UVG-MM2014

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16 Nov 2009

Tarea 4a.

Sección 4.1

1. $\sum_{n=1}^{\infty} \frac{3^n}{n} x^n$; $\lim_{n\to\infty} \sum_{n=1}^{\infty} \frac{3^n}{n} x^n = x \cdot \frac{\frac{3^{n+1}}{n+1}}{\frac{3^n}{n}} = x \cdot \frac{3^{n+1} n}{3^n (n+1)} = 3x \frac{n}{n+1}$

converge para

$|3x| < 1 \iff x < 1/3 \quad x > -1/3$

$\boxed{R: 1/3 \text{ , intervalo de convergencia: } [-1/3, 1/3)}$

3. $\sum_{k=1}^{\infty} \frac{(-1)^k}{10^k} (x-2)^k$; $\lim_{k\to+\infty} \sum_{k=1}^{\infty} \frac{(-1)^k}{10^k}(x-2)^k = |x-2| \frac{\frac{(-1)^{k+1}}{10^{k+1}}}{\frac{-1^k}{10^k}}$

$= -|x-2| \frac{10^k}{10^{k+1}} = -\frac{|x-2)}{10}$

converge ssi $-|x-2| < 10 \iff |x-2| > 10$

$x-2 < -10 \quad \wedge \quad x-2 > 10 \iff x < -8 \quad \wedge \quad x > 12$

$\boxed{R: 10 \text{ , Intervalo: } (-8, 12)}$

4. $\sum_{k=0}^{\infty} k!(x-1)^k$ ~~si~~ converge $\iff \lim_{k\to+\infty} \frac{(k+1)!}{k!} |x-1| < 1 \iff \lim_{k\to+\infty} (k+1)|x-1| < 1$

$\boxed{\text{siempre diverge, } R=0 \text{ , Intervalo: } () \text{ , converge (solo) para } x = 1}$

6. $e^{-x}\cdot\cos x = \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \cdots\right)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\cdots\right.$

$= 1 - x + \left(\frac{1}{2} - \frac{1}{2}\right)x^2 - \left(\frac{1}{2}\right)x^3 + \left(\frac{1}{4!} + \frac{1}{2!}\right)x^4 \cdots$

7. $\frac{1}{\cos x} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$$\boxed{= 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots}$$

$$\begin{array}{l|l} & 1 + \frac{x^2}{2!} + \frac{5x^4}{24} + \cdots \\ \hline 1 & 1 \\ & -\left(\cancel{1} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\cdots\right) \\ \hline & \cancel{\frac{x^2}{2!}} - \frac{x^4}{4!} + \frac{x^6}{6!} - \cdots \\ & -\left(\cancel{\frac{x^2}{2!}} - \frac{x^4}{2!\cdot 2!} + \frac{x^6}{4!\,2!}\cdots\right) \\ \hline & \frac{5x^4}{24} - \frac{24x^6}{720}\cdots \end{array}$$

$\boxed{\text{Intervalo: } (-\pi/2, \pi/2)}$

8. $\frac{1-x}{2+x} =$

$$\begin{array}{r|l} & 1/2 - 3/4x + 3/8x^2 - 3/16x^3 \\ \hline 2+x & 1 - x \\ & -(1 + 1/2x) \\ \hline & -3/2x \\ & -(-3/2x - 3/4x^2) \\ \hline & 3/4x^2 \\ & -(3/4x^2 + 3/8x^3) \\ \hline & -3/8x^3 \\ & -(-3/8x^3 - 3/16x^4) \end{array}$$

$$\boxed{= \frac{1}{2} - 3/4x + 3/8x^2 - 3/16x^3\cdots}$$

Intervalo: $(-2, 2)$

14. Comprobar que

$y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2}x^{2n}$ es solución de $xy'' + y' + xy = 0$

$\Rightarrow y' = \sum_{n=0} \frac{2n(-1)^n}{2^{2n}(n!)^2}x^{2n-1} \Rightarrow y'' = \sum_{n=0}^{+\infty} \frac{2n(2n-1)(-1)^n}{2^{2n}(n!)^2}x^{2n-2}$

Probando:

$xy'' + y' + xy = \sum_{n=1} \frac{2n(2n-1)(-1)^n}{2^{2n}(n!)^2}x^{2n-1} + \sum_{n=1} \frac{2n(-1)^n}{2^{2n}(n!)^2}x^{2n-1} + \sum_{n=0}^{+\infty} \frac{(-1)^n}{2^{2n}(n!)^2}x^{2n+1} = 0$

$\Leftrightarrow \sum_{n=1}^{\infty}\left[\frac{2n(2n-1)(-1)^n}{2^{2n}(n!)^2} + \frac{2n(-1)^n}{2^{2n}(n!)^2} + \frac{(-1)^{n-1}}{2^{2(n-1)}((n-1)!)^2}\right]x^{2n-1} = 0$

$\Leftrightarrow \sum_{n=1}^{\infty}\left[\frac{2n(-1)^n(2n)}{2^{n}(n!)^2} - \frac{(-1)^n}{2^{2n-2}[(n-1)!]^2}\right]x^{2n-1} = 0 \Leftrightarrow \sum_{n=1}^{\infty}(-1)^n\left[\frac{4n^2}{2^{2n}(n!)^2} - \frac{1}{2^{2n-1}[(n-1)!]^2}\right]$

$\Leftrightarrow \sum_{n=1}^{\infty}(-1)^n\left[\frac{2^{-2}(4n^2) - n^2}{2^{2n}\cdot 2^{-2}(n!)^2}\right]x^{2n-1} = 0$ ✓ $\boxed{\text{cierto}}$

15. $(x^2-25)y'' + 2xy' + y = \cdot$

respecto a $x=0$, $x = \pm 5$ es un pto singular. $\Rightarrow R = 5.$

respecto a $x=1$, un límite inferior es $4. \Rightarrow R=4.$

16. $(x^2-2x+10)y'' + xy' - 4y = 0$

$x = 1+3i$ y $x_2 = 1-3i$ son ptos singulares

para $x=0$, el límite inferior es $+\sqrt{10}$

para $x=1$, el límite inferior es 3

25. $y'' - (x+1)y' - y = 0$

$$\sum_{n=2} n(n-1)C_n x^{n-2} - \sum_{n=1} C_n x^n - \sum_{n=1} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^n = 0$$

$$2C_2 - C_1 - C_0 + \sum_{n=0}^{\infty}\left[(n+2)(n+1)C_{n+2} - nC_n - (n+1)C_{n+1} - C_n\right]x^n = 0$$

$$2C_2 - C_1 - C_0 = 0 \Rightarrow 2C_2 = C_0 + C_1 \Rightarrow C_2 = \frac{C_0 + C_1}{2}$$

Ec. Rec.

$$C_{n+2} = \frac{(n+1)C_{n+1} + C_n(n+1)}{(n+1)(n+2)} = \frac{\cancel{(n+1)}(C_{n+1} + C_n)}{\cancel{(n+1)}(n+2)} = \frac{C_{n+1} + C_n}{n+2}$$

con $C_0 = 1$ y $C_1 = 0$, $C_2 = 1/2$, $C_3 = 1/6$, $C_4 = 1/6$ ..

$$\Rightarrow y = 1 + 1/2\, x^2 + 1/2\, x^3 + 1/3\, x^4$$

26. $\sum_{n=0} n(n-1)C_n x^n + \sum_{n=2} n(n-1)C_n x^{n-2} - 6\sum_{n=0}^{+\infty} C_n x^n = 0$

$$\sum_{n=0}^{\infty}\left[n(n-1)C_n + (n+2)(n+1)C_{n+2} - 6C_n\right]x^n = 0$$

$$C_{n+2} = \frac{6C_n - n(n-1)C_n}{(n+1)(n+2)} = \frac{[n(1-n)+6]C_n}{(n+1)(n+2)} \Rightarrow$$

$C_2 = 3C_0$, $C_3 = C_1$

$C_4 = C_0$, $C_5 = 0$

$C_6 = -1/6\, C_0$

$$\Rightarrow y_1 = x + x^3, \quad y_2 = 1 + 1/3\, x^2 + x^4 - 1/5\, x^6$$

40. $(x^2-1)y'' + xy' - y = 0$

$$\sum_{n=2}^{\infty} c_n n(n-1)x^n - \sum_{n=2} c_n \cdot n(n-1)x^{n-2} + \sum_{n=1} nx^n c_n - \sum_{n=0} c_n x^n = 0$$

$$-2c_2 - c_0 - 6c_3 x + \sum_{n=2}^{\infty} \left[-c_{n+2}(n+2)(n+1) + nc_n - c_n + c_n(n^2-n) \right.$$

$$c_3 = 0 \;,\; c_2 = -1/2 \Rightarrow c_{n+2} = \frac{(n^2-1)c_n}{(n+2)(n+1)} = \frac{n-1}{n+2} c_n$$

$$\boxed{\Rightarrow y(x) = c_0 + c_1 x - 1/2\, c_0 x^2 - 1/8\, c_0 x^4 \ldots}$$

34. $y'' + e^x y' - y = 0 = \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right) \sum_{n=1} n c_n x^{n-1}$

$$= 2c_2 - 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + \ldots + \left(1 + x + \frac{x^2}{2!} \ldots\right)(c_1 + 2c_2 + 3c_3 x^2 \ldots)$$

$$= (2c_2 + c_1 - c_0) * (6c_3 + 2c_2)x + [c_1 + (2c_2 + c_1)x + (3c_3 + 2c_2 + c_1/2)x^2 .$$

$$c_0 - c_1 x - c_2 x^2 \ldots$$

$$= (2c_2 + c_1 - c_0) * (6c_3 + 2c_2)x + (12c_4 + 3c_3 + c_2 + c_1/2)x^2 + \ldots = 0$$

$$\Leftrightarrow 2c_2 + c_1 - c_0 = 0 \;\wedge\; 6c_3 + 2c_2 = 0 \;\wedge\; 12c_4 + 3c_3 + c_2 + c_1/2$$

$$\Rightarrow c_0 = 0, c_1 \neq 0, c_2 = -1/2\, c_1, c_3 = 1/6\, c_1, c_4 = -1/24\, c_1$$

$$\boxed{\begin{aligned} y_1 &= c_0 + 1/2\, x^2 c_0 - 1/6\, x^3 c_0 + \ldots \\ y_2 &= c_1 x - 1/2\, c_1 x^2 + 1/6\, c_1 x^3 - 1/24\, c_1 x^4 \ldots \end{aligned}}$$

37. $\phi(x) = \frac{\operatorname{sen} x}{x}$ que es una función analítica, entonces $x=0$ es un punto ordinario.