

Applied Mathematics Exercises for 15. and 16. of February

Lucy Gschwantner

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1 8.26

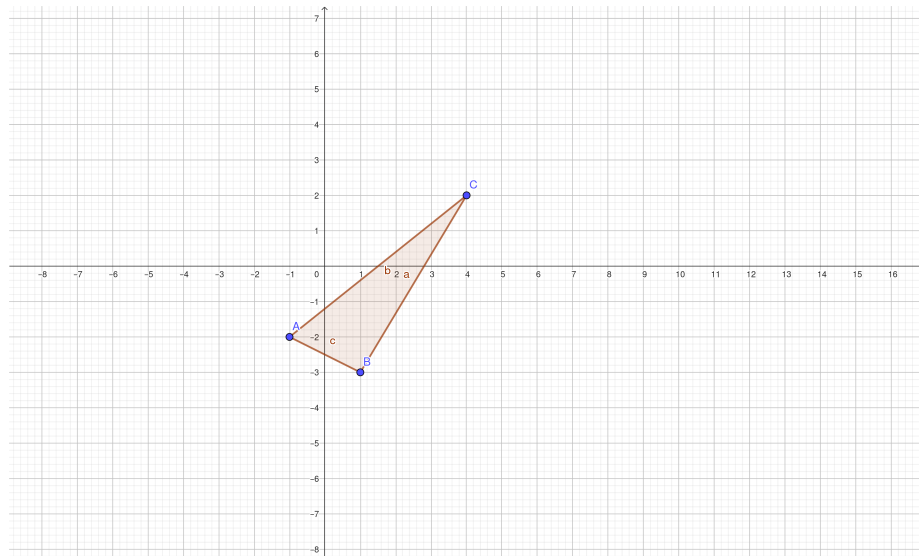
1.1 b)

Specification: Check if these points form a right angled triangle:

$$A(-1|-2)$$

$$B(1|-3)$$

$$C(4|2)$$



Definitions:

$$\vec{a} = \vec{BC}$$

$$\vec{b} = \vec{CA}$$

$$\vec{c} = \vec{AB}$$

Exercises:

$$\vec{a} \cdot \vec{b} = a_x * b_x + a_y * b_y \quad (1)$$

$$\vec{a} \cdot \vec{b} = -1 * 1 + -2 * -3 \quad (2)$$

$$\vec{a} \cdot \vec{b} = -1 + 6 \quad (3)$$

$$\vec{a} \cdot \vec{b} = 5 \quad (4)$$

$$\vec{a} \cdot \vec{c} = a_x * c_x + a_y * c_y \quad (5)$$

$$\vec{a} \cdot \vec{c} = -1 * 4 + -2 * 2 \quad (6)$$

$$\vec{a} \cdot \vec{c} = -4 + -4 \quad (7)$$

$$\vec{a} \cdot \vec{c} = -8 \quad (8)$$

$$\vec{a} \cdot \vec{c} = b_x * c_x + b_y * c_y \quad (9)$$

$$\vec{a} \cdot \vec{c} = 1 * 4 + -3 * 2 \quad (10)$$

$$\vec{a} \cdot \vec{c} = 4 + -6 \quad (11)$$

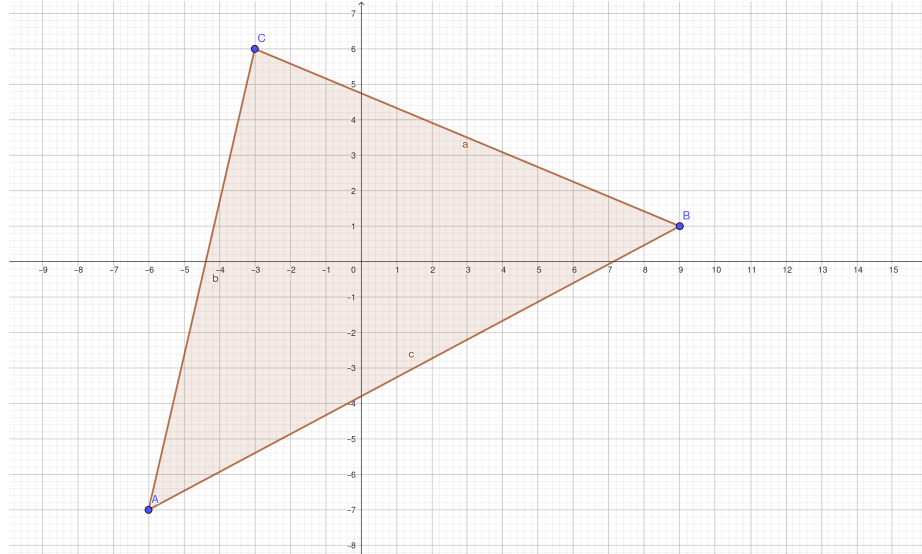
$$\vec{a} \cdot \vec{c} = -2 \quad (12)$$

Answer: The triangle is not a right angle as none of the angles α , β , γ are 90° . This is shown by the fact that none of the dot products of the triangles sides are 0.

2 8.29

2.1 1)

Specification: Find the mistake in the following equations:



$$\vec{c} = \vec{AB} \quad (13)$$

$$= \begin{pmatrix} 15 \\ 8 \end{pmatrix} \quad (14)$$

$$c = |\vec{c}| \quad (15)$$

$$= 17cm \quad (16)$$

$$\vec{a} = \vec{BC} \quad (17)$$

$$= \begin{pmatrix} -12 \\ 5 \end{pmatrix} \quad (18)$$

$$a = |\vec{a}| \quad (19)$$

$$= 13cm \quad (20)$$

$$(21)$$

$$\cos(\beta) = \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| * |\vec{b}|} \quad (22)$$

$$= \frac{\binom{15}{8} * \binom{-12}{5}}{13 * 17} \quad (23)$$

$$= -\frac{140}{221} \quad (24)$$

$$= -0.633... \quad (25)$$

$$\beta = \arccos(-0.633...) \quad (26)$$

$$\approx 129.31^\circ \quad (27)$$

$$A = \frac{a * c * \sin(\beta)}{2} \quad (28)$$

$$= \frac{13 * 17 * \sin(129.31^\circ)}{2} \quad (29)$$

$$\approx 85.5cm^2 \quad (30)$$

Answer: The obtuse value of β (129.31°) was used instead of the acute value of β (50.69°). This didn't affect the result because $\sin(\beta)$ returns the same value for the obtuse and the acute value of β

$$\sin(\alpha) = \sin(180^\circ - \alpha) \tag{31}$$

$$\sin(129.31^\circ) = 0.77368 \tag{32}$$

$$\sin(50.69^\circ) = 0.77368 \tag{33}$$

3 8.31

Specification: An isosceles triangle ABC, which is named in the mathematically positive direction, has the base AB with $A(-2|-1)$, $B(4|7)$ and the height $h = 10E$. Calculate the missing point C, the angle ϕ which is enclosed by \vec{AB} and \vec{AC} , and the area A_{rea} of the triangle.

Definitions:

$$h = 10E \quad (34)$$

$$A = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad (35)$$

$$B = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \quad (36)$$

$$\vec{c} = \vec{AB} \quad (37)$$

$$c_m = \vec{OA} + \frac{1}{2}\vec{c} \quad (38)$$

Exercise:

$$\vec{c} = \vec{B} - \vec{A} \quad (39)$$

$$\vec{c} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad (40)$$

$$\vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \quad (41)$$

$$\vec{c}_m = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ 8 \end{pmatrix} \quad (42)$$

$$\vec{c}_m = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (43)$$

$$\vec{c}_m = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (44)$$

$$\vec{c}_n = \begin{pmatrix} -8 \\ 6 \end{pmatrix} \quad (45)$$

$$\vec{c}_{n0} = \frac{1}{|\vec{c}_n|} \vec{c}_n \quad (46)$$

$$\vec{c}_{n0} = \frac{1}{\sqrt{(-8)^2 + 6^2}} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \quad (47)$$

$$\vec{c}_{n0} = \frac{1}{10} \begin{pmatrix} -8 \\ 6 \end{pmatrix} \quad (48)$$

$$\vec{c}_{n0} = \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \quad (49)$$

$$\vec{OC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + 10 \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} \quad (50)$$

$$\vec{OC} = \begin{pmatrix} -7 \\ 9 \end{pmatrix} \quad (51)$$

4 8.33

4.1 1)

Specification: Given are the points $pA(120|10)$, $pB_1(150|140)$, and $pB_2(500|250)$. Calculate the angle β that encloses the horizontal and the vector $pA\vec{p}B_1$.

Definitions: Let \vec{i} be the horizontal enclosing β .

Exercise:

$$\vec{AB} = \begin{pmatrix} 150 \\ 140 \end{pmatrix} - \begin{pmatrix} 120 \\ 10 \end{pmatrix} \quad (52)$$

$$\vec{AB} = \begin{pmatrix} 30 \\ 130 \end{pmatrix} \quad (53)$$

$$\vec{H} = \vec{A} + \vec{i} \quad (54)$$

$$\vec{H} = \begin{pmatrix} 120 \\ 10 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (55)$$

$$\vec{H} = \begin{pmatrix} 121 \\ 10 \end{pmatrix} \quad (56)$$

$$\cos(\beta) = \frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|} \quad (57)$$

$$\beta = \arccos\left(\frac{\vec{AB} \cdot \vec{H}}{|\vec{AB}| * |\vec{H}|}\right) \quad (58)$$

$$\beta = \arccos\left(\frac{\begin{pmatrix} 30 \\ 130 \end{pmatrix} \cdot \begin{pmatrix} 121 \\ 10 \end{pmatrix}}{\sqrt{30^2 + 130^2} * \sqrt{121^2 + 10^2}}\right) \quad (59)$$

$$\beta = \arccos\left(\frac{30 * 121 + 130 * 10}{133.416640641 * 121.412519947}\right) \quad (60)$$

$$\beta = 72.2809315108^\circ \quad (61)$$

Answer: The angle β that is enclosed by the horizontal and the vector \vec{AB} has a value of 72.2809315108° .

4.2 2)

Specification: Calculate the angle between the vectors $B_1\vec{A}$ and $B_1\vec{B}_2$.

Exercise:

$$B_1 \vec{A} = \begin{pmatrix} 120 \\ 10 \end{pmatrix} - \begin{pmatrix} 150 \\ 140 \end{pmatrix} \quad (62)$$

$$B_1 \vec{A} = \begin{pmatrix} -30 \\ 110 \end{pmatrix} \quad (63)$$

$$B_1 \vec{B}_2 = -1.0 \quad (64)$$

$$B_1 \vec{B}_2 = \begin{pmatrix} 350 \\ 110 \end{pmatrix} \quad (65)$$

$$\cos(\alpha) = \frac{B_1 \vec{A} \cdot B_1 \vec{B}_2}{|B_1 \vec{A}| * |B_1 \vec{B}_2|} \quad (66)$$

$$\alpha = \arccos\left(\frac{\begin{pmatrix} -30 \\ 110 \end{pmatrix} \cdot \begin{pmatrix} 350 \\ 110 \end{pmatrix}}{\left| \begin{pmatrix} -30 \\ 110 \end{pmatrix} \right| * \left| \begin{pmatrix} 350 \\ 110 \end{pmatrix} \right|}\right) \quad (67)$$

$$\alpha = \arccos\left(\frac{1600}{41830.6108012}\right) \quad (68)$$

$$\alpha = 87.80793028^\circ \quad (69)$$

Answer: The angle between the vectors $B_1 \vec{A}$ and $B_1 \vec{B}_2$ is 87.80793028° .

5 8.37

6 8.38

6.1 a)

7 8.39

7.1 b)