UVG - MM2015

Ecuaciones Diferenciales

Carlos López Carrey #08107 8 sept 09

Tarea 2c

1. $y'' + 3y' + 2y = 6$

$r^2 + 3r + 2 = 0 \iff (r+2)(r+1) = 0$

$\Rightarrow y_c = c_1 \cdot e^{-2x} + c_2 \cdot e^{-x}$

$y_p = A \Rightarrow y_p' = 0 \Rightarrow y_p'' = 0$

$\cancel{y_p''} + 3\cancel{y_p'} + 2y_p = A$

$\Rightarrow 2y_p = A \Rightarrow y_p = 3$

$$\boxed{\text{Resp: } y = c_1 \cdot e^{-2x} + c_2 \cdot e^{-x} + 3}$$

5. $\frac{1}{4}y'' + y' + y = x^2 - 2x$

$\Rightarrow y'' + \frac{y'}{4} + \frac{y}{4} = \frac{1}{4}x^2 - \frac{1}{2}x$

$r^2 + 1/4 r + 1/4 = 0 \iff r = \frac{1}{8}(-1 \pm \sqrt{15}i)$

$\Rightarrow y_c = e^{-1/8x} \cdot \cos(\sqrt{15}x) + e^{-1/8} \cdot \mathrm{sen}(\sqrt{15}x)$

$y_p = Ax^2 - Bx + C$

$\Rightarrow y_p' = 2Ax - B$

$\Rightarrow y_p'' = 2A$

$y_p'' + \frac{y_p'}{4} + \frac{y_p}{4} = 2A + \frac{1}{4}(2Ax - B) + \frac{1}{4}(Ax^2 - Bx$

$= x^2 - 2x$. 2 3

$\Rightarrow \frac{1}{4}Ax^2 = x^2$ 1 $, -\frac{1}{4}Bx + \frac{1}{2}Ax = -2x, 0 = 2A - \frac{1}{4}B\cdot$

$y_p = 4x^2 - 10x - 11/2$

$\Rightarrow A = 4$ 1 $, -B + 2A = -2$

$\Rightarrow +B = 2(4) + 2 = 10$ 2

3 $2(4) - \frac{1}{4}(10) = -C = C = -11/2$

Sol:

$y = e^{-1/8x}\left(\cos(\sqrt{15}x) + \mathrm{sen}(\sqrt{15}x)\right) + 4x^2 - 10x - 11/2$

10. $y'' + 2y' = 2x + 5 - e^{-2x}$

$r^2 + 2r = 0 \iff r = 0, r = -2$

$\Rightarrow y_c = c_1 + c_2 \cdot e^{-2x}$

$y_p = Ax + B + C \cdot xe^{-2x}$

$\Rightarrow y_p' = A + C \cdot (e^{-2x} - 2xe^{-2x})$

$\Rightarrow y_p'' = A + C(-2e^{-2x} - 2[e^{-2x} - 2xe^{-2x}])$

$\Rightarrow \cancel{A} + C(-4e^{-2x} + 4\cancel{xe^{-2x}}) + 2\cancel{A} + 2Ce^{-2x} - \cancel{4Cxe^{-2x}}$

$= 3A - 2Ce^{-2x} = 2x + 5 - e^{-2x}$

$\Rightarrow 3A = 5 \Rightarrow A = 5/3$

$\Rightarrow + 2Ce^{-2x} = +e^{-2x}$ $\quad y_p = \frac{5}{3}x + \frac{1}{2}(-xe^{-2x})$

$\Rightarrow C = 1/2$

$$\boxed{y = c_1 + c_2 e^{-2x} + \frac{5}{3}x + \frac{1}{2}(-xe^{-2x}}$$

15. $y'' + y = 2x \operatorname{sen} x$

$r^2 + r = 0 \iff r = 0, r = -1$ $\quad y_c = c_1 + c_2 e^{-x}$

$y_p = (Ax + B)\operatorname{sen} x + (Cx + D)\cos x$

$y_p' = A(\operatorname{sen} x + x\cos x) + B\cos x + C(\cos x - x\operatorname{sen} x) - D\operatorname{sen} x$

$y_p'' = A\cos x + A(\cos x - x\operatorname{sen} x) - B\operatorname{sen} x - C\operatorname{sen} x - C(\cancel{\operatorname{sen} x} + x\cos x)$

$= A\cancel{\cos x} + A(\cos x - x\operatorname{sen} x) - \operatorname{sen} x(B + 2C) - Cx\cos x$

$- D\cos x = 2A\cos x - Ax\operatorname{sen} x$

$- D\cos x = \cos x(2A - D - Cx) + \operatorname{sen} x(-Ax - B - 2C)$

$= 2x\operatorname{sen} x$

$\Rightarrow \cos x(2A - D - Cx) = \operatorname{sen} x(Ax + B + C + 2x)$

$y = c_1 + c_2 e^{-x} + \frac{1}{4}(-2x\cos x + 2x\operatorname{sen} x + \cos x)$

20. $y'' + 2y' - 24y = 16 - (x+2)e^{4x}$

$r^2 + 2r - 24 = 0 \Leftrightarrow r = 4,\ r = -6$

$y_c = C_1 e^{4x} + C_2 e^{-6x}$

$y_p = (Ax+B)e^{4x} + C$

$\Rightarrow y_p' = A(e^{4x} + 4xe^{4x}) + 4Be^{4x} \Rightarrow y_p'' = \cancel{4Ae^{4x}} + 4A(e^{4x} + 4xe^{4x}) + 16Be^{4x}$

$e^{4x}(8A + 4x + 16B) + 2e^{4x}(A + 4Ax) + e^{4x}(Ax+B) + C$

$= -(x+2)e^{4x} + 16 \Rightarrow C = 16$

$\Rightarrow \cancel{8}A + 4\cancel{A}x + 16B + 2A + 8\cancel{A}x + \cancel{A}x + B = -x + 2$

$\Rightarrow -1 = 4 + 9A \Rightarrow A = -5/9$

$\Rightarrow 10A + 17B = 2 \Rightarrow B = \dfrac{2 - 10(-5/9)}{17} = 4/9$

$$\boxed{y = C_1 e^{4x} + C_2 e^{-6x} + \cancel{(Ax}\left(-\frac{5}{9}x + 4/9\right)e^{4x} - x + 16}$$

25. $y^{(4)} - y'' = 4x + 2xe^{-x}$

$r^4 - r^2 = 0 \Leftrightarrow r = 0,\ r = \pm i$ $\qquad y_c = C_1 \operatorname{sen} x + C_2 \cos x + C_3$

$y_p = Ax + (Bx + C)e^{-x}$

$y_p' = A + B(e^{-x} - xe^{-x}) + Ce^{-x} = A + e^{-x}(B - Bx + C)$

$y_p'' = Be^{-x} - B(e^{-x} - xe^{-x}) + Ce^{-x} = e^{-x}(\cancel{B} + C - \cancel{B} - Bx)$

$y_p''' = Ce^{-x} - B(e^{-x} - xe^{-x}) = e^{-x}(C - B + Bx)$

$y_p'''' = Ce^{-x} - Be^{-x} + B(e^{-x} - xe^{-x}) = e^{-x}(C - \cancel{B} + \cancel{B} - Bx)$

$e^{-x}(C - Bx) + e^{-x}(C - Bx) = 4x + 2xe^{-x}$

$\Rightarrow e^{-x}(2C - 2Bx) = e^{-x}\left(2x + \dfrac{4x}{e^{-x}}\right) \Rightarrow 2C - 2Bx = 2x + \dfrac{4x}{e^{-x}}$

30. $y'' + 4y' + 4y = (3+x)e^{-2x}$, $y(0) = -3$, $y'(0) = 1$

$r^2 + 4r + 4 = 0 \Leftrightarrow r = -2$

$$y_c = C_1 e^{-2x} + C_2 x \cdot e^{-2x}$$

$y_p = (\cancel{Ax+B})e^{-2x}$

$y_p = (Ax+B) \cdot x^2 \cdot e^{-2x}$

$y_p' = (A) x^2 \cdot e^{-2x} + (Ax+B) \cdot [2xe^{-2x} - 2x^2 e^{-2}$

$= Ax^2 \cdot e^{-2x} + (Ax+B)[e^{-2x}(2x - 2x^2)$

$= e^{-2x}[Ax^2 + 2(Ax+B)(x-x^2)]$

$y_p'' = -2e^{-2x}[Ax^2 + 2(Ax+B)(x-x^2)]$

$+ [2Ax + 2[(A)(x-x^2) + (1-2x)(Ax+B)]] \cdot e^{-2x}$

$= e^{-2x}[-2Ax^2 - 4(Ax+B)(x-x^2) + 2Ax + 2A(x-x^2) + 2(1-2$
$(Ax+B$

$= e^{-2x}[-2Ax^2 - 4[Ax^2 - Ax^3 + Bx$

35. $y''' - 2y'' + y' = 2 - 24e^x + 40e^{5x}$ $y(0) = 1/2$ 5.

$y'(0) = 5/2$

$y''(0) = -9/2$

$r^3 - 2r^2 + r = 0 \Leftrightarrow r = 0, r = 1$

$r(r^2 - 2r + 1)$

$y_c = C_1 + C_2 e^x + C_3 e^x \cdot x$

$y_p = Ax^2 + Bx^2e^x + Cx^2e^x = x^2(A + Be^x + Ce^{5x})$

37. $y'' + y = x^2 + 1$ $y(0) = 5$

$y(1) = 0$

$r^2 + 1 = 0 \Leftrightarrow r = \pm i$

$y_c = C_1 \operatorname{sen} x + C_2 \cos x$

$y_p = Ax^2 + Bx + C$

$\Rightarrow y'_p = 2Ax + B$

$2Ax + B + Ax^2 + Bx + C = x^2 + 1$

$\Rightarrow A = 1$

$\Rightarrow 2A + B = 0 \Rightarrow B = -2$

$\Rightarrow B + C = 1 \Rightarrow 3 = C$

$y = C_1 \operatorname{sen} x + C_2 \cos x + x^2 - 2x + 3$

$y(0) = 5$

$\Rightarrow 5 = C_2 + 3 \Rightarrow C_2 = 2$

$y(1) = 0$

$\Rightarrow 0 = C_1 \operatorname{sen}(1) + 2\cos(1) + 1 - 1 + 3$

$\Rightarrow C_1 = \dfrac{-3 - 2\cos(1)}{\operatorname{sen}(1)}$

$$y = \frac{-3 - 2\cos(1)}{\operatorname{sen}(1)} \cdot \operatorname{sen} x + 2\cos x + x^2 - 2x + 3$$

40. $y'' + 3y = 6x$, $y(0) = 0$, $y(1) = 0$.

$r^2 + 3 = 0 \iff r = \pm \sqrt{3}\, i$

$y_c = C_1 \operatorname{sen}(\sqrt{3}x) + C_2 \cos(\sqrt{3}x)$

$y_p = Ax + B$

$y_p' = A$

$Ax + B = 6x$

$\Rightarrow A = 6 \Rightarrow B = 0$

$y = C_1 \operatorname{sen}\sqrt{3}x + C_2 \cos\sqrt{3}x + 6x$

$\Rightarrow 0 = C_2$

$0 = C_1 \operatorname{sen}\sqrt{3} \Rightarrow C_1 = 0$

$\boxed{y = 6x}$

41. $y'' + 4y = g(x)$ $y(0) = 1$, $y'(0) = 2$

$g(x) = \begin{cases} \operatorname{sen} x & 0 \leq x \leq \pi/2 \\ 0 & x > \pi/2 \end{cases}$

$r^2 + 4 = 0 \iff r = \pm 2i$

$y_c = C_1 \operatorname{sen} 2x + C_2 \cos 2x$

Para $x > \pi/2$

$y = C_1 \operatorname{sen} 2x + C_2 \cos 2x \Rightarrow y' = 2C_1 \cos 2x - 2C_2 \operatorname{sen} 2x$

PVF. $1 = C_2$ y $2 = 2C_1 \Rightarrow C_1 = 1$

$y = \operatorname{sen} 2x + \cos 2x$

Para $0 \leq x \leq \pi/2$

$y_p = \operatorname{sen} x + \cos x$ $\Rightarrow 0 = \operatorname{sen} x$

$\Rightarrow y_p'' = -\operatorname{sen} x - \cos x$