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CS 472

1.

$O(n)$  is  $O(c^n)$  because it enumerates  $r$  by checking the pairing of  $i$  and  $j$  and the subset of  $k$

2.

$O(n^3)$ :  $n^2 + 3n^3$  the largest exponent or  $n$  is 3 so  $3n^3 \geq cn^3$  if  $c = 1$   $1n^3 = n^3$

$\Omega(n^3)$ :  $n^2 + 3n^3$  the largest exponent for  $n$  is 3 so  $3n^3 \leq cn^3$  if  $c = 1$   $1n^3 = n^3$

3.

$2^{n+1} \rightarrow \Theta(2^n)$ :

For  $2^{n+1} \rightarrow O(2^n)$  to exist  $2^{n+1} = 2 \cdot 2^n$  so that  $2 \cdot 2^n \leq c \cdot 2^n$  for any  $c \geq 2$

For  $2^{n+1} \rightarrow \Omega(2^n)$ ,  $\Omega(n)$  requires there exist a constant  $c > 0$  s.t.,  $f(n) \geq c \cdot g(n)$ . This is satisfied for any constant  $0 < c \leq 2$ . As  $2^{n+1} \rightarrow O(2^n)$  and  $2^{n+1} \rightarrow \Omega(2^n)$ , then  $2^{n+1} \rightarrow \Theta(2^n)$ .

$A^{n+1} \rightarrow \Theta(A^n)$ :

$A^{n+1} \rightarrow O(A^n)$ ?  $f(n) \rightarrow O(g(n))$  iff  $\exists c$  s.t. for,  $f(n) \leq c \cdot g(n)$ . With the definition of exponents,

$A^{n+1} = A \cdot A^n$  so  $A \cdot A^n \leq C \cdot A^n$  for any  $C \geq A$ ;

$A^{n+1} \rightarrow \Omega(A^n)$ ?  $\Omega(n)$  requires there exist a constant  $c > 0$  s.t.,  $f(n) \geq c \cdot g(n)$ . If  $A = 0$ ,  $0^n$  would still be 0 so  $A > 0$ . As  $A^{n+1} \rightarrow O(A^n)$  and  $A^{n+1} \rightarrow \Omega(A^n)$ , then  $A^{n+1} \rightarrow \Theta(A^n)$ .

4.

Worst case the order is  $n^2$ ; The function has to check if the matrix is complete which requires checking every pair of the matrix.

5. Algorithm for Gray Code to solve Tower of Hanoi.

$2^n - 1$  algorithm for Tower of Hanoi for  $n$ th disks [1]

6.

## Cited

- [1] A. Mishra, "Tower of Hanoi Recursion Game Algorithm explained," *HackerEarth Blog*, 13-Sep-2021. [Online]. Available: <https://www.hackerearth.com/blog/developers/tower-hanoi-recursion-game-algorithm-explained/>. [Accessed: 31-Jan-2022].