Statistical Inference Course Project

Author: Dale Hunscher

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Part 1: Simulation Exercise

Overview

Per the description of the project in Coursera:

> In this part of the project, you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

As stated above, this part of the project is an exploration of the Central Limit theorem. We'll compare the mean and variance of a series of randomly generated sample distributions with their theorical values computed using the same lambda value. Then we will use histograms to do a visual comparison of the sample distribution with the normal paradigm.

Setup

First we set our working directory and load libraries:

Next we set the random seed so our project results will be reproducible.

Set the variables we'll need based on the project instructions:

Explorations Part One: Comparing Sample and Theory

Compute the exponential sample data set, and then the sample and theoretical means and variances.

Comparing means: first the sample and theoretical means…

```
## [1] 5.032989
```

```
## [1] 5
```

…and the sample and theoretical variances…

```
## [1] 0.6276361
```

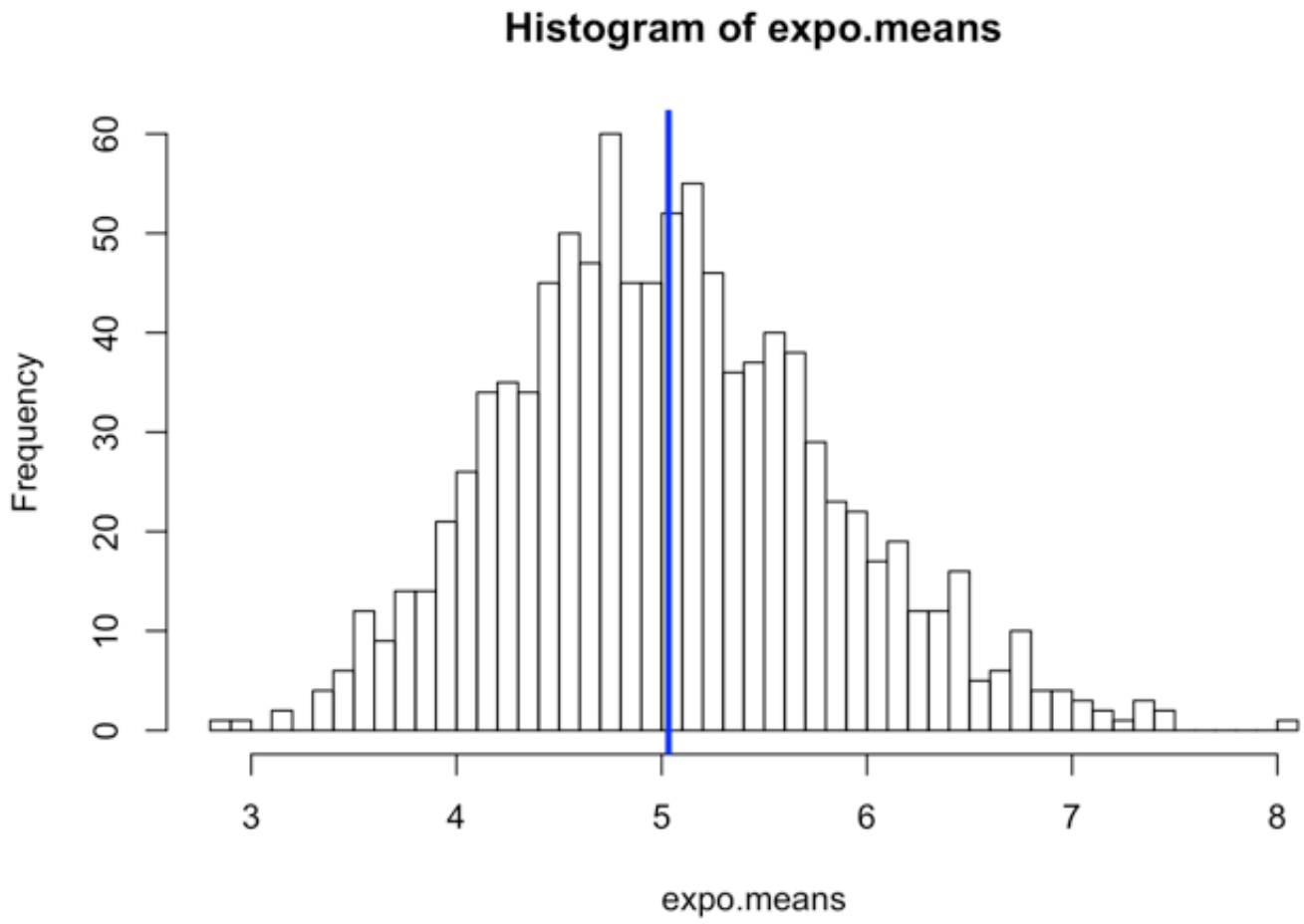
```
## [1] 0.625
```

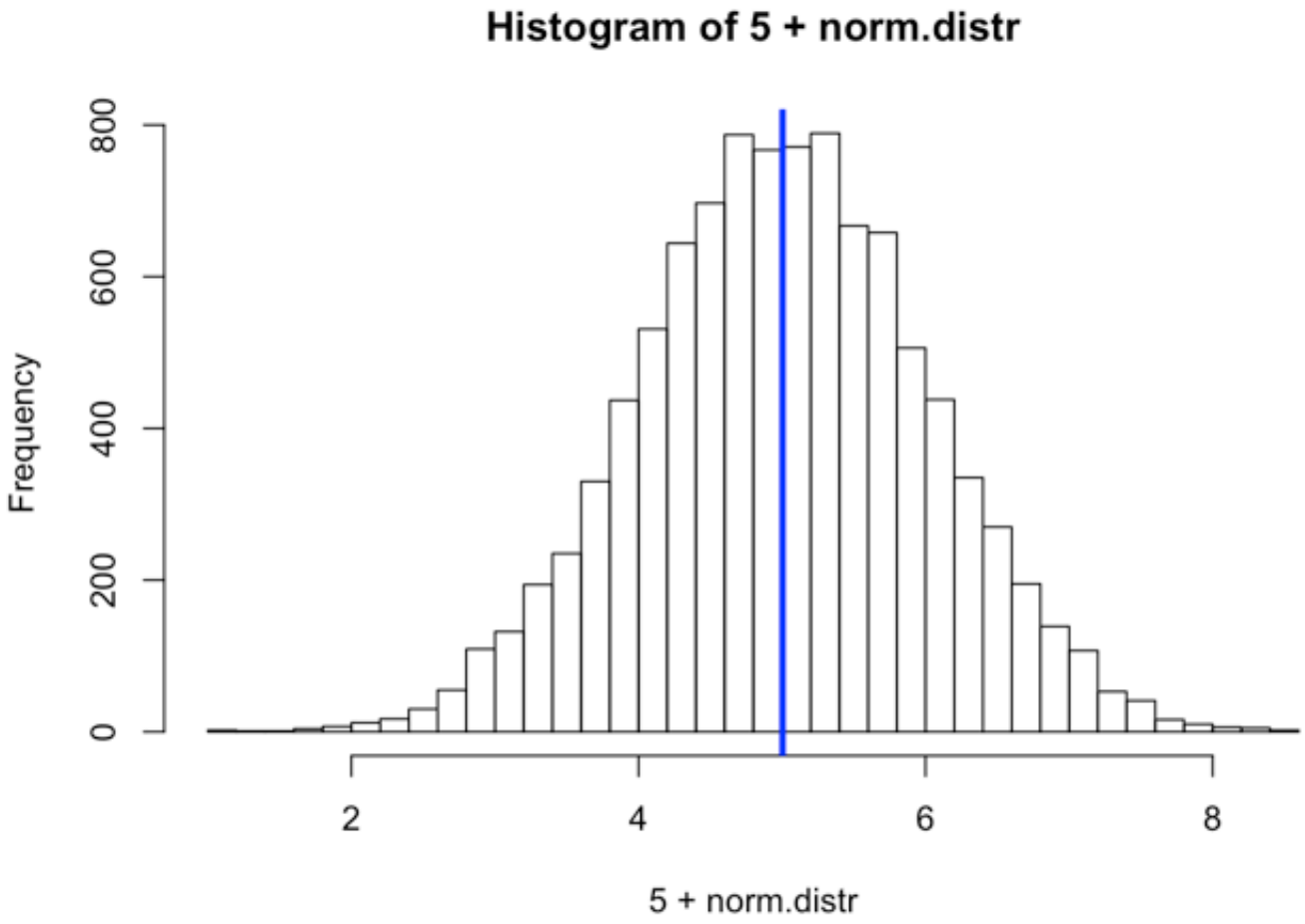
We can see that the mean and variance values computed from the sample data sets are very close to their theoretical equivalents.

Explorations Part Two: A Visual Comparison

The first histogram below histogram shows the distribution of the sample means. The vertical bar marks the mean of the distribution of sample means.

For comparison purposes, we'll compute a random sample drawn from the normal distribution and show a histogram. We'll up the sample "n" to 10,000 to ensure it's close to the paradigm for a normal distribution. We'll also add 5 (theoretical mean of the sample exponential distribution) so the histograms' scales are comparable.





We can see that the distribution of even a small set of means of random sample data sets drawn from the exponential distribution approximates the normal paradigm, and the means are very close to the theoretical value for the exponential distribution.

The right tail on the sample distribution is larger than we would expect based on the normal distribution. According to the Law of Large Numbers, the distribution of sample means will become closer to the normal distribution as the size of each sample data set increases (for example, from 40 to 1000). The page limit for the project precludes a visual demonstration of this effect, which in any case is not in the project scope.

Conclusions

- The sample mean and variance values for are quite close to the theoretical mean and variance coomputed using the same lambda value. This is the expected result, because the sample mean and variance are unbiased estimators of their theoretical equivalents.
- A histogram showing the distribution of means of 1000 sample 40-value data sets drawn from the exponential distribution are visually similar to a histogram of the normal distribution.
- These conclusions demonstrate the Central Limit Theorem in action.