UVG - MM2014

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30/10/09

Tarea 3c

7) $\mathcal{L}\{t \cdot e^{2t} \operatorname{sen} 6t\} = \frac{d}{dt}\mathcal{L}\{e^{2t}\operatorname{sen}6t\} = \frac{d}{dt}\left[\frac{6}{(s-2)^2+36}\right]$

$$\boxed{= \frac{12s-24}{[(s-2)^2+36]^2}}$$

10) $y' - y = t \cdot e^t \operatorname{sen} t$; $y(0)=0$

$sY(s) - Y(s) = \frac{d}{dt}\left(\frac{1}{(s-1)^2+1}\right) = \frac{(2s-1)1}{[(s-1)^2+1]^2}$

$= \frac{(2s-1)}{[(s-1)^2+1]^2} = Y(s)[s-1] \Rightarrow Y(s) = \frac{2s-1}{[(s-1)^2+1]^2(s-1)}$

por Heavyside:

$Y(s) = \frac{1}{s-1} + \frac{1}{[(s-1)^2+1]} - \frac{s}{[(s-1)^2+1]} + \frac{3}{[(s-1)^2+1]^2} - $

$\rightarrow \boxed{y(t) = e^t - e^t\cos t - t \cdot e^t \cos t}$

13) $y'' + 16y = f(t)$ $y(0)=0$, $y'(0)=1$

$f(t) = \begin{cases} \cos 4t & 0 \leq t \leq \pi \\ 0 & t \geq \pi \end{cases} = \cos 4t - \cos 4t\, \mathcal{U}(t-\pi)$

$s^2Y(s) - \cancel{sy(0)} - y'(0) + 16Y(s) = \frac{s}{s^2+16} - e^{-\pi}\left(\frac{s}{s^2+16}\right)$

$Y(s)[s^2+16] = \frac{s}{s^2+16} - \frac{e^{-\pi}s}{s^2+16} + 1 \Rightarrow Y(s) = \frac{s}{[s^2+16]^2} - e^{-\pi s}\left[\frac{s}{(s}\right.$

$$\boxed{\Rightarrow y(t) = 1/4 \operatorname{sen} 4t + 1/8 \cos 4t - 1/8(t-\pi)\cos 4(t-\pi)\,\mathcal{U}}$$

19) $\mathcal{L}\{1 * t^3\} = \mathcal{L}\{1\}\,\mathcal{L}\{t^3\} = \left(\frac{1}{s}\right)\left(\frac{6}{s^4}\right) = \boxed{\frac{6}{s^5}}$

21) $\mathcal{L}\{e^{-t} * e^{-t}\cos t\} \boxed{= \left(\frac{1}{s+1}\right)\left(\frac{(s-1)}{(s-1)^2+1}\right)}$

22) $\mathcal{L}\{e^{2t} * \operatorname{sen} t\} = \boxed{\left(\frac{1}{s-2}\right)\left(\frac{1}{s^2+1}\right)}$

28) $\mathcal{L}\{\int_0^t \operatorname{sen}\tau \cdot \cos(t-\tau)\,d\tau\} = \mathcal{L}\{\operatorname{sen}t * \cos t\} = \boxed{\left(\frac{1}{s^2+1}\right)}$

31) $\mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} = \int_0^t e^{\tau}(1)\,d\tau \boxed{= e^t - 1}$

33) $\mathcal{L}^{-1}\left\{\frac{1}{s^3(s-1)}\right\} = e^t * \left(\frac{1}{2}t^2\right) = \frac{1}{2}\int_0^t e^{\tau}(t-\tau)^2\,d\tau$

$= \frac{1}{2}\int_0^t e^{\tau}(t^2 - 2t\tau + \tau^2) = \frac{1}{2}\left[-t^2 - 2t + 2e^t - 2\right]$

$\boxed{= e^t - \frac{1}{2}t^2 - t - 1}$

45) $y'(t) = 1 - \operatorname{sen} t - \int_0^t y(\tau)\,d\tau \quad ; \; y(0) = 0$

$sY(s) - y(0)^{=0} = \frac{1}{s} - \frac{1}{s^2+1} - \frac{Y(s)}{s}$

$Y(s)\left[s + \frac{1}{s}\right] = \frac{1}{s} - \frac{1}{s^2+1} \Rightarrow Y(s) = \left(\frac{1}{s^2+1}\right) - \left(\frac{s}{s^2+1}\right.$

$= \frac{1}{s^2+1} - \frac{s}{(s^2+1)^2} \quad \boxed{\Rightarrow y(t) = \operatorname{sen}t - \frac{1}{2}t \cdot \operatorname{sen}(t)}$

48) $L = 0.005, \quad R = 1, \quad C = 0.002$

$E(t) = 100\,[\,t - (t-1)\mathcal{U}(t-1)\,]$

$Lq'' + Rq' + \frac{1}{C}q = E(t) \qquad i(0) = 0$

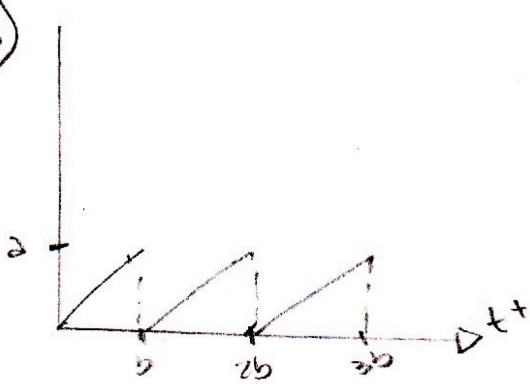
$0.005\,q'' + q' + 500q = 100\,[\,t - (t-1)\mathcal{U}(t-1)\,]$

$0.005\,[\,s^2 Q(s) - \cancel{sq(0)}^{0} - \cancel{q'(0)}^{0}\,] + sQ(s) - \cancel{q(0)}^{0} + 500\,Q(s) =$
$100\,[\,1/s - e^{-s}(1/s)\,]$

$Q(s)\,[\,0.005s^2 + s + 500\,] = 100\,[\,1/s - e^{-s}(1/s)\,]$

$\Rightarrow Q(s) = \dfrac{100\,[\,1/s - e^{-s}(1/s)\,]}{0.005s^2 + s + 500}$

50)



$\mathcal{L}\{f(t)\} = \dfrac{1}{1-e^{-sb}} \displaystyle\int_0^b e^{-st}\,\frac{a}{b}\,t\,dt$

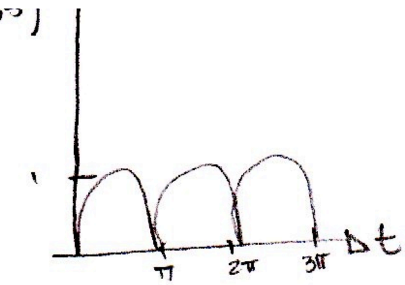
$\dfrac{a/b}{1-e^{-sb}} \displaystyle\int_0^b t e^{-st}\,dt$

$= \dfrac{a/b}{1-e^{-sb}} \left[\, \dfrac{-t}{s} e^{-st} + \dfrac{e^{-st}}{-s^2} \,\right]\Bigg|_0^b$

$= \dfrac{a/b}{1-e^{-sb}} \left(-\dfrac{be^{-sb}}{s} - \dfrac{e^{-sb}}{s^2} + 1/s^2 \right) = \dfrac{a}{s}\left(\dfrac{-e^{-sb}}{1-e^{-sb}} + \dfrac{(1-e^{-sb})}{(1-e^{-sb})sb} \right.$

$= \dfrac{a}{s}\left(\dfrac{1}{sb} - \dfrac{e^{-sb}/e^{-sb}}{1/e^{-sb} - e^{-sb}/e^{-sb}} \right) = \boxed{\dfrac{a}{s}\left[\, \dfrac{1}{sb} - \dfrac{1}{e^{sb} - 1} \,\right]}$

5b)



$$T = \pi$$

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-\pi s}}\int_0^{\pi} e^{-st}\,\text{sen}(t)\,dt$$

$$= \frac{-e^{-st}}{s}\,\text{sent} + \frac{1}{s}\int e^{-st}\cos t\,dt$$

$$= \frac{e^{-st}\,\text{sent}}{s} + \frac{1}{s}\left(\frac{-e^{-st}\cos t}{s} - \frac{a}{s}\right)$$

$$\int e^{-st}\,\text{sent}\,dt\,(1 + 1/s^2) = -\frac{e^{-st}\,\text{sen}(t)}{s} - \frac{e^{-st}\cos(t)}{s^2}$$

$$\Rightarrow \int e^{-st}\,\text{sent}\,dt = \frac{-e^{-st}(s\cdot\text{sent} + \cos t)}{s^2+1}$$

$$\Rightarrow \mathcal{L}\{f(t)\} = \frac{1}{1-e^{-\pi s}}\left(\frac{-e^{-st}(\text{sen}\,t + \cos t)}{s^2+1}\right)\Bigg|_0^{\pi}$$

$$= \frac{1}{1-e^{-\pi s}}\left(\frac{-e^{-\pi s}(\pi\,\cancel{\text{sen}\,\pi}^{\,0} + \cos(\pi))}{s^2+1} + \frac{\cos 0}{s^2+1}\right)$$

$$\boxed{= \frac{1}{e-e^{-\pi s}}\left[\frac{e^{-\pi s}+1}{s^2+1}\right]}$$

5a)

$$\mathcal{L}^{-1}\left\{\ln\left(\frac{s-3}{s+1}\right)\right\} - \frac{d}{ds}(F(s)) \quad -\frac{d}{ds}\left(\ln\left[\frac{(s-3)}{(s+1)}\right]\right)$$

$$\frac{-d}{ds}\left(\ln\frac{(s-3)}{(s+1)}\right) = \frac{1}{s-3} + \frac{1}{s+1}$$

$$\mathcal{L}\{f\cdot t\} = \frac{-d}{ds}F(s) \Leftarrow f(t) = 1/t\;\mathcal{L}^{-1}\left\{\frac{-d}{ds}F(s)\right\}$$

$$\boxed{f(t) = 1/t\,(e^{3t} - e^{-t})}$$

6) $ty'' + (1-t)y'' + ny = 0$

$n = 0, 1, 2 \ldots$

$\frac{1}{s^2}\left[sY(s) - sy(0) - y'(0)\right] + (1/s - 1/s^2)\left[sY(s) - y(0)\right] + n$

$Y(s)\left[1 + (1 - 1/s) + n\right] = 1/s + C_1/s^2 + (1/s - 1/s^2)$

$Y(s)\left[\frac{2s + ns - 1}{s}\right] = Y(s)\left[\frac{s(2+n) - 1}{s}\right] = \frac{2}{s} + \frac{C_1 - 1}{s^2}$

$\Rightarrow Y(s) = \frac{2(n+2)}{s - (1/n+2)} + \frac{(C_1 - 1)(n+2)}{s(s - (1/n+2))}$

$\boxed{(C_1 - 1)(n+2) = C_2 \quad 1/n+2 = C_3}$

$Y(s) = \frac{2(n+2)}{s - C_3} + \frac{C_2}{s(s - C_3)}$

$\Rightarrow \quad y(t) = 2(n+2)e^{C_3} + C_2\left(\int_0^t e^{C_3}\,dt\right) = \boxed{2(n+2)e^{C_3} + \frac{C}{C}}$

3) $y'' + y = \delta(t - 2\pi) \quad y(0) = 0 \quad y'(0)$

$\left[s^2Y(s) - sy(0) - y'(0)\right] + Y(s) = e^{-2\pi s}$

$Y(s)\left[s^2 + 1\right] = e^{-2\pi s} + 1 \Rightarrow Y(s) = e^{-2\pi s}\left(\frac{1}{s^2+1}\right)$

$\Rightarrow \boxed{y(t) = \operatorname{sen}(t - 2\pi)\,\mathcal{U}(t - 2\pi) + \operatorname{sen} t}$

5) $y'' + y = \delta(t - 1/2\pi) + \delta(t - 3/2\pi) \quad y(0) = 1, \; y'(0) = 1$

$\left[s^2Y(s) - sy(0) - y'(0)\right] + Y(s) = e^{-1/2\pi s} + e^{-3/2\pi s}$

$\Rightarrow Y(s) = e^{-\pi/2 s}\left(\frac{1}{s^2+1}\right) + e^{-3/2\pi s}\left(\frac{1}{s^2+1}\right) + \frac{s}{s^2+1}$

$\Rightarrow \boxed{y(t) = \operatorname{sen}(t - 1/2\pi)\,\mathcal{U}(t - 1/2\pi) + \operatorname{sen}(t - 3/2\pi)\,\mathcal{U}(t - }$

12. $y'' - ty' + 6y = e^{t} + \delta(t-2) + \delta(t-4) \quad y(0) = y'(0) = 0$

$$\left[s^2 Y(s) - \cancel{sy(0)} - \cancel{y'(0)}\right] - 7\left[sY(s) - \cancel{y(0)}\right] + 6Y(s) = \ldots$$

$$\ldots \frac{1}{s-1} + e^{-2s} + e^{-4s}$$

Voyage:

$$Y(s) = \frac{1}{(s-6)(s-1)^2} + \frac{e^{-2s}}{(s-6)(s-1)} + \frac{e^{-4s}}{(s-6)(s-1)}$$

$$Y(s) = \frac{-1}{25}\left(\frac{1}{s-1}\right) - \frac{1}{5}\left(\frac{1}{(s-1)^2}\right) + \frac{1}{25}\left(\frac{1}{s-6}\right) + e^{-2s}\left[\frac{1}{5}\left(\frac{1}{s-6}\right.\right.$$

$$+ e^{-4s}\left[\frac{1}{5}\left(\frac{1}{s-6}\right) - \frac{1}{5}\left(\frac{1}{s-1}\right)\right]$$

$$\boxed{\Rightarrow y(t) = \frac{-1}{25}e^{t} + \frac{1}{25}e^{6t} + \frac{1}{5}\left[e^{6(t-2)} - e^{t-2}\right]\mathcal{U}(t-2) + \frac{1}{5}\left[e^{6(t-4} \ldots \mathcal{U}(t-4) - 1/5\, te^{t}}$$

14) $y(0) = 0,\ y'(0) = 0,\ y(L) = 0,\ y'(L) = 0$

$$EI\frac{d^4y}{dx^4} = w_0\,\delta(x - 1/2 L)$$

$$s^4 Y(s) - \cancel{s^3 y(0)} - \cancel{s^2 y'(0)} - s'' y(0) - y'''(0) = \frac{w_0}{EI} e^{-1/2}$$

$$Y(s) = \frac{c_1}{s^3} + \frac{c_2}{s^4} + \frac{c\, e^{-1/2 Ls}}{s^4}$$

$$\boxed{\Rightarrow y(t) = 1/2\, c_1 t + 1/6\, c_2 t^3 + 1/6\, c\,(t - 1/2 L)^3\, \mathcal{U}(t - 1/2 L)}$$