IQmodelo User Manual

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Contents

1 Front Matter

1.1 Legal disclaimer

1.2 Contact for bugs or questions

This software is a work in progress. To report bugs or for questions, please contact

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1.3 Acknowledgement

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2 Introduction

The IQmodelo package (http://github.com/DIDSR/IQmodelo) provides statistical software for task-based image quality assessments using mathematical model observers. All software is written for MATLAB® and is compatible with release R2007b and newer. In addition to the standard MATLAB® installation, the functions for parametric analysis methods require the Statistics Toolbox™. All of the functions in this package can be downloaded as a single zip file from the aforementioned website.

The functions in the IQmodelo package are described below, organized by method. To illustrate application of the functions, several demo scripts are provided, as described in Chapter 5.

Some Notation:

In the function descriptions below, the number of class 1 (signal-absent) and class 2 (signal-present) images are denoted by m and n respectively. The routines for channelized Hotelling observers use p to denote the number of channels. Also, functions designed for analyzing multiple imaging scenarios (e.g., different readers or modalities) use q to denote the number of scenarios.

All functions that return confidence intervals estimate $1-\alpha$ confidence intervals, where α is the sum of two parameters, α_1 and α_2, which control the lower and upper significance levels, respectively, i.e., $\alpha = \alpha_1 + \alpha_2$. By specifying α_1 and α_2, the user can choose to estimate either two-sided or one-sided confidence intervals. For example, for a conventional two-sided 95% interval, take $\alpha_1 = \alpha_2 = 0.025$. On the other hand, to get a one-sided 95% interval with only a lower bound (useful for superiority and non-inferiority testing), take $\alpha_1 = 0.05$ and $\alpha_2 = 0$.

3 Analysis Functions: Parametric Methods for Linear Observers

3.1 Channelized Hotelling observers

3.1.1 exactCI_CHO.m

Description: For a channelized Hotelling observer (CHO), returns exact confidence intervals for SNR and AUC as described in [1].

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`v1` ($p \times m$ matrix of class 1 channel output vectors)

`v2` ($p \times n$ matrix of class 2 channel output vectors)

Outputs:

`ret` (structure containing point and interval estimates of SNR and AUC)

3.1.2 diffCI_CHO.m

Description: For CHOs, returns conservative confidence intervals for a difference of SNR or AUC values. This method uses exact intervals for each imaging scenario obtained with `exactCI_CHO.m` together with the Bonferroni inequality, as described in the appendix.

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`vA1` ($p_A \times m_A$ matrix of class 1 channel output vectors for scenario A)

`vA2` ($p_A \times n_A$ matrix of class 2 channel output vectors for scenario A)

`vB1` ($p_B \times m_B$ matrix of class 1 channel output vectors for scenario B)

`vB2` ($p_B \times n_B$ matrix of class 2 channel output vectors for scenario B)

Outputs:

`ret` (structure containing confidence intervals for $\mathrm{SNR}_B - \mathrm{SNR}_A$ and $\mathrm{AUC}_B - \mathrm{AUC}_A$)

3.2 Channelized Hotelling observers with known difference of class means

3.2.1 `exactCI_CHO_km.m`

Description: For a channelized Hotelling observer (CHO) with known difference of class means, returns exact confidence intervals for SNR and AUC as described in [2].

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`delta_mu` ($p \times 1$ vector corresponding to known difference of class means in channel space)

`v1` ($p \times m$ matrix of class 1 channel output vectors)

`v2` ($p \times n$ matrix of class 2 channel output vectors)

Outputs:

`ret` (structure containing point and interval estimates of SNR and AUC)

3.2.2 `diffCI_CHO_km.m`

Description: For CHOs with known difference of class means, returns conservative confidence intervals for a difference of SNR or AUC values. This method uses exact intervals for each imaging scenario obtained with `exactCI_CHO_km.m` together with the Bonferroni inequality, as described in the appendix.

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`delta_muA` ($p_A \times 1$ vector corresponding to known difference of class means in channel space for scenario A)

`delta_muB` ($p_B \times 1$ vector corresponding to known difference of class means in channel space for scenario B)

`vA1` ($p_A \times m_A$ matrix of class 1 channel output vectors for scenario A)

`vA2` ($p_A \times n_A$ matrix of class 2 channel output vectors for scenario A)

`vB1` ($p_B \times m_B$ matrix of class 1 channel output vectors for scenario B)

`vB2` ($p_B \times n_B$ matrix of class 2 channel output vectors for scenario B)

Outputs:

`ret` (structure containing confidence intervals for $\mathrm{SNR}_B - \mathrm{SNR}_A$ and $\mathrm{AUC}_B - \mathrm{AUC}_A$)

3.3 Known-template linear observers

3.3.1 `exactCI_kt.m`

Description: For a linear observer defined by a known (fixed) template, returns exact confidence intervals for SNR and AUC as described in [3].

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`x` ($1 \times m$ vector of class 1 ratings)

`y` ($1 \times n$ vector of class 2 ratings)

Outputs:

`ret` (structure containing point and interval estimates of SNR and AUC)

3.3.2 `diffCI_kt.m`

Description: For a linear observer defined by a known (fixed) template, returns conservative confidence intervals for a difference of SNR or AUC values. This method uses exact intervals for each imaging scenario obtained with `exactCI_kt.m` together with the Bonferroni inequality, as described in the appendix.

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`xA` ($1 \times m_A$ vector of class 1 ratings for scenario A)

`yA` ($1 \times n_A$ vector of class 2 ratings for scenario A)

`xB` ($1 \times m_B$ vector of class 1 ratings for scenario B)

`yB` ($1 \times n_B$ vector of class 2 ratings for scenario B)

Outputs:

`ret` (structure containing confidence intervals for $\mathrm{SNR}_B - \mathrm{SNR}_A$ and $\mathrm{AUC}_B - \mathrm{AUC}_A$)

3.4 Known-template linear observers with known difference of class means

3.4.1 `exactCI_ktkm.m`

Description: For a linear observer defined by a known (fixed) template with known difference of class means, returns exact confidence intervals for SNR and AUC as described in [4].

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`delta` (known difference of class means)

`x` ($1 \times m$ vector of class 1 ratings)

`y` ($1 \times n$ vector of class 2 ratings)

Outputs:

`ret` (structure containing point and interval estimates of SNR and AUC)

3.4.2 `diffCI_ktkm.m`

Description: For a linear observer defined by a known (fixed) template, and known difference of class means, returns an approximate confidence interval for a difference of SNR or AUC values, as described in [4].

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`deltaA` (known difference of class means for scenario A)

`deltaB` (known difference of class means for scenario B)

`xA` ($1 \times m$ vector of class 1 ratings for scenario A)

`yA` ($1 \times n$ vector of class 2 ratings for scenario A)

`xB` ($1 \times m$ vector of class 1 ratings for scenario B)

`yB` ($1 \times n$ vector of class 2 ratings for scenario B)

Outputs:

`ret` (structure containing confidence intervals for $\mathrm{SNR}_B - \mathrm{SNR}_A$ and $\mathrm{AUC}_B - \mathrm{AUC}_A$)

4 Analysis Functions: General Methods

4.1 Nonparametric ROC analysis with fixed observers

4.1.1 `fastDeLong.m`

Description: For ROC analysis, implements a fast rank-based algorithm for the Mann-Whitney AUC estimator and for the DeLong covariance matrix estimator [5], as described in [6]. The output is identical (up to numerical error) to EROCcov.m in the case of ROC analysis, but the execution time is much faster for large sample sizes. This function assumes that variability is due to cases only.

Inputs:

`X` ($q \times m$ matrix of class 1 ratings)

`Y` ($q \times n$ matrix of class 2 ratings)

Outputs:

`AUC` ($q \times 1$ vector of AUC estimates)

`S` ($q \times q$ covariance matrix)

4.1.2 `npAUC_CI.m`

Description: For an ROC assessment, returns a confidence interval for a single AUC or for a difference of AUCs. This function requires `fastDeLong.m`, and assumes that variabilty is due to cases only. The confidence interval for a single AUC is computed using the logit transformation method recommended by Pepe [7, p. 107]. For a difference of AUCs, the usual method based on a normal approximation is used.

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`X` ($q \times m$ matrix of class 1 ratings)

`Y` ($q \times n$ matrix of class 2 ratings)

Note: $q \in \{1, 2\}$ is the number of fixed imaging scenarios or readers

Outputs:

AUC ($q \times 1$ vector of AUC estimates)

AUC_CI (confidence interval for a single AUC ($q = 1$) or for a difference ($q = 2$)

Note: The interval estimate for a difference is for performance of the second scenario minus the first.

4.2 Nonparametric LROC and EROC analysis with fixed observers

4.2.1 EROCcov.m

Description: Computes U-statistic estimates of the area under ROC, LROC, or EROC curves, and the corresponding Sen-type covariance matrix estimate for q imaging scenarios as described in [5, 8, 9]. This function assumes that variability is due to cases only.

Inputs:

X ($q \times m$ matrix of class 1 ratings)

Y ($q \times n$ matrix of class 2 ratings)

U ($q \times n$ matrix of utilities for class 2 images)

Note: For ROC analysis, set U = ones(q,n). For LROC analysis, each entry of U is one if the lesion was localized correctly, and zero otherwise. For EROC analysis, U is a defined by a utility function for parameter estimation.

Outputs:

AUC ($q \times 1$ vector of AUC estimates)

S ($q \times q$ covariance matrix)

4.2.2 npAEROC_CI.m

Description: For LROC/EROC assessment, returns an approximate confidence interval for a single AUC or for a difference of AUCs. This function requires the function EROCcov.m, and assumes that variability is due to cases only.

Inputs:

alpha1 (lower significance level)

alpha2 (upper significance level)

X ($q \times m$ matrix of class 1 ratings)

Y ($q \times n$ matrix of class 2 ratings)

`U` ($q \times n$ matrix of utilities for class 2 images)

Note: For LROC analysis, each entry of U is one if the lesion was localized correctly, and zero otherwise. For EROC analysis, U is a defined by a utility function for parameter estimation.

Note: $q \in \{1, 2\}$ is the number of fixed imaging scenarios or readers

Outputs:

`AUC` ($q \times 1$ vector of AUC estimates)

`AUC_CI` (confidence interval for a single AUC ($q = 1$) or for a difference ($q = 2$)

Note: The interval estimate for a difference is for performance of the second scenario minus the first.

4.3 Binomial proportions with fixed observers

The functions in this section can be used to analyze any figure of merit estimated as a proportion, such as the percent correct in a multiple alternative forced choce (MAFC) assessment or the proportion of correct marks in an LROC experiment. This function assumes that variability is due to cases only.

4.3.1 `binPropCov.m`

Description: Estimate covariance matrix for a vector of binomial proportions using an unbiased estimator, as described in [10].

Inputs:

`X` ($n \times q$ matrix of success scores, where n =number of cases and q =number of fixed imaging scenarios or readers)

Outputs:

`p` ($q \times 1$ vector of percent correct estimates)

`S` ($q \times q$ covariance matrix estimate)

4.3.2 `binProp_CI.m`

Description: Returns a confidence interval for a single binomial proportion or for a difference of binomial proportions. This function requires binPropCov.m, and assumes that variabilty is due to cases only.

Inputs:

`alpha1` (lower significance level)

`alpha2` (upper significance level)

`X` ($n \times q$ matrix of success scores, where n =number of cases and q =number of fixed imaging scenarios or readers)

Outputs:

`p` ($q \times 1$ vector of percent correct estimates)

`p_CI` (confidence interval for single binomial proportion ($q = 1$) or for a difference ($q = 2$))

Note: The interval estimate for a difference is for performance of the second scenario minus the first.

4.4 ANOVA-based multi-reader multi-case analysis with random readers

4.4.1 `ORHmrmc.m`

Description: Implements the Obuchowski-Rockette-Hillis (ORH) method for multi-reader multi-case (MRMC) analysis, i.e., the OR method [11] with Hillis' degrees of freedom [12, 13]). The notation generally follows Hillis' papers [12, 13], where t is the number of tests (modalities), r is the number of readers, θ_{ij} is the figure of merit for the i'th test and the j'th reader, and $\widehat{\theta}_{ij}$ is the estimate of θ_{ij}.

Inputs:

`thetaHat` ($t \times r$ matrix of estimated values for θ_{ij}),

`S` ($tr \times tr$ fixed-reader covariance matrix for the vector

$$[\widehat{\theta}_{11}, \widehat{\theta}_{12}, \ldots, \widehat{\theta}_{1r}, \widehat{\theta}_{21}, \ldots, \widehat{\theta}_{2r}, \ldots, \widehat{\theta}_{t1}, \ldots, \widehat{\theta}_{tr}].\quad)$$

NOTE: This covariance matrix can be estimated with `fastDeLong.m`, `EROCcov.m`, or `binPropCov.m` for ROC, LROC/EROC and MAFC analysis, respectively.

`alpha1` (lower significance level)

`alpha2` (upper significance level)

Outputs:

`ret` (structure containing point and interval estimates)

fields:

`mth` ($t \times 1$ vector of reader-averaged estimates for θ_i)

`CIsingle` ($t \times 2$ matrix of confidence intervals for θ_i estimated using only corresponding data)

`CIsingle_all` ($t \times 2$ matrix of confidence intervals for θ_i estimated using all data)

`CIdiff` ($t - 1 \times 2$ matrix of confidence intervals for $\theta_i - \theta_1$)

5 Demos

5.1 Known-location signal detection: one imaging scenario

5.1.1 `demo1a.m`

Description: Demonstration of point and interval estimators for the area under the ROC curve for a fixed-location signal detection task with one imaging scenario. This script uses the function `create_images1.m` to create a set of random images with a specified set of random number generator seeds. The channel matrix for a channelized Hotelling observer is created with the function `make_channels.m` for two difference channel types. For each analysis method, the demo script outputs AUC point estimates and two-sided confidence intervals to the screen.

5.1.2 `demo1b.m`

Description: Same as `demo1a.m`, but outputs a 1-sided confidence interval with each method.

5.2 Known-location signal discrimination: two imaging scenarios

5.2.1 `demo2a.m`

Description: For a fixed-location signal detection task, demonstration for two imaging scenarios (called A and B), in which each analysis method is used to obtain a two-sided confidence interval estimate for the AUC difference $\mathrm{AUC}_B - \mathrm{AUC}_A$. This script uses the function `create_images2.m` to create two correlated sets of random images, one for each imaging scenario, with a specified set of random number generator seeds. The channel matrix for a channelized Hotelling observer is created with the function `make_channels.m` for two difference channel types.

5.2.2 `demo2b.m`

Description: Same as `demo2a.m`, but outputs a 1-sided confidence interval with each method.

5.3 Unknown-location signal detection: one imaging scenario

5.3.1 `demo3.m`

Description: Demonstration of nonparametric point and interval estimators for the area under the LROC curve for an unknown-location detection task with one imaging scenario. This script uses the function `create_images1.m` to create a set of random images with a specified set of random number generator seeds. The observer is a scanning-linear observer with a fixed template that takes a uniform value inside a circular disk roughly the same size as the signal. The demo outputs a point estimate and a two-sided confidence interval for the probability of correct localization and for LROC area to the screen.

5.4 Unknown-location signal detection: two imaging scenarios

5.4.1 `demo4.m`

Description: Demonstration of nonparametric point and interval estimators for the area under the LROC curve for an unknown-location detection task with two imaging scenarios. This script uses the function `create_images2.m` to create two correlated sets of random images, one for each imaging scenario, with a specified set of random number generator seeds. The observer is a scanning-linear observer with a fixed template that takes a uniform value inside a circular disk roughly the same size as the signal. The demo outputs point estimates for the probability of correct localization and LROC area, as well as two-sided confidence intervals for the inter-scenario differences of each figure of merit to the screen.

5.5 Supporting functions

5.5.1 `create_images1.m`

Description: Generate random images for one imaging senario.

Inputs:

`params` a structure with the following fields:

- `sp` (boolean variable specifying if signal-present images are desired)
- `Nx` (number of pixels in x dimension)
- `Ny` (number of pixels in y dimension)
- `Rbg1` (range, i.e., twice amplitude, of background noise component 1)
- `Rbg2` (range, i.e., twice amplitude, of background noise component 2)
- `zc` (2×1 vector of signal center coordinates in the range $-Nx/2$ to $Nx/2$)
- `sigScale` (standard deviation of guassian signal, in pixels)
- `A` (signal amplitude)
- `sd` (seed for random number generator)

Outputs:

`g` (random Nx $\times$ Ny image)

`s` (Nx $\times$ Ny image of signal)

5.5.2 `create_images2.m`

Description: Generate correlated random images for two imaging scenarios applied to the same case, where both scenarios have the same overall task-performance.

Inputs:

`params` a structure with the following fields:

- `sp` (boolean variable specifying if signal-present images are desired)
- `Nx` (number of pixels in x dimension)
- `Ny` (number of pixels in y dimension)
- `Rbg1` (range, i.e., twice amplitude, of background noise component 1)
- `Rbg2` (range, i.e., twice amplitude, of background noise component 2)
- `zc` (2×1 vector of signal center coordinates in the range $-Nx/2$ to $Nx/2$)
- `sigScale` (standard deviation of guassian signal, in pixels)
- `A` (signal amplitude)
- `sd` (seed for random number generator)

Outputs:

`gA` (random Nx $\times$ Ny image for scenario A)

`gB` (random Nx $\times$ Ny image for scenario B)

`s` (Nx $\times$ Ny image of signal)

5.5.3 `make_channels.m`

Description: Construct (normalized) channel matrix for a 2-D image. For details and references see [14]

Inputs:

`chanType` (flag specifying type of channels)

channels are specified as follows, where p is the number of channels:

- `chanType` = 1: Gabor ($p = 40$)
- `chanType` = 2: DOG ($p = 3$)
- `chanType` = 3: dense DOG ($p = 10$)
- `chanType` = 4: square ($p = 4$)

`Nx` (number of x pixels)

`Ny` (number of y pixels)

`dx` (pixel size in cm)

Output:

`U` ($\mathtt{Nx} * \mathtt{Ny} \times p$ channel matrix)

A Conservative confidence intervals for a difference via Bonferroni

Given a method to estimate confidence intervals for θ_A and θ_B, Bonferroni's inequality justifies contruction of a conservative confidence interval for the difference $\theta_B - \theta_A$ in terms of differences of one-sided interval endpoints. This construction is described below, and is used in the functions `diffCI_CHO.m`, `diffCI_CHO_km.m`, and `diffCI_kt.m`. Recall that for two events, E_1 and E_2, Bonferroni's inequality [15, p. 11] takes the form

$$P(E_1 \cap E_2) \geq P(E_1) + P(E_2) - 1. \tag{A.1}$$

First, we obtain a lower bound on the differece $\theta_B - \theta_A$ with significance level α_1. Denoting the domain of θ_A and θ_B as (Θ_L, Θ_U), let $(\Theta_L, U_{A1}]$ and $[L_{B1}, \Theta_U)$ be one-sided $1 - \alpha_1/2$ confidence intervals for θ_A and θ_B, respectively. By definition,

$$P(\theta_A \leq U_{A1}) = 1 - \alpha_1/2 \qquad \text{and} \qquad P(\theta_B \geq L_{B1}) = 1 - \alpha_1/2. \tag{A.2}$$

Applying the Bonferroni inequality, it follows that

$$P(\theta_A \leq U_{A1} \text{ and } \theta_B \geq L_{B1}) \geq 1 - \alpha_1. \tag{A.3}$$

Next, observe that since $\theta_A \leq U_{A1}$ and $\theta_B \geq L_{B1}$ together imply that $\theta_B - \theta_A \geq L_{B1} - U_{A1}$, the former event is a subset of the latter. Consequently,

$$P(\theta_B - \theta_A \geq L_{B1} - U_{A1}) \geq P(\theta_A \leq U_{A1} \text{ and } \theta_B \geq L_{B1}) \geq 1 - \alpha_1. \tag{A.4}$$

We can similarly obtain an upper bound on the difference $\theta_B - \theta_A$ with significance level α_2. Namely, let $[L_{A2}, \Theta_U)$ and $(\Theta_L, U_{B2}]$ be one-sided $1 - \alpha_2/2$ confidence intervals for θ_A and θ_B, respectively. By definition,

$$P(\theta_A \geq L_{A2}) = 1 - \alpha_2/2 \qquad \text{and} \qquad P(\theta_B \leq U_{B2}) = 1 - \alpha_2/2. \tag{A.5}$$

Applying the Bonferroni inequality, it follows that

$$P(\theta_A \geq L_{A2} \text{ and } \theta_B \leq U_{B2}) \geq 1 - \alpha_2. \tag{A.6}$$

Also, since $\theta_A \geq L_{A2}$ and $\theta_B \leq U_{B2}$ together imply that $\theta_B - \theta_A \leq U_{B2} - L_{A2}$, the former event is a subset of the latter. Consequently,

$$P(\theta_B - \theta_A \leq U_{B2} - L_{A2}) \geq P(\theta_A \geq L_{A2} \text{ and } \theta_B \leq U_{B2}) \geq 1 - \alpha_2. \tag{A.7}$$

Thus, from (A.4) and (A.7), it follows that the interval $[L_{B1} - U_{A1},\ U_{B2} - L_{A2}]$ is a conservative $1 - (\alpha_1 + \alpha_2)$ confidence interval for $\theta_B - \theta_A$ with lower and upper significance levels α_1 and α_2, respectively.

Bibliography

[1] A. Wunderlich, F. Noo, B. D. Gallas, and M. E. Heilbrun, “Exact confidence intervals for channelized Hotelling observer performance in image quality studies,” *IEEE Trans. Med. Imag.*, vol. 34, no. 2, pp. 453–464, Feb 2015.

[2] A. Wunderlich and F. Noo, “New theoretical results on channelized Hotelling observer performance estimation with known difference of class means,” *IEEE Trans. Nucl. Sci.*, vol. 60, no. 1, pp. 182–193, Feb 2013.

[3] A. Wunderlich and F. Noo, “Confidence intervals for performance assessment of linear observers,” *Med. Phys.*, vol. 38, no. S1, pp. S57–S68, July 2011.

[4] A. Wunderlich and F. Noo, “On efficient assessment of image-quality metrics based on linear model observers,” *IEEE Trans. Nucl. Sci.*, vol. 59, no. 3, pp. 568–578, June 2012.

[5] E. R. DeLong, D. M. DeLong, and D. L. Clarke-Pearson, “Comparing the areas under two or more correlated receiver operating characteristic curves: A nonparametric approach,” *Biometrics*, vol. 44, no. 3, pp. 837–845, Sept 1988.

[6] X. Sun and W. Xu, “Fast implementation of DeLong’s algorithm for comparing the areas under correlated receiver operating characteristic curves,” *IEEE Signal Process. Lett.*, vol. 21, no. 11, pp. 1389–1393, Nov 2014.

[7] M. S. Pepe, *The Statistical Evaluation of Medical Tests for Classification and Prediction.* Oxford Univ. Press, 2003.

[8] A. Wunderlich and F. Noo, “A nonparametric procedure for comparing the areas under correlated LROC curves,” *IEEE Trans. Med. Imag.*, vol. 31, no. 11, pp. 2050–2061, Nov 2012.

[9] A. Wunderlich and B. Goossens, “Nonparametric estimation receiver operating characteristic analysis for performance evaluation on combined detection and estimation tasks,” *SPIE J. Med. Imag.*, vol. 1, no. 3, p. 031002, Oct 2014.

[10] F. Noo, A. Wunderlich, D. Heuscher, K. Schmitt, and Z. Yu, “A nonparametric approach for statistical comparison of results from alternative forced choice experiments,” in *Proc. SPIE Medical Imaging Conf.*, vol. 8673, Feb 2013, p. 86730F.

[11] N. A. Obuchowski and H. E. Rockette, “Hypothesis testing of diagnostic accuracy for multiple readers and multiple tests: An ANOVA approach with dependent observations,” *Commun. Stat. Simulat.*, vol. 24, no. 2, pp. 285–308, 1995.

[12] S. L. Hillis, "A comparison of denominator degrees of freedom methods for multiple observer ROC analysis," *Stat. Med.*, vol. 26, no. 3, pp. 596–619, Feb 2007.

[13] S. L. Hillis, "A marginal-mean ANOVA approach for analyzing multireader multicase radiological imaging data," *Stat. Med.*, vol. 33, no. 2, pp. 330–360, Jan 2014.

[14] A. Wunderlich and F. Noo, "Evaluation of the impact of tube current modulation on lesion detectability using model observers," in *Proc. Intl. Conf. IEEE Eng Med. Biol. Soc.*, Aug 2008, pp. 2705–2708.

[15] G. Casella and R. L. Berger, *Statistical Inference*, 2nd ed. Duxbury, 2001.