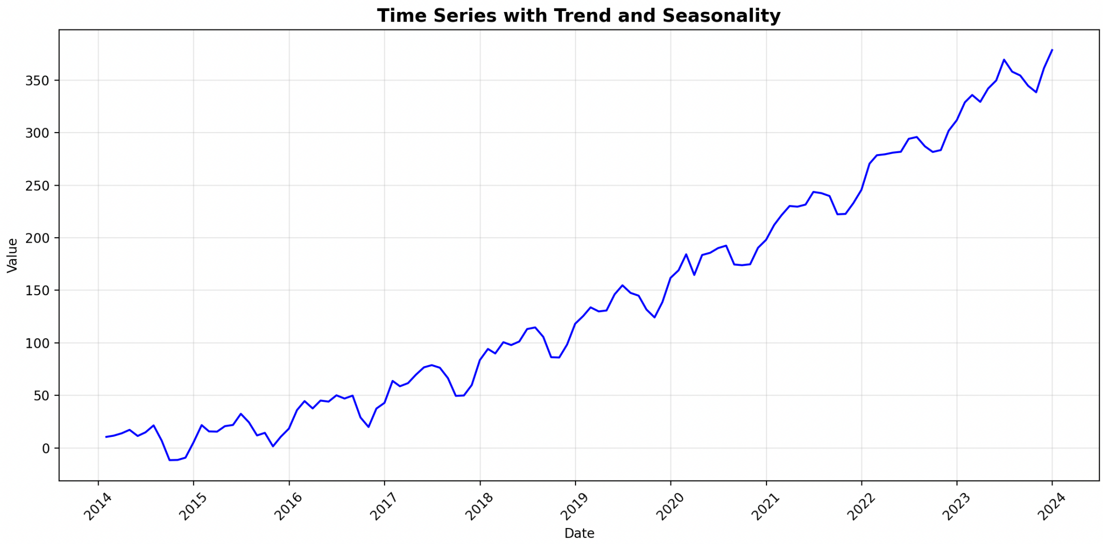
1. To decompose the follow time series sequence which is generated by the given code:




```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Set parameters
np.random.seed(42)
n = 120  # 10 years of monthly data
t = np.arange(n)

# 1. Trend component (linear + quadratic trend)
trend = 0.8 * t + 0.02 * t**2

# 2. Seasonal component (annual cycle)
seasonal = 15 * np.sin(2 * np.pi * t / 12) + 8 * np.cos(2 * np.pi * t / 6)

# 3. Noise component
noise = np.random.normal(0, 5, n)

# 4. Combine all components
time_series = trend + seasonal + noise

# Create DataFrame with dates
dates = pd.date_range(start='2014-01-01', periods=n, freq='M')
df = pd.DataFrame({
    'date': dates,
    'value': time_series
})
```

Please decompose the trend and seasonal components from the sequence, plot them and write down the equation for this time series data.

2. Consider the time series

$$x_t = \beta_1 + \beta_2 t + w_t,$$

where β_1 and β_2 are known constants and w_t is a white noise process with variance σ_w^2.

(a) Determine whether x_t is stationary.
(b) Show that the process $y_t = x_t - x_{t-1}$ is stationary.

(c) Show that the mean of the moving average

$$v_t = \frac{1}{2q+1} \sum_{j=-q}^{q} x_{t-j}$$

is $\beta_1 + \beta_2 t$, and give a simplified expression for the autocovariance function.

3. Let wt, for t = 0, ±1, ±2, . . . be a normal white noise process, and consider the series

$$x_t = w_t w_{t-1}.$$

Determine the mean and autocovariance function of x_t, and state whether it is stationary.

4. For the AR(2) model given by xt = −.9xt−2 + wt, find the roots of the autoregressive polynomial, and then sketch the ACF, ρ(h).

5. Identify the following models as ARMA(p, q) models (watch out for parameter redundancy), and determine whether they are causal and/or invertible:

(a) $x_t = .80x_{t-1} - .15x_{t-2} + w_t - .30w_{t-1}.$
(b) $x_t = x_{t-1} - .50x_{t-2} + w_t - w_{t-1}.$