

Tarea #2b.

Sección 4.3.

13. $3y'' + 2y' + 2y = 0$

$3r^2 + 2r + 2 = 0$ raíces: $\frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot 2}}{6} = \frac{-1 \pm \sqrt{-20}}{3}$

$y = C_1 e^{-1/3x} \cdot \cos\left(\frac{\sqrt{5}}{3}x\right) + C_2 e^{-1/3x} \cdot \sin\left(\frac{\sqrt{5}}{3}x\right)$

14. $2y'' - 3y' + 4y = 0$

$2r^2 - 3r + 4$

$\frac{3 \pm \sqrt{9 - 4 \cdot 2 \cdot 4}}{4} = \frac{3 \pm \sqrt{-23}}{4} = \frac{3}{4} \pm \frac{\sqrt{23}}{4}i$

$y = e^{3/4x} \left(\cos\left(\frac{\sqrt{23}}{4}x\right) + \sin\left(\frac{\sqrt{23}}{4}x\right) \right)$

27. $U^{(5)} + 5U^{(4)} - 2U^{(3)} - 10U'' + U' + 5U = 0$

$p^5 + 5p^4 - 2p^3 - 10p^2 + p + 5 = 0$

1 | 1 5 -2 -10 1 5
1 | 1 6 4 -6 -5

raíces: 1, -1, -5
#2, #2, #1

-1 | 1 6 4 -6 -5 | 0
-1 | -1 -5 1 5

-5 | 1 5 -1 -5 | 0
-5 | -5 0 5

$y = C_1 e^r + C_2 x e^r + C_3 e^{-r} + C_4 x e^{-r} + C_5 e^{-5r}$

1 | 1 0 -1 | 0
1 | 1 1

1 | 1 1 | 0

-1 | -1 | 0

1 | 0

$$28. \quad 2x^{(5)} - 7x^{(4)} + 12x''' + 8x'' = 0$$

$$p(q) = 2q^5 - 7q^4 + 12q^3 + 8q^2$$

$$\begin{array}{r|rrrrrr} 0 & 2 & -7 & 12 & 8 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 \\ \hline & 2 & -7 & 12 & 8 & 0 & 0 \end{array}$$

$$\text{raíces: } 2, -8, 16, 2 \pm 2i$$

$$\begin{array}{r|rrrr} 0 & 2 & -7 & 12 & 8 \\ -1/2 & & -1 & 4 & -8 \\ \hline & 2 & -8 & 16 & 0 \end{array}$$

$$2q^2 - 8q + 16 = 0$$

$$8 \pm \sqrt{64 - 4 \cdot 16 \cdot 2} = 2 \pm 2i$$

$$y = c_1 e^{2x} + c_2 e^{-8x} + c_3 e^{16x} + c_4 e^{2x} \cos(2x) + c_5 e^{2x} \sin(2x)$$

$$36. \quad y''' + 2y'' - 5y' - 6y = 0$$

$$r^3 + 2r^2 - 5r - 6 = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -5 & -6 \\ 2 & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$$\text{raíces: } 2, -3, -1$$

$$\textcircled{1} \quad y = c_1 e^{2x} + c_2 e^{-3x} + c_3 e^{-x}$$

$$\textcircled{2} \Rightarrow y' = 2c_1 e^{2x} - 3c_2 e^{-3x} - c_3 e^{-x}$$

$$r^2 + 4r + 3 = (r+3)(r+1) \quad \textcircled{3} \Rightarrow y'' = 4c_1 e^{2x} + 9c_2 e^{-3x} + c_3 e^{-x}$$

P.V.F.

$$y(0) = 1 \quad \textcircled{1} \quad 1 = c_1 + c_2 + c_3 \Rightarrow c_3 = 1 - c_1 - c_2$$

$$y'(0) = 1 \quad \textcircled{2} \quad 1 = 2c_1 - 3c_2 - c_3 \Rightarrow 1 = 2c_1 - 3c_2 - 1 + c_1 + c_2$$

$$\textcircled{3} \quad 1 = 4c_1 + 9c_2 + c_3 \Rightarrow 1 = 3c_1 - 2c_2 - 1$$

$$\Rightarrow 2 + 2c_2 = c_1$$

$$\Rightarrow 1 = 4\left(\frac{2+2c_2}{3}\right) + 9c_2 + 1 - c_1 - c_2$$

$$\Rightarrow 0 = 8 + 8c_2 + 27c_2 + 3 - 2 - 2c_2 - c_2$$

$$\Rightarrow 33c_2 = -6 \Rightarrow c_2 = -6/33 \Rightarrow c_1 = 6/11 \Rightarrow c_3 = 7/11$$

$$y = \frac{6}{11} e^{2x} - \frac{6}{33} e^{-3x} + \frac{7}{11} e^{-x}$$

$$40. y'' - 2y' + 2y = 0$$

$$r^2 - 2r + 2 = 0$$

$$\text{raíces: } 1 \pm i$$

$$y = c_1 e^x \cdot \cos(x) + c_2 e^x \cdot \sin(x)$$

$$y(0) = 1 \Rightarrow 1 = c_1 \cdot e^0$$

$$y(\pi) = 1 \Rightarrow 1 = -c_1 \cdot e^\pi$$

No tiene solución para el P.V.F.

43.-48.

	raíces	solución
a) $y'' - 3y' - 4y = 0$	4, -1	$y = c_1 e^{4x} + c_2 e^{-x}$
b) $y'' + 4y = 0$	$\pm 2i$	$y = c_1 \cos(2x) + c_2 \sin(2x)$
c) $y'' + 2y' + y = 0$	-1, -1	$y = c_1 \cdot e^{-x} + c_2 \cdot e^{-x} = e^{-x}(c_1 + c_2)$
d) $y'' + y = 0$	$\pm i$	$y = c_1 \cos x + c_2 \sin x$
e) $y'' + 2y' + 2y = 0$	$-1 \pm i$	$y = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x$
f) $y'' - 3y' + 2y = 0$	2, 1	$y = c_1 e^{2x} + c_2 e^x$

43. f), es exponencial

47. d) cos y sen normal

44. a) Graficadora

48. b) cos y sen de $2x$.

45. e) M.A.S amortiguado

46. c) (solo una raíz)

49. $m_1 = 4$ factores $(m-4)$
 $m_2 = -5$ $(m+5)^2$
 $m_3 = -5$

$$(m-4)(m+5)^2 = m^3 + 6m^2 - 15m - 100$$

$$y''' + 6y'' - 15y' - 100 = 0$$

No, no es única, puede ser una qe sea L.D. a este

54. $y'' + \lambda y = 0$

$$r^2 + 1 = 0$$

\Rightarrow raíces: $\pm i$

$$y = C_1 e^x \cos x + C_2 e^x \sin x$$

$$0 = C_1 e^x \cos x$$

$$y(0) = 0$$

a) si

$$0 = C_2 e^x \sin x$$

$$y(\pi/2) = 0$$

b) si

59. $3.15y^{(4)} - 5.34y'' + 6.33y' - 2.03y = 0$

$$3.15r^4 - 5.34r^2 + 6.33r - 2.03 = 0$$

Rootes: -1.74806

0.501219

$0.6342 \pm 0.588965i$

$$y = C_1 e^{-1.74x} + C_2 e^{0.5x} + C_3 e^{0.63x} \cos x + C_4 e^{0.63x} \sin x$$