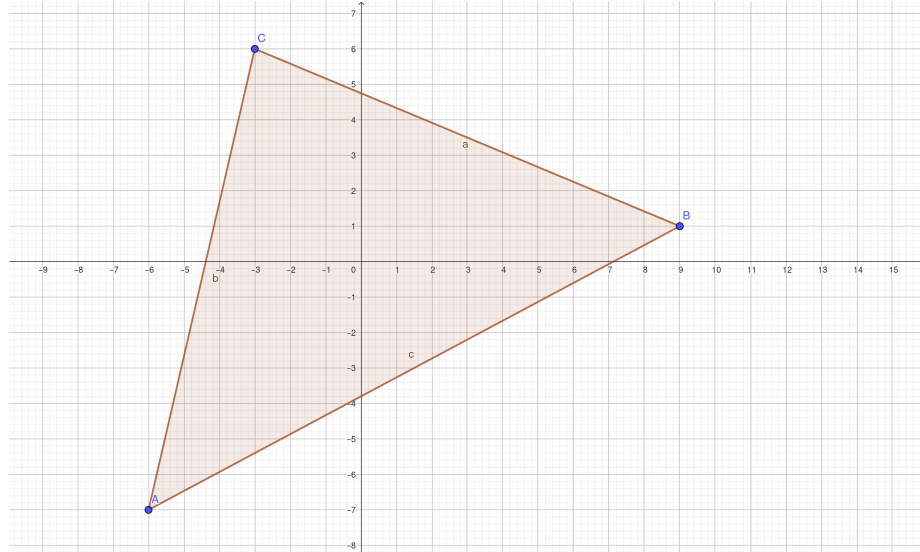
$$\vec{a} \cdot \vec{c} = -2 \quad (12)$$

**Answer:** The triangle is not a right angle as none of the angles  $\alpha$ ,  $\beta$ ,  $\gamma$  are  $90^\circ$ . This is shown by the fact that none of the dot products of the triangles sides are 0.

8.29

1)

**Specification:** Find the mistake in the following equations:



$$\begin{aligned}\vec{c} &= \vec{AB} \\ &= \begin{pmatrix} 15 \\ 8 \end{pmatrix} \\ c &= |\vec{c}| \\ &= 17cm\end{aligned}$$

$$\begin{aligned}\vec{a} &= \vec{BC} \\ &= \begin{pmatrix} -12 \\ 5 \end{pmatrix} \\ a &= |\vec{a}| \\ &= 13cm\end{aligned}$$

$$\begin{aligned}
\cos(\beta) &= \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| * |\vec{b}|} \\
&= \frac{\begin{pmatrix} 15 \\ 8 \end{pmatrix} * \begin{pmatrix} -12 \\ 5 \end{pmatrix}}{13 * 17} \\
&= -\frac{140}{221} \\
&= -0.633...
\end{aligned}$$

$$\begin{aligned}
\beta &= \arccos(-0.633...) \\
&\approx 129.31^\circ
\end{aligned}$$

$$\begin{aligned}
A &= \frac{a * c * \sin(\beta)}{2} \\
&= \frac{13 * 17 * \sin(129.31^\circ)}{2} \\
&\approx 85.5 cm^2
\end{aligned}$$

**Answer:** The obtuse value of  $\beta$  ( $129.31^\circ$ ) was used instead of the acute value of  $\beta$  ( $50.69^\circ$ ). This didn't affect the result because  $\sin(\beta)$  returns the same value for the obtuse and the acute value of  $\beta$

$$\sin(\alpha) = \sin(180^\circ - \alpha)$$

$$\sin(129.31^\circ) = 0.77368$$

$$\sin(50.69^\circ) = 0.77368$$

## 8.31

**Specification:** An isosceles triangle ABC, which is named in the mathematically positive direction, has the base AB with  $A(-2|-1)$ ,  $B(4|7)$  and the height  $h = 10E$ . Calculate the missing point C, the angle  $\phi$  which is enclosed by  $\vec{AB}$  and  $\vec{AC}$ , and the area  $A_{rea}$  of the triangle.

**Definitions:**

$$h = 10E$$

$$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

$$\vec{c} = \vec{AB}$$

$$\vec{c}_m = \vec{OA} + \frac{1}{2}\vec{c}$$

**Exercise:**

$$\begin{aligned}\vec{c} &= \vec{B} - \vec{A} \\ \vec{c} &= \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \\ \vec{c} &= \begin{pmatrix} 6 \\ 8 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\vec{c}_m &= \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ 8 \end{pmatrix} \\ \vec{c}_m &= \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \\ \vec{c}_m &= \begin{pmatrix} 1 \\ 3 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\vec{c}_n &= \begin{pmatrix} -8 \\ 6 \end{pmatrix} \\ c_{n0} &= \frac{1}{|\vec{c}_n|} \vec{c}_n \\ c_{n0} &= \frac{1}{\sqrt{(-8)^2 + 6^2}} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \\ c_{n0} &= \frac{1}{10} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \\ c_{n0} &= \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\vec{OC} &= \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 10 \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \\ \vec{OC} &= \begin{pmatrix} -7 \\ 9 \end{pmatrix}\end{aligned}$$

## 8.33

1)

**Specification:** Given are the points  $pA(120|10)$ ,  $pB_1(150|140)$ , and  $pB_2(500|250)$ . Calculate the angle  $\beta$  that encloses the horizontal and the vector  $pA\vec{p}B_1$ .

**Definitions:** Let  $\vec{i}$  be the horizontal enclosing  $\beta$ .

**Exercise:**

$$\vec{AB} = \begin{pmatrix} 150 \\ 140 \end{pmatrix} - \begin{pmatrix} 120 \\ 10 \end{pmatrix}$$

$$\vec{AB} = \begin{pmatrix} 30.0 \\ 130.0 \end{pmatrix}$$

$$\vec{H} = \vec{A} + \vec{i}$$

$$\vec{H} = \begin{pmatrix} 120 \\ 10 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec{H} = \begin{pmatrix} 121 \\ 10 \end{pmatrix}$$

$$\cos(\beta) = \frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|}$$

$$\beta = \arccos\left(\frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|}\right)$$

$$\beta = \arccos\left(\frac{\begin{pmatrix} 30.0 \\ 130.0 \end{pmatrix} \cdot \begin{pmatrix} 121 \\ 10 \end{pmatrix}}{\sqrt{30.0^2 + 130.0^2} * \sqrt{121^2 + 10^2}}\right)$$

$$\beta = \arccos\left(\frac{30 * 121 + 130 * 10}{133.416640641 * 121.412519947}\right)$$

$$\beta = 72.2809315108^\circ$$

**Answer:** The angle  $\beta$  that is enclosed by the horizontal and the vector  $\vec{AB}$  has a value of  $72.2809315108^\circ$ .

**2)**

**Specification:** Calculate the angle between the vectors  $\vec{B_1A}$  and  $\vec{B_1B_2}$ .



**Exercise:**

$$\vec{B_1A} = \begin{pmatrix} 120 \\ 10 \end{pmatrix} - \begin{pmatrix} 150 \\ 140 \end{pmatrix}$$

$$\vec{B_1A} = \begin{pmatrix} -30.0 \\ -130.0 \end{pmatrix}$$

$$\vec{B_1B_2} = -1.0$$

$$\vec{B_1B_2} = \begin{pmatrix} 350.0 \\ 110.0 \end{pmatrix}$$

$$\cos(\alpha) = \frac{\vec{B_1A} \cdot \vec{B_1B_2}}{|\vec{B_1A}| * |\vec{B_1B_2}|}$$