IMPLEMENTACIÓN DE MÉTODOS COMPUTACIONALES
EJERCICIO 1 - INTRODUCCIÓN A LOS AUTÓMATAS

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febrero, 2021

Demuestra cada uno de las siguientes proposiciones:

1. Para todo $n \geq 1$: $\sum_1^n (2i-1) = n^2$
2. Para todo $n \geq 1$: $\sum_1^n (4i-1) = n(2n+1)$
3. Para todo $n \geq 1$: $\sum_1^n (2i-1) * 3^i = (n-1) * 3^{n+1} + 3$
4. Para todo $n \geq 1$: $\sum_1^n i^2 = \frac{n(n+1)(2n+1)}{6}$

Bibliografía

- Hopcroft, J. E., Motwani, R., & Ullman, J. D. (2014). Introduction to Automata Theory, Languages, and Computation: Pearson New International Edition: Vol. 3rd ed. Pearson.

1) Para todo $n \geq 1$

$$\sum_{i=1}^{n} (2i-1) = n^2$$

Caso base: $n = 1$

$$(2\cdot 1 - 1) = 1^2$$
$$2 - 1 = 1$$
$$1 = 1$$

Caso base: $n=2$

$$(2\cdot 1 - 1) + (2\cdot 2 - 1) = 2^2$$
$$1 + 3 = 4$$
$$4 = 4$$

Para $k \geq 1$, considerar que $\sum_{i=1}^{k} (2i-1) = n^2$

Paso inductivo: Probar $k+1$

$$\sum_{i=1}^{k+1} (2i-1) = (k+1)^2$$

$$\sum_{i=1}^{k} (2i-1) + (2(k+1)-1) = (k+1)^2$$

$$[\ (k)^2 + (2(k+1)-1)] = (k+1)^2$$

$$k^2 + 2k + 2 - 1 = (k+1)^2$$

$$k^2 + 2k + 1 = (k+1)^2$$

$$(k+1)(k+1) = (k+1)^2$$

$$(k+1)^2 = (k+1)^2$$

2. Para todo $n \geq 1$: $\sum_1^n (4i - 1) = n(2n + 1)$

Para $n = 1$

$$(4 \cdot 1 - 1) = 1(2 \cdot 1 + 1)$$
$$(4 - 1) = 2 + 1$$
$$3 = 3$$

Para $n = 2$

$$(4 \cdot 1 - 1) + (4 \cdot 2 - 1) = 2(2 \cdot 2 + 1)$$
$$(4 - 1) + (8 - 1) = 2(5)$$
$$3 + 7 = 10$$
$$10 = 10$$

Para $k \geq 1$ considerar que $\sum_{i=1}^{k} (4i - 1) = k(2k + 1)$

Paso inductivo: Probar $k + 1$

$$\sum_{i=1}^{k+1} (4i - 1) = k + 1(2(k+1) + 1)$$

$$\sum_{i=1}^{k} (4i - 1) + (4(k+1) - 1) = k + 1(2(k+1) + 1$$

$$[k(2k + 1)] + [4(k + 1) - 1] = k + 1(2(k+1) + 1$$

$$2k^2 + k + 4k + 3 = \quad {''}$$

$$2k^2 + 5k + 3 = k + 1(2k + 3)$$

$$2k^2 + 5k + 3 = (2k^2 + 3k + 2k + 3)$$

$$2k^2 + 5k + 3 = 2k^2 + 5k + 3$$

3) Para $n \geq 1$

$$\sum_{i=1}^{n} (2i-1) * 3i = (n-1) * 3^{n+1} + 3$$

Caso base $(n=1)$

$$(2*1-1) * 3 * 1 = \cancel{(1-1)}^{0} * 3^{1+1} + 3$$
$$(2-1) * 3 = 3$$
$$1 * 3 = 3$$
$$3 = 3$$

Para $n=2$

$$(2 \cdot 1 - 1) \cdot 3^1 = (1-1) \cdot 3^{1+1} + 3$$
$$(2-1) \cdot 3 = 3$$
$$1 \cdot 3 = 3$$
$$3 = 3$$

¿Cuál es el último término?
¿Paso Inductivo?

Para $n=2$

$$(2 \cdot 1 - 1) \cdot 3^2 + (2 \cdot 2 - 1) \cdot 3^2 = (2-1) \cdot 3^{2+1} + 3$$
$$(1 \cdot 3) + (3 \cdot 9) = 3^3 + 3$$
$$3 + 27 = 27 + 3$$
$$30 = 30$$

PARA $k \geq 1$, considerar que $\sum_{i=1}^{k} (2i-1) \cdot 3^i = (k-1) \cdot 3^{k+1} + 3$

PASO Inductivo: Probar $k+1$

$$\sum_{i=1}^{k+1} (2i-1) \cdot 3^i = k \cdot 3^{k+2} + 3$$

$$\sum_{i=1}^{k} (2i-1) \cdot 3^i + (2(k+1)-1) \cdot 3^{k+1} = k \cdot 3^{k+2} + 3$$

$$[(k-1)\cdot 3^{k+1}+3]+[(2(k+1)-1)\cdot 3^{k+1}] = k\cdot 3^{k+1}+3$$

$$(k-1)\cdot 3^{k+1}+3+(2k+2-1)\cdot 3^{k+1}$$

$$(k-1)\cdot 3^{k+1}+3+2k\cdot 3^{k+1}+2\cdot 3^{k+1}-1\cdot 3^{k+1}$$

$$(k-1+2k+2-1)\cdot 3^{k+1}+3$$

$$3k\cdot 3^{k+1}+3$$

$$k\cdot 3^{1}\cdot 3^{k+1}+3$$

$$k\cdot 3^{k+2}+3 = k\cdot 3^{k+2}+3$$

4. Para todo $n \geq 1$: $\sum_1^n i^2 = \frac{n(n+1)(2n+1)}{6}$

Para $n = 1$ Para $n = 2$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} 1^2 + 2^2 = \frac{2(2+1)(2\cdot 2+1)}{6}$$

$$1^2 = \frac{1(1+1)(2\cdot 1+1)}{6} \qquad 5 = \frac{\cancel{(6)}(5)}{\cancel{6}}$$

$$1 = \frac{(2)(3)}{6}$$

$$1 = 1$$

Para $k \geq 1$, considerar que $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$

Para $k+1$

$$\sum_{i=1}^{k+1} i^2 = \frac{k+1((k+1)+1)(2(k+1)+1)}{6}$$

$$\sum_{i=1}^{k} i^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$$

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$$

$$\frac{k(k+1)(2k+1) + 6(k+1)^2}{6} =$$

$$\frac{(k^2+k)(2k+1)+6(k^2+2k+1)}{6}$$

$$\frac{\cancel{2k^3}+\cancel{k^2}+2k^2+\cancel{k}+\cancel{6k^2}+\cancel{12k}+6}{6}$$

$$\frac{2k^3+9k^2+13k+6}{6}=\frac{(k+1)(k+2)(2k+3)}{6}$$

$$=\frac{(k+1)(2k^2+3k+4k+6)}{6}$$

$$=\frac{(k+1)(2k^2+7k+6)}{6}$$

$$=\frac{2k^3+7k^2+6k+2k^2+7k+6}{6}$$

$$\frac{2k^3+9k^2+13k+6}{6}=\frac{2k^3+9k^2+13k+6}{6}$$