

calzone: A Python package for measuring calibration of probabilistic models for classification

Kwok Lung Fan¹, Gene Pennello¹, Qi Liu¹, Nicholas Petrick¹, Ravi K. Samala¹, Frank W. Samuelson¹, Yee Lam Elim Thompson¹, and Qian Cao¹¶

¹ U.S. Food and Drug Administration ¶ Corresponding author

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Summary

calzone is a Python package for evaluating the calibration of probabilistic outputs of classifier models. It provides a set of functions for visualizing calibration and computing calibration metrics given a representative dataset with the model's predictions and the true class labels. The metrics provided in calzone include: expected calibration error (ECE), maximum calibration error (MCE), Hosmer-Lemeshow (HL) test, integrated calibration index (ICI), Spiegelhalter's Z-statistic and Cox's calibration slope/intercept. The package is designed with versatility in mind. Many metrics allow users to adjust binning schemes and choose between top-class or class-wise calculations.

Statement of need

Classification is one of the most common applications in machine learning. Metrics associated with discrimination performance (resolution), such as area under the curve (AUC), sensitivity (Se, true positive rate), and specificity (Sp, 1 - false positive rate) are typically used to characterize classification performance Hastie et al. (2001). These metrics may be sufficient if the outputs of the model are not meant to be interpreted as a probability.

However, Diamond (1992) showed that the resolution (i.e., high performance) of a model does not indicate the reliability/calibration of the model. Calibration is the agreement between predicted and true probabilities, $P(D = 1|\hat{p} = p) = p$, defined as moderate calibration by Van Calster & Steyerberg (2018), also known as model reliability. Bröcker (2009) later showed that any proper scoring rule can be decomposed into the resolution and reliability. Thus, a model with high resolution may still lack reliability. In high-risk medical applications such as computer-aided diagnosis, reliability enables the correct interpretation of model output, and for downstream treatment decisions.

While existing libraries such as scikit-learn include basic tools like reliability diagrams and expected calibration error, they lack support for more comprehensive and flexible evaluation metrics—such as reliability diagrams with error bars, class-conditional calibration error, different binning schemes or statistical significance testing for miscalibration. This is also the case with other calibration-focused libraries, such as ml-calibration, uncertainty-toolbox, and pycaleva. For example, ml-calibration provides advanced controls for plotting reliability diagrams and computing smooth expected calibration error but does not include statistical tests for miscalibration (Blasiok & Nakkiran, 2024). The uncertainty-toolbox focuses on calibration methods rather than assessment (Chung et al., 2021). The pycaleva package overlaps with many functionalities in calzone, but it does not support Cox's calibration analysis, Wald intervals for reliability, or custom curve fitting methods for expected calibration error (Martin Weigl, 2022). In contrast, calzone emphasizes the evaluation of calibration. It

42 features a comprehensive set of calibration metrics, statistical tests (e.g., hypothesis testing
43 for miscalibration), and visualization tools tailored for many types of classification tasks (e.g.,
44 multi-class metrics). The package is designed to help users not only visualize miscalibration
45 but also quantify and statistically validate it in a consistent and interpretable way.

46 Software description

47 Input data

48 To evaluate the calibration of a model, users need a representative dataset from the intended
49 population. The dataset should contain the true class labels and the model's predicted
50 probabilities. In calzone, the dataset can be a CSV file or two NumPy arrays containing the
51 true labels and predicted probabilities.

52 Reliability Diagram

53 The reliability diagram is a graphical representation of the calibration (Bröcker & Smith, 2007;
54 Murphy & Winkler, 1977). It groups the predicted probabilities into bins and plots the mean
55 predicted probability against the empirical frequency in each bin. The reliability diagram can
56 be used to qualitatively assess the calibration of the model. The confidence intervals of the
57 empirical frequency are calculated using Wilson's score interval (Wilson, 1927).

```
from calzone.utils import reliability_diagram
from calzone.vis import plot_reliability_diagram
reliability, confidence, bin_edges, bin_counts = reliability_diagram(
    labels,
    probs,
    num_bins=15,
    class_to_plot=1
)

plot_reliability_diagram(
    reliability,
    confidence,
    bin_counts,
    error_bar=True,
    title='Reliability diagram'
)
```

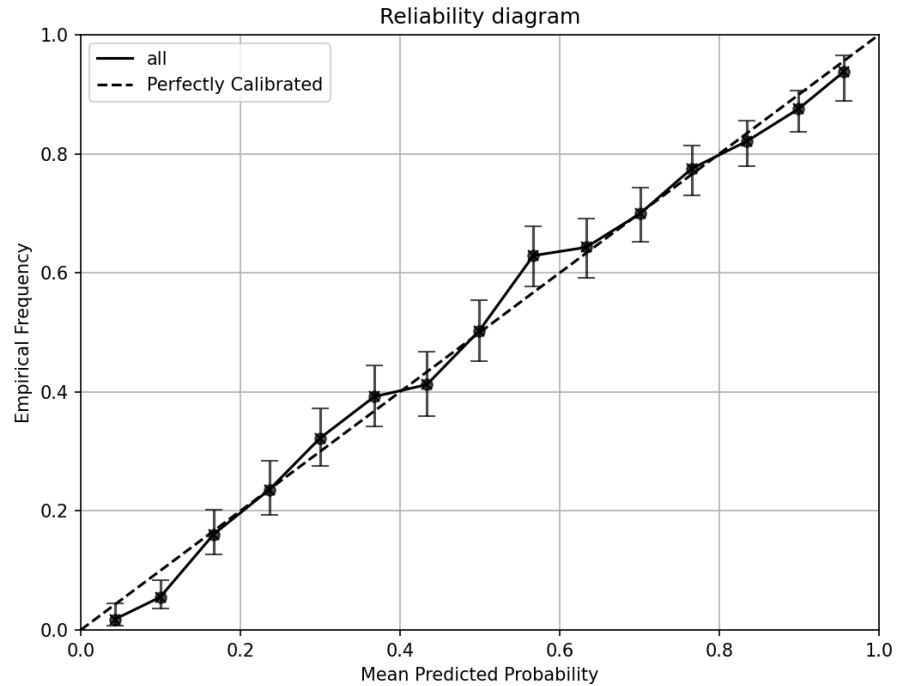


Figure 1: Reliability Diagram for class 1 with simulated data.

Calibration metrics

calzone provides functions to compute various calibration metrics, including methods to compute expected calibration error and statistical tests to assess calibration. These functions provide quantitative metrics for users to evaluate the calibration performance of the model. The `CalibrationMetrics()` class allows the user to compute the calibration metrics in a more convenient way. The following are metrics that are currently supported in calzone:

Expected Calibration Error (ECE) and Maximum Calibration Error (MCE)

Expected calibration error (ECE) and maximum calibration error (MCE) (Guo et al., 2017; Pakdaman Naeini et al., 2015) measure the average and maximum deviation between predicted and true probabilities. calzone supports two binning strategies for ECE: equal-width binning (ECE-H), which divides the probability range [0, 1] into bins of equal width, and equal-count binning (ECE-C), which divides predictions into bins containing approximately the same number of samples. Users can compute these metrics for the top-class (highest probability) or class-of-interest (one-vs-rest classification).

Hosmer-Lemeshow statistic (HL)

The Hosmer-Lemeshow (HL) test (Hosmer & Lemeshow, 1980) evaluates model calibration using a chi-square test comparing observed and expected events in bins. The null hypothesis is that the model is well calibrated. calzone supports equal-width (ECE-H) and equal-count (ECE-C) binning. The test statistic is:

$$HL = \sum_{m=1}^M \frac{(O_{1,m} - E_{1,m})^2}{E_{1,m} \left(1 - \frac{E_{1,m}}{N_m}\right)} \sim \chi_{M-2}^2$$

77 where $E_{1,m}$ and $O_{1,m}$ are the expected and observed events in the m^{th} bin, N_m is the total
 78 observations in the bin, and M is the number of bins. For validation sets, the degrees of
 79 freedom change from $M - 2$ to M (Hosmer Jr et al., 2013). The increase in degree of freedom
 80 for validation samples has often been overlooked but it is crucial for the test to maintain the
 81 correct Type I error rate. In calzone, the default is $M - 2$, adjustable via the `df` parameter.

82 Cox's calibration slope/intercept

83 Cox's calibration slope/intercept assesses model calibration without binning (Cox, 1958). A
 84 logistic regression is fit with predicted odds ($\frac{p}{1-p}$) as the independent variable and the outcome
 85 as the dependent variable. Perfect calibration is indicated by a slope of 1 and intercept of 0. To
 86 test calibration, fit the intercept with slope fixed at 1; if the intercept differs from 0, the model
 87 is not calibrated. Similarly, fit the slope with intercept fixed at 0; if the slope differs from 1,
 88 the model is not calibrated. Alternatively, fit both simultaneously using a bivariate distribution
 89 (McCullagh & Nelder, 1989). This feature is not in calzone, but users can manually test using
 90 the covariance matrix.

91 A slope >1 indicates overconfidence at high probabilities and underconfidence at low prob-
 92 abilities, while a slope <1 indicates the opposite. A positive intercept indicates general
 93 overconfidence. Even with ideal slope and intercept, non-linear miscalibration may still exist.

94 Integrated calibration index (ICI)

95 The integrated calibration index (ICI) measures the average deviation between predicted
 96 and true probabilities using curve smoothing techniques (Austin & Steyerberg, 2019). It is
 97 calculated as:

$$ICI = \frac{1}{n} \sum_{i=1}^n |f(p_i) - p_i|$$

98 where f is the fitting function and p is the predicted probability. Typically, Locally Weighted
 99 Scatterplot Smoothing (LOWESS) is used, but any curve fitting method can be applied.
 100 calzone supports both Cox regression-based ICI and LOWESS-based ICI, allowing users to
 101 choose their preferred method. Users should visualize the fitting results to avoid overfitting or
 102 underfitting, as flexible methods like LOWESS are sensitive to span and delta parameters.

103 Spiegelhalter's Z-test

104 Spiegelhalter's Z-test is a test of calibration proposed by Spiegelhalter in 1986 (Spiegelhalter,
 105 1986). It uses the fact that the Brier score can be decomposed into:

$$B = \frac{1}{N} \sum_{i=1}^N (x_i - p_i)^2 = \frac{1}{N} \sum_{i=1}^N (x_i - p_i)(1 - 2p_i) + \frac{1}{N} \sum_{i=1}^N p_i(1 - p_i)$$

106 And the test statistic (TS) of Z test is defined as:

$$Z = \frac{B - E(B)}{\sqrt{\text{Var}(B)}} = \frac{\sum_{i=1}^N (x_i - p_i)(1 - 2p_i)}{\sum_{i=1}^N (1 - 2p_i)^2 p_i(1 - p_i)}$$

107 and it is asymptotically distributed as a standard normal distribution.

108 Metrics class

109 calzone also provides a class called `CalibrationMetrics()` to calculate all the metrics men-
 110 tioned above. The function will return a dictionary containing the metrics' names and their
 111 values. The metrics can be specified as a list of strings. The string 'all' can be used to calculate
 112 all the metrics.

```
from calzone.metrics import CalibrationMetrics

metrics = CalibrationMetrics(class_to_calculate=1)

metrics.calculate_metrics(
    labels,
    probs,
    metrics='all'
)
```

Other features

Confidence intervals

calzone also provides functionality to compute confidence intervals for all metrics using bootstrapping. The user can specify the number of bootstrap samples and the confidence level.

```
from calzone.metrics import CalibrationMetrics

metrics = CalibrationMetrics(class_to_calculate=1)

CalibrationMetrics.bootstrap(
    labels,
    probs,
    metrics='all',
    n_samples=1000
)
```

and a structured NumPy array will be returned.

Subgroup analysis

calzone will perform subgroup analysis by default in the command line user interface. If the user input CSV file contains a subgroup column, the program will compute metrics for the entire dataset and for each subgroup. A detailed description of the input format can be found in the documentation.

Prevalence adjustment

calzone offers prevalence adjustment to correct for differences in disease prevalence between training and testing data. Calibration is based on posterior probability, so a shift in prevalence can cause miscalibration. The adjusted probability is calculated as:

$$P'(D = 1 | \hat{p} = p) = \frac{\eta' / (1 - \eta')}{(1/p - 1)(\eta / (1 - \eta))} = p'$$

where η is the testing data prevalence, η' is the training data prevalence, and p is the predicted probability. The optimal η' is found by minimizing cross-entropy loss, or users can specify η' directly if known (Chen et al., 2018; Gu & Pepe, 2010; Horsch et al., 2008; Tian et al., 2020).

Multiclass extension

calzone supports multiclass classification using a 1-vs-rest approach or top-class calibration. In top-class calibration, class 1 probability is the highest predicted probability, and class 0 is 1 minus this probability. Metrics interpretation may change in this transformation.

135 Verification of methods

136 To ensure the accuracy and reliability of the metrics implemented in calzone, we performed
137 comprehensive validation against established external packages. Reliability diagrams were
138 compared with `sklearn.calibration.calibration_curve()` (Pedregosa et al., 2011), top-
139 class ECE and Spiegelhalter's Z scores were validated against MAPE (Taquet et al., 2022), and
140 the Hosmer-Lemeshow statistic was checked against ResourceSelection (Lele et al., 2024) in
141 R. Additional tests were conducted using the `relplot` and `pycalleva` Python packages to further
142 confirm metric consistency. All differences were within 0.1%, demonstrating strong agreement.
143 These validation tests are documented in `test_results.py`. Furthermore, synthetic data tests
144 (see `test_metrics.py`) were used to confirm the expected behavior of the calibration metrics
145 under controlled conditions.

146 Command line interface

147 calzone offers a command line interface for visualizing calibration curves, calculating metrics,
148 and confidence intervals. Run `python cal_metrics.py -h` for help.

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158 Conflicts of interest

159 The authors declare no conflicts of interest.

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