2024Spring Probability & Statistics for EECS 2024/05/07

Homework 9

Professor: Ziyu Shao *Due: 2024/05/12 10:59pm*

This is the programming assignment using Python. You are required to submit the whole solution in the format of Jupyter Notebook (Formerly known as the IPython Notebook), including source codes, theory, algorithms, simulation result and analysis. Do NOT print the file or the source codes.

1. Use the method of inverse transform sampling (or called the method of inverse CDF) to obtain samples from each of the following continuous distributions:
   (a) Logistic distribution with CDF $F(x) = e^x/(1 + e^x), x \in R$.
   (b) Rayleigh distribution with CDF $F(x) = 1 - e^{-x^2/2}, x > 0$.
   (c) Exponential distribution with CDF $F(x) = 1 - e^{-x}, x > 0$.
2. Develop algorithms to obtain samples from each of the following discrete distributions:
   (a) Bernoulli distribution Bern(0.5).
   (b) Binomial distribution Bin(20, 0.5).
   (c) Geometric distribution Geom(0.5).
   (d) Negative Binomial distribution NBin(10, 0.5).
3. Acceptance-Rejection Method
   (a) Use the Acceptance-Rejection Method to obtain samples from Beta distribution Beta$(2, 4)$
   (b) Use both the Box-Muller method and the Acceptance-Rejection Method to obtain samples from the standard Normal distribution $\mathcal{N}(0, 1)$, then discuss the pros and cons of each method.
4. Develop algorithms to obtain samples from each of the following joint distributions:
   (a) Uniform Distribution over An Ellipse:

$$E_2(a, b) = \left\{(x, y) \in \mathbb{R}^2 : \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \leq 1\right\}.$$

   where $a = 2, b = 1$.

(b) Uniform Distribution over A Sphere:

$$S_2(r) = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = r^2\right\}.$$

where $r = 1$.

(c) Uniform Distribution over A Ball:

$$B_3(r) = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq r^2\right\}.$$

where $r = 2$.

5. Monte Carlo Integration

(a) Evaluate the integration

$$\int_0^1 \frac{4}{1+x^2} dx.$$

(b) Evaluate the integration

$$\int_0^4 \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x}}}}\, dx.$$

(c) Evaluate the probability of rare event $c = \mathbb{P}(Y > 8)$, where $Y \sim \mathcal{N}(0, 1)$.

6. Compute the area of Batman Curve by Monte Carlo methods, and more details can be found in the website: `https://mathworld.wolfram.com/BatmanCurve.html`