Applied Mathematics Exercises for 15. and 16. of February

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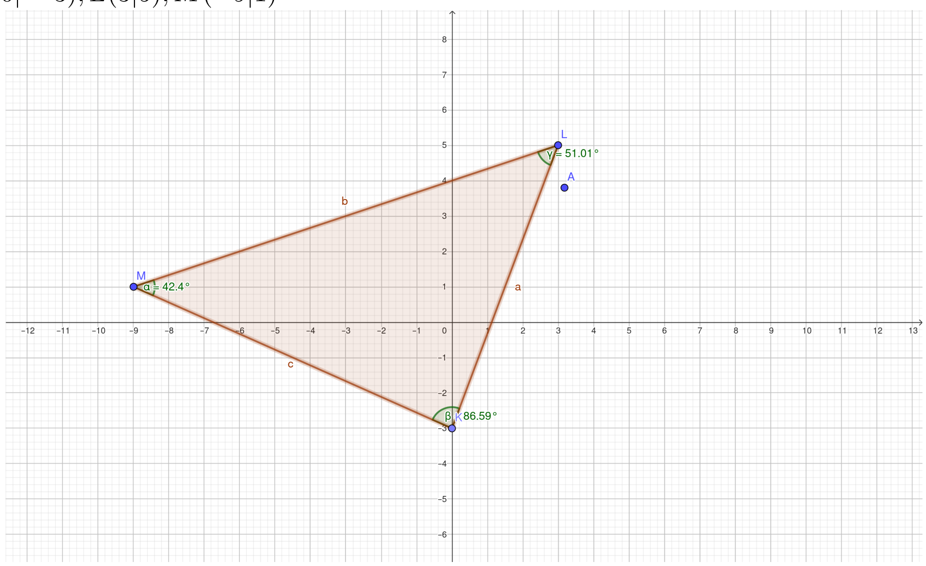
Contents

1 8.23

1.1 b)

Requirement: Calculate the size of the interior angles of the triangle with the given vertices.
$K(0|-3), L(3|5), M(-9|1)$



Definitions:

$$\vec{a} = \vec{KL} \quad (1)$$

$$\vec{b} = \vec{LM} \quad (2)$$

$$\vec{c} = \vec{MK} \quad (3)$$

Exercises:

$$\vec{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{4}$$

$$\vec{a} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \tag{5}$$

$$\vec{b} = \begin{pmatrix} -9 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{6}$$

$$\vec{b} = \begin{pmatrix} -12 \\ -4 \end{pmatrix} \tag{7}$$

$$\vec{c} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} - \begin{pmatrix} -9 \\ 1 \end{pmatrix} \tag{8}$$

$$\vec{c} = \begin{pmatrix} 9 \\ -4 \end{pmatrix} \tag{9}$$

$$\cos(\alpha) = \frac{\vec{b} \cdot -\vec{c}}{|\vec{b}| * |\vec{c}|} \tag{10}$$

$$\cos(\alpha) = \frac{\begin{pmatrix} -12 \\ -4 \end{pmatrix} \cdot - \begin{pmatrix} 9 \\ -4 \end{pmatrix}}{\sqrt{12^2 + 4^2} * \sqrt{9^2 + 4^2}} \tag{11}$$

$$\cos(\alpha) = \frac{92}{124.579292019} \tag{12}$$

$$\alpha = \arccos(0.738485493929) \tag{13}$$

$$\alpha = 42.39744^\circ \tag{14}$$

$$\cos(\beta) = \frac{\vec{a} \cdot -\vec{c}}{|\vec{a}| * |\vec{c}|} \tag{15}$$

$$\cos(\beta) = \frac{\begin{pmatrix} 3 \\ 8 \end{pmatrix} \cdot - \begin{pmatrix} 9 \\ -4 \end{pmatrix}}{\sqrt{3^2 + 8^2} * \sqrt{9^2 + 4^2}} \tag{16}$$

$$\cos(\beta) = \frac{5}{84.1486779456} \tag{17}$$

$$\beta = \arccos(0.0594186399842) \tag{18}$$

$$\beta = 86.59356^\circ \tag{19}$$

$$\gamma = 180^\circ - (\alpha + \beta) \tag{20}$$

$$\gamma = 180^\circ - (42.39744^\circ + 86.59356^\circ) \tag{21}$$

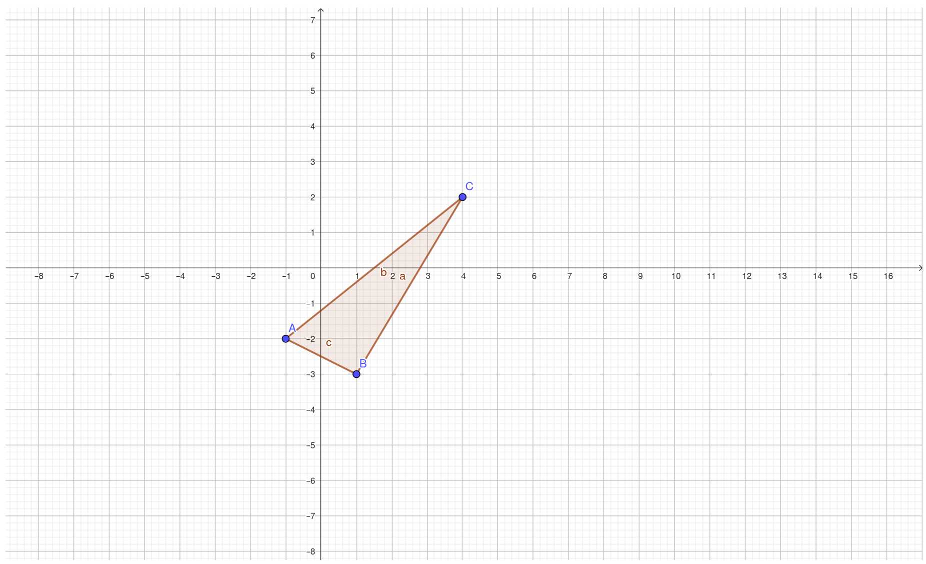
$$\gamma = 51.00901^\circ \tag{22}$$

2 8.26

2.1 b)

Requirements: Check if these points form a right angled triangle:

$$A(-1|-2) \qquad B(1|-3) \qquad C(4|2)$$



Definitions:

$$\vec{a} = B\vec{C}$$
$$\vec{b} = C\vec{A}$$
$$\vec{c} = A\vec{B}$$

Exercises:

$$\vec{a} \cdot \vec{b} = a_x * b_x + a_y * b_y \tag{23}$$
$$\vec{a} \cdot \vec{b} = -1 * 1 + -2 * -3 \tag{24}$$
$$\vec{a} \cdot \vec{b} = -1 + 6 \tag{25}$$
$$\vec{a} \cdot \vec{b} = 5 \tag{26}$$

$$\vec{a} \cdot \vec{c} = a_x * c_x + a_y * c_y \tag{27}$$
$$\vec{a} \cdot \vec{c} = -1 * 4 + -2 * 2 \tag{28}$$
$$\vec{a} \cdot \vec{c} = -4 + -4 \tag{29}$$
$$\vec{a} \cdot \vec{c} = -8 \tag{30}$$

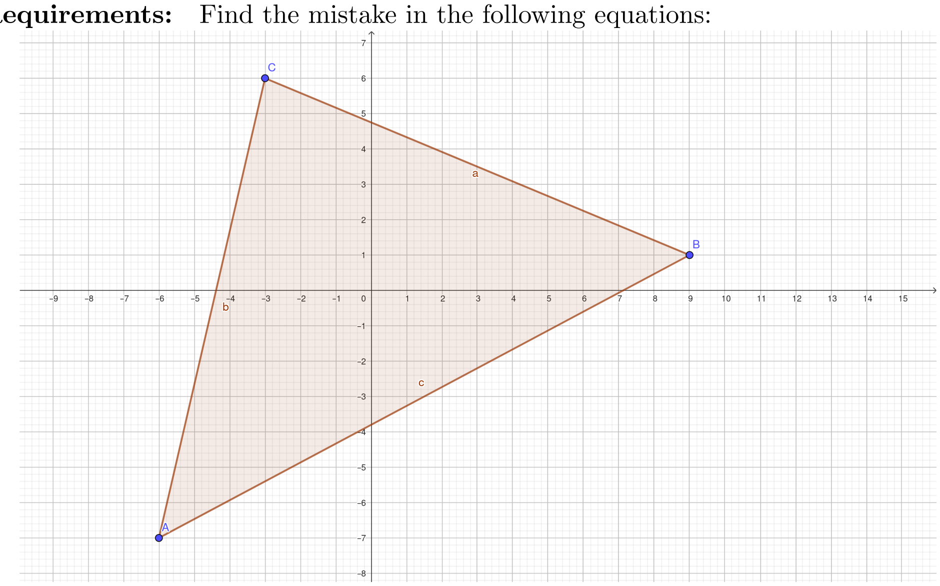
$$\vec{a} \cdot \vec{c} = b_x * c_x + b_y * c_y \tag{31}$$
$$\vec{a} \cdot \vec{c} = 1 * 4 + -3 * 2 \tag{32}$$
$$\vec{a} \cdot \vec{c} = 4 + -6 \tag{33}$$
$$\vec{a} \cdot \vec{c} = -2 \tag{34}$$

Answer: The triangle is not a right angle as none of the angles α, β, γ are $90°$. This is shown be the fact that none of the dot products of the triangles sides are 0.

3 8.29

3.1 1)

Requirements: Find the mistake in the following equations:



$$\vec{c} = \vec{AB} \tag{35}$$

$$= \begin{pmatrix} 15 \\ 8 \end{pmatrix} \tag{36}$$

$$c = |\vec{c}| \tag{37}$$

$$= 17cm \tag{38}$$

$$\vec{a} = \vec{BC} \tag{39}$$

$$= \begin{pmatrix} -12 \\ 5 \end{pmatrix} \tag{40}$$

$$a = |\vec{a}| \tag{41}$$

$$= 13cm \tag{42}$$

$$\tag{43}$$

$$
\begin{aligned}
\cos(\beta) &= \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| * |\vec{b}|} && (44)\\
&= \frac{\begin{pmatrix} 15 \\ 8 \end{pmatrix} * \begin{pmatrix} -12 \\ 5 \end{pmatrix}}{13 * 17} && (45)\\
&= -\frac{140}{221} && (46)\\
&= -0.633... && (47)
\end{aligned}
$$

$$
\begin{aligned}
\beta &= \arccos(-0.633...) && (48)\\
&\approx 129.31^\circ && (49)
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{a * c * \sin(\beta)}{2} && (50)\\
&= \frac{13 * 17 * \sin(129.31^\circ)}{2} && (51)\\
&\approx 85.5 cm^2 && (52)
\end{aligned}
$$

Answer: The obtuse value of β ($129.31°$) was used instead of the acute value of β ($50.69°$). This didn't affect the result because $\sin(\beta)$ returns the same value for the obtuse and the acute value of β

$$\sin(\alpha) = \sin(180° - \alpha) \tag{53}$$

$$\sin(129.31°) = 0.77368 \tag{54}$$
$$\sin(50.69°) = 0.77368 \tag{55}$$

4 8.31

Specification: An isosceles triangle ABC, which is named in the mathematically positive direction, has the base AB with $A(-2|-1)$, $B(4|7)$ and the height h = 10E.

Requirements: Calculate the missing point C, the angle ϕ which is enclosed by $\vec{AB}$ and $\vec{AC}$, and the area A_{rea} of the triangle.

Definitions:

$$h = 10E \tag{56}$$

$$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{57}$$

$$B = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \tag{58}$$

$$\vec{c} = \vec{AB} \tag{59}$$

$$\vec{c_m} = \vec{OA} + \frac{1}{2}\vec{c} \tag{60}$$

Exercise:

Point C:

$$\vec{c} = \vec{OB} - \vec{OA} \tag{61}$$

$$\vec{c} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{62}$$

$$\vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{63}$$

$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{64}$$

$$\vec{c_m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{65}$$

$$\vec{c_m} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{66}$$

$$\vec{c_n} = \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{67}$$

$$\vec{c_{n0}} = \frac{1}{|\vec{c_n}|} \vec{c_n} \tag{68}$$

$$\vec{c_{n0}} = \frac{1}{\sqrt{(-8)^2 + 6^2}} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{69}$$

$$\vec{c_{n0}} = \frac{1}{10} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{70}$$

$$\vec{c_{n0}} = \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \tag{71}$$

$$\vec{OC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 10 \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \tag{72}$$

$$\vec{OC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix} \tag{73}$$

Angle ϕ:

$$\vec{AC} = \vec{OC} - \vec{OA} \tag{74}$$

$$\vec{AC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{75}$$

$$\vec{AC} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} \tag{76}$$

$$\cos(\phi) = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| * |\vec{AC}|} \tag{77}$$

$$\cos(\phi) = \frac{\begin{pmatrix} 6 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 10 \end{pmatrix}}{\sqrt{6^2 + 8^2} * \sqrt{(-5)^2 + 10^2}} \tag{78}$$

$$\cos(\phi) = \frac{50}{111.803398875} \tag{79}$$

$$\phi = \arccos(0.4472135955) \tag{80}$$

$$\phi = 63.43495^\circ \tag{81}$$

Area:

$$A = \frac{1}{2} c h_c \tag{82}$$

$$A = 5c \tag{83}$$

$$c = |\vec{c}| \tag{84}$$

$$c = \sqrt{6^2 + 8^2} \tag{85}$$

$$c = 10 \tag{86}$$

$$A = 50E^2 \tag{87}$$

Answer: The point C is at the location $(-7|9)$, the angle between $\vec{AB}$ and $\vec{AC}$ is 63.43495°, and the area of the triangle is $50E^2$.

5 8.33

Specification: In a rowing competition on a lake, two buoys at positions $B_1(150|140)$ and $B_2(500|250)$ must be passed. Then the team has to return to the starting point. A rowing team starts at point $A(120|10)$.
Note: Units in meters

5.1 1)

Requirements Calculate the minimum angle to the horizontal at which the team should approach the first buoy B_1.

Definitions:

$$\vec{a} = A\vec{B}_1 \tag{88}$$

Exercises:

$$\cos(\alpha) = \frac{\vec{i} \cdot \vec{a}}{|\vec{i}| * |\vec{a}|} \tag{89}$$

$$\cos(\alpha) = \frac{\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 130 \end{pmatrix}}{\sqrt{30^2 + 130^2}} \tag{90}$$

$$\cos(\alpha) = \frac{30}{133.416640641} \tag{91}$$

$$\alpha = \arccos(0.224859506699) \tag{92}$$

$$\alpha = 77.00538° \tag{93}$$

Answer: The minimum angle that the team has to approach the buoy at is 77.00538°.

5.2 2)

Requirements Determine the angle between $B_1\vec{A}$ and $B_1\vec{B}_2$.

Definitions:

$$\vec{a} = A\vec{B}_1 \tag{94}$$

$$\vec{b} = B_1\vec{B}_2 \tag{95}$$

Exercise:

$$\cos(\beta) = \frac{\vec{a} \cdot -\vec{b}}{|\vec{a}| * |\vec{b}|} \tag{96}$$

$$\cos(\beta) = \frac{\begin{pmatrix} 30 \\ 130 \end{pmatrix} \cdot - \begin{pmatrix} 350 \\ 110 \end{pmatrix}}{\sqrt{30^2 + 130^2} * \sqrt{350^2 + 110^2}} \tag{97}$$

$$\cos(\beta) = \frac{-24800}{48947.72722} \tag{98}$$

$$\beta = \arccos(-0.506662952675) \tag{99}$$

$$\beta = 120.4418° \tag{100}$$

Answer: The angle between $\vec{B_1A}$ and $\vec{B_1B_2}$ is $120.4418°$.

5.3 3)

Requirements Calculate the water area enclosed by the course AB_1B_2.

Definitions:

$$\vec{a} = \vec{AB_1} \tag{101}$$

$$\vec{b} = \vec{B_1B_2} \tag{102}$$

Exercises:

$$A = \frac{1}{2}|a_x b_y - a_y b_x| \tag{103}$$

$$A = \frac{1}{2}|30 * 110 - 130 * 350| \tag{104}$$

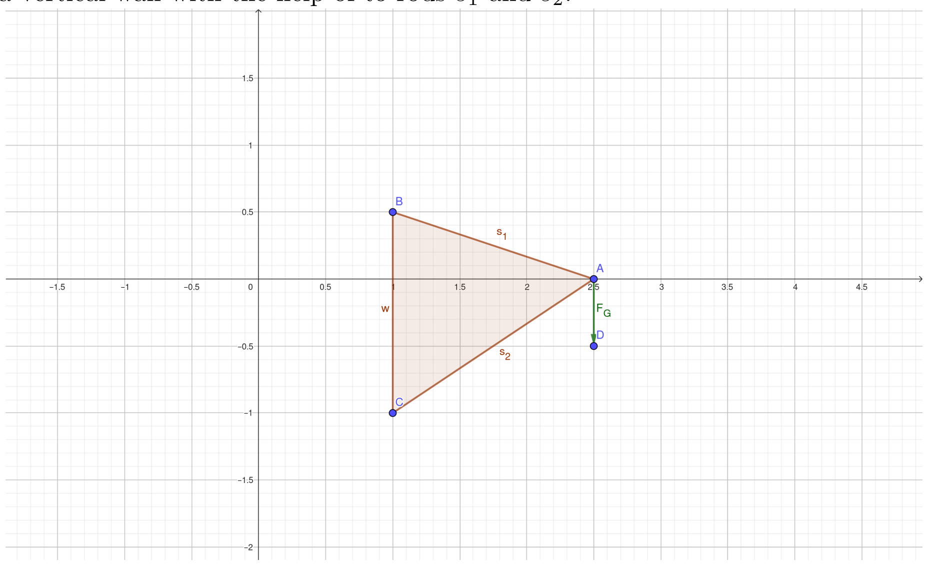
$$A = \frac{1}{2}| - 42200| \tag{105}$$

$$A = 21100 \tag{106}$$

Answer: The water area enclosed by the course AB_1B_2 is $21100m^2$.

6 8.37

Specification A Lamp with the weight $F_G = |\vec{F_G}| = 45N$ should be mounted to a vertical wall with the help of to rods s_1 and s_2.



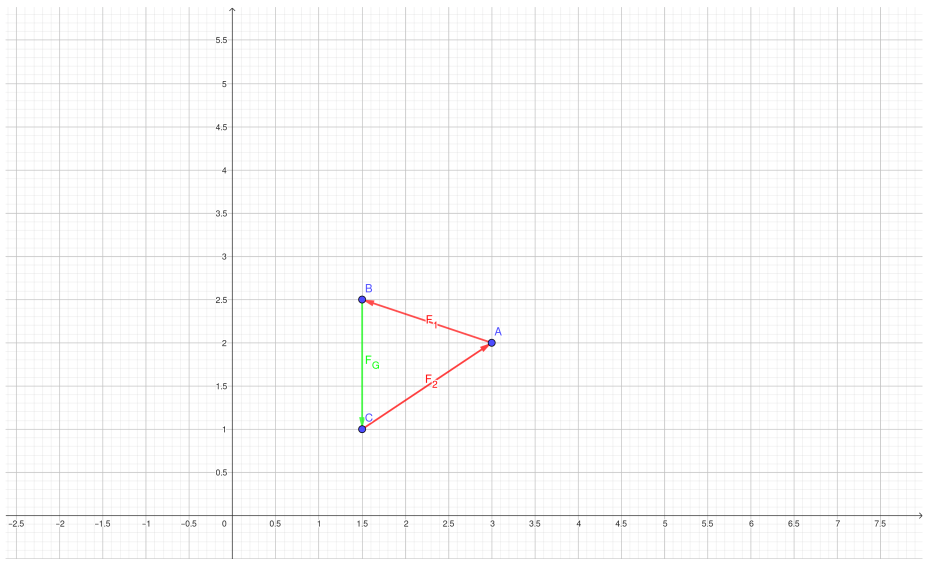
6.1 1)

Requirements: Sketch the force triangle of the forces acting on the point A where the lamp is mounted. Denote the directions of the forces.

Answer: The direction of the force $\vec{F_1}$ is taken from the direction of $s_1 = \begin{pmatrix} -1.5 \\ 0.5 \end{pmatrix}$.

The same goes for the force $\vec{F_2}$: $s_2 = \begin{pmatrix} 1.5 \\ 1 \end{pmatrix}$.

The weight force points vertically down with $45N$: $\vec{F_G} = \begin{pmatrix} 0 \\ -45 \end{pmatrix}$.



6.2 2)

Requirements: Calculate the magnitudes of the forces in the rods s_1 and s_2.
Note: The sum of the forces acting on A has to be the zero vector.

$$\vec{F_G} + \vec{F_1} + \vec{F_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{107}$$

$$\vec{F_1} + \vec{F_2} = -\vec{F_G} \tag{108}$$

$$r * \begin{pmatrix} -1.5 \\ -.5 \end{pmatrix} + t * \begin{pmatrix} 1.5 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 45 \end{pmatrix} \tag{109}$$

I. $-1.5r + 1.5t = 0 \Rightarrow t = r$
II. $0.5r + t = 45$
I. in II. $1.5r = 45 \Rightarrow r = 30, t = 30$

$$\vec{F_1} = r + s_1 = \begin{pmatrix} -45 \\ 15 \end{pmatrix} \tag{110}$$

$$|\vec{F_1}| = 47.343...N \approx 47N \tag{111}$$

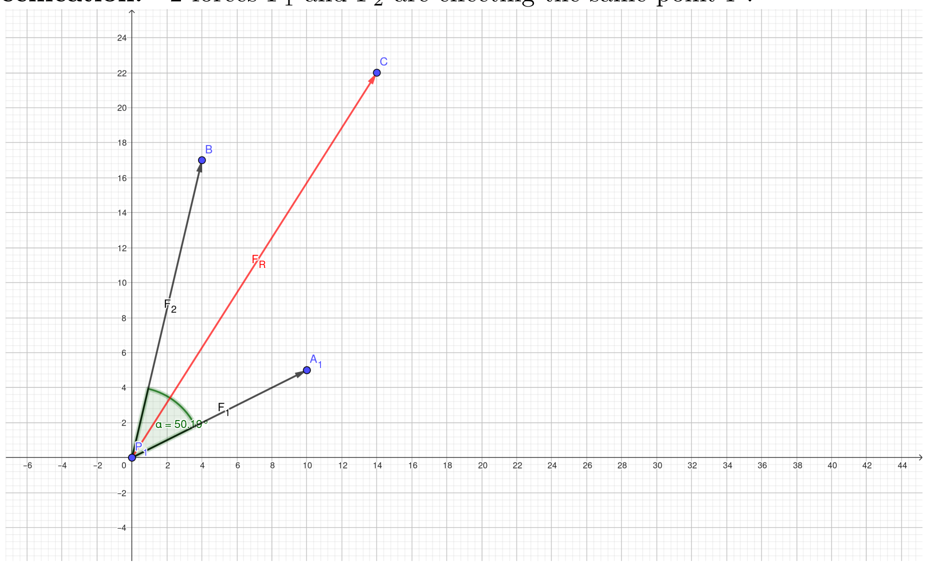
$$\vec{F_2} = r + s_2 = \begin{pmatrix} 45 \\ 30 \end{pmatrix} \tag{112}$$

$$|\vec{F_2}| = 54.083...N \approx 54N \tag{113}$$

7 8.38

7.1 a)

Specification: 2 forces $\vec{F_1}$ and $\vec{F_2}$ are effecting the same point P.



Requirements: Calculate the resulting force $\vec{F_R}$ and the angle ϕ between $\vec{F_1}$ and $\vec{F_2}$ (unit = Newton).

Definitions:

$$\vec{F_1} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} \tag{114}$$

$$\vec{F_2} = \begin{pmatrix} 4 \\ 17 \end{pmatrix} \tag{115}$$

Exercises:

$$\vec{F_R} = \vec{F_1} + \vec{F_2} \tag{116}$$

$$\vec{F_R} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 17 \end{pmatrix} \tag{117}$$

$$\vec{F_R} = \begin{pmatrix} 14 \\ 22 \end{pmatrix} \tag{118}$$

$$|\vec{F_R}| = \sqrt{14^2 + 22^2} \tag{119}$$

$$|\vec{F_R}| = 26.0768096208 \tag{120}$$

$$\cos(\phi) = \frac{\vec{F_1} \cdot \vec{F_2}}{|\vec{F_1}| * |\vec{F_2}|} \tag{121}$$

$$\cos(\phi) = \frac{\begin{pmatrix} 10 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 17 \end{pmatrix}}{\sqrt{125} * \sqrt{305}} \tag{122}$$

$$\cos(\phi) = \frac{125}{195.256241898} \tag{123}$$

$$\phi = \arccos(0.640184399663) \tag{124}$$

$$\phi = 50.19443^\circ \tag{125}$$

Answer: The resulting force F_R has a magnitude of 26.0768096208 Newton and the angle ϕ between the forces $\vec{F_1}$ and $\vec{F_2}$ is 50.19443°.

8 8.39

8.1 b)

Specification: The forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ form a force triangle.

Requirements: The forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ form a force triangle. Find the missing force graphically and through calculation, such that the force triangle is balanced. (units = kN)

Definitions:

$$\vec{F_1} = \begin{pmatrix} 20 \\ 30 \end{pmatrix} \tag{126}$$

$$\vec{F_2} = \begin{pmatrix} -10 \\ 15 \end{pmatrix} \tag{127}$$

$$\vec{F_3} = ? \tag{128}$$

Exercise:

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{129}$$

$$\vec{F_1} + \vec{F_2} = \begin{pmatrix} 20 \\ 30 \end{pmatrix} + \begin{pmatrix} -10 \\ 15 \end{pmatrix} \tag{130}$$

$$\vec{F_1} + \vec{F_2} = \begin{pmatrix} 10 \\ 45 \end{pmatrix} \tag{131}$$

$$\begin{pmatrix} 10 \\ 45 \end{pmatrix} + \vec{F_3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{132}$$

$$\vec{F_3} = \begin{pmatrix} -10 \\ -45 \end{pmatrix} \tag{133}$$

Answer: The force F_3 has the value $\begin{pmatrix} -10 \\ -45 \end{pmatrix}$.