The Journal of Open Source Software

calzone: A Python package for measuring calibration of probabilistic models for classification

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DOI: 10.xxxxxx/draft

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Editor: Open Journals

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Submitted: 01 January 1970
Published: unpublished

Summary

calzone is a Python package for evaluating the calibration of probabilistic outputs of classifier models. It provides a set of functions and classes for visualizing calibration and computing calibration metrics given a representative dataset with the model's predictions and true class labels. The metrics provided in calzone include: Expected Calibration Error (ECE), Maximum Calibration Error (MCE), Hosmer-Lemeshow (HL) statistic, Integrated Calibration Index (ICI), Spiegelhalter's Z-statistics and Cox's calibration slope/intercept. The package is designed with versatility in mind. For many of the metrics, users can adjust the binning scheme and toggle between top-class or class-wise calculations.

Statement of need

Classification is one of the most fundamental tasks in machine learning. Classification models are often evaluated by a proper scoring rule, such as cross-entropy or mean square error. Examination of the discrimination performance (resolution), such as AUC or Se/Sp are also used to evaluate the model performance. However, the reliability or calibration performance of the model is often overlooked.

Bröcker (2009) has shown that the proper scoring rule can be decomposed into the resolution and reliability. That means even if the model has high resolution (high AUC), it may not be a reliable or calibrated model. In many high-risk machine learning applications, such as medical diagnosis, the reliability of the model is of paramount importance.

We refer to calibration as the agreement between the predicted probability and the true posterior probability of a class-of-interest, $P(D = 1|\hat{p} = p) = p$. This is also termed as moderate calibration by Calster & Steyerberg (2018) .

In the calzone package, we provide a set of functions and classes for calibration visualization and metrics computation. Existing libraries such as scikit-learn are often not dedicated to calibration metrics computation and don't provide calibration metrics computation that are widely used in the statistical literature. Other libraries are focused on implementing calibration methods instead of ways to evaluate calibration [TODO: cite].

Functionality

Reliability Diagram

The reliability diagram is a graphical representation of the calibration of a classification model (Bröcker & Smith, 2007). It groups the predicted probabilities into bins and plots the mean predicted probability against the empirical frequency in each bin. The reliability diagram can be used to assess the calibration of the model and to identify any systematic errors in the predictions. In addition, `calzone` gives the option to also plot the confidence interval of the empirical frequency in each bin. The confidence intervals are calculated using Wilson's score interval (Wilson, 1927). We provide an example analsis in the `example_data` folder using beta-binomial distribution (Griffiths, 1973). Users can generate simulated data using the `fake_binary_data_generator` class in the `utils` module.

```
from calzone.utils import reliability_diagram
from calzone.vis import plot_reliability_diagram

wellcal_dataloader = data_loader(
    data_path="example_data/simulated_welldata.csv"
)

reliability, confidence, bin_edges, bin_counts = reliability_diagram(
    wellcal_dataloader.labels,
    wellcal_dataloader.probs,
    num_bins=15,
    class_to_plot=1
)

plot_reliability_diagram(
    reliability,
    confidence,
    bin_counts,
    error_bar=True,
    title='Class 1 reliability diagram for well calibrated data'
)
```

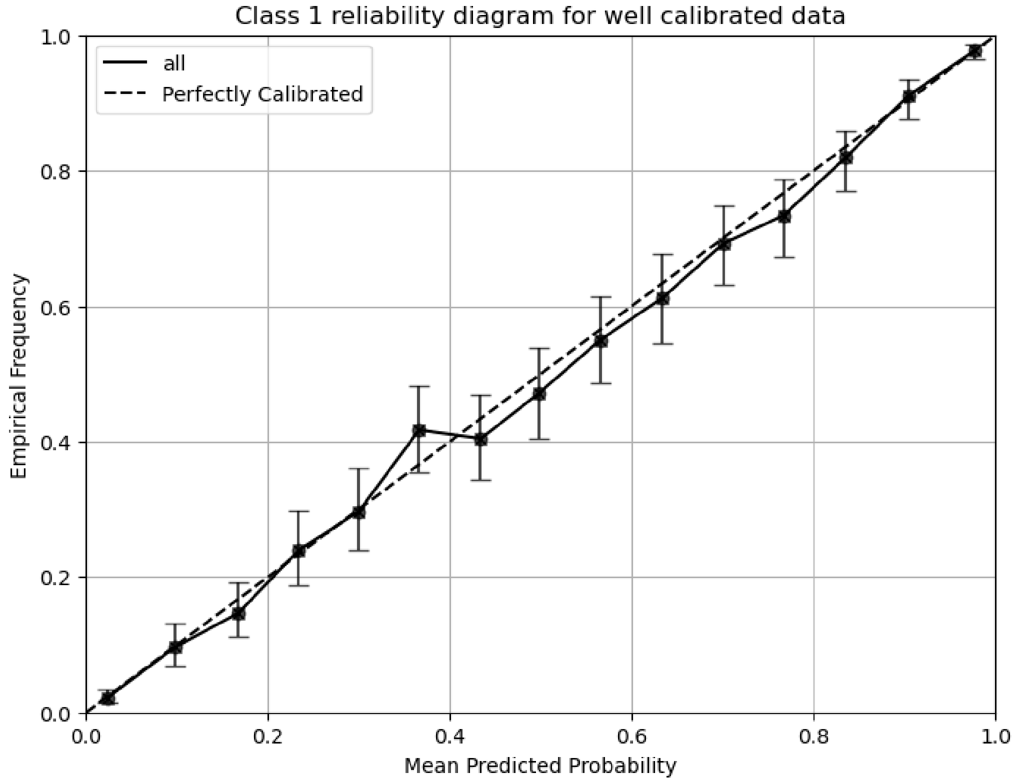

Figure 1: Reliability Diagram for well calibrated data

Calibration metrics

`calzone` provides functions to compute various calibration metrics. The `CalibrationMetrics()` class allows the user to compute the calibration metrics in a more convenient way. The following are metrics that are currently supported in `calzone`:

Expected Calibration Error (ECE) and Maximum Calibration Error (MCE)

Expected Calibration Error (ECE), Maximum Calibration Error (MCE) and other binning-based methods (Guo et al., 2017; Pakdaman Naeini et al., 2015) aim to measure the average deviation between predicted probability and true probability. We provide the option to use equal-width binning or equal-count binning, labeled as ECE-H and ECE-C respectively. Users can also choose to compute the metrics for the class-of-interest or the top-class. In the case of class-of-interest, `calzone` will evaluate the calibration of a one-vs-rest classification problem. The following snipped demonstrates how these metrics are calculated in our package:

```
from calzone.metrics import calculate_ece_mce

reliability, confidence, bin_edges, bin_counts = reliability_diagram(
    wellcal_dataloader.labels,
    wellcal_dataloader.probs,
    num_bins=10,
    class_to_plot=1,
    is_equal_freq=False
)

ece_h_classone, mce_h_classone = calculate_ece_mce(
```

```
    reliability,
    confidence,
    bin_counts=bin_counts
)
```

Hosmer-Lemeshow statistic (HL)

The Hosmer-Lemeshow (HL) statistical test is for evaluating the calibration of a probabilistic model. It is a chi-square-based test that compares the observed and expected number of events in each bin. The null hypothesis is that the model is well calibrated. HL-test first bins data into predicted probability bins (equal-width H or equal-count C) and the test statistic is calculated as:

$$\text{HL} = \sum_{m=1}^{M} \frac{(O_{1,m} - E_{1,m})^2}{E_{1,m}(1 - \frac{E_{1,m}}{N_m})} \sim \chi^2_{M-2}$$

where $E_{1,m}$ is the expected number of class-of-interest events in the mth bin, $O_{1,m}$ is the observed number of class-of-interest events in the mth bin, N_m is the total number of observations in the mth bin, and M is the number of bins. In calzone, the HL-test can be computed as follows:

```python
from calzone.metrics import hosmer_lemeshow_test

HL_H_ts, HL_H_p, df = hosmer_lemeshow_test(
    reliability,
    confidence,
    bin_count=bin_counts
)
```

In calzone, user can sepecify the degree of freedom of the HL test by setting the df parameter.This is useful because when performing the HL test on validation sets that are not used in training, the degree of freedom of the HL test changes from $M-2$ to M [TODO: cite]. Intuitively, $\frac{(O_{1,m}-E_{1,m})^2}{E_{1,m}(1-\frac{E_{1,m}}{N_m})}$ is the difference squared divided by the variance of a binomial distribution and follows a chi-square distribution with 1 degree of freedom. Hence, the sum of M chi-square distributions with 1 degree of freedom is a chi-square distribution with M degrees of freedom if the data has no effect on the model.

Cox's calibration slope/intercept

Cox's calibration slope/intercept is a non-parametric method for assessing the calibration of a probabilistic model (COX, 1958). A new logistic regression model is fitted to the data, with the predicted odds ($\frac{p}{1-p}$) as the dependent variable and the true probability as the independent variable. The slope and intercept of the regression line are then used to assess the calibration of the model. A slope of 1 and intercept of 0 indicates perfect calibration. To test whether the model is calibrated, fix the slope to 1 and fit the intercept. If the intercept is significantly different from 0, the model is not calibrated. Then, fix the intercept to 0 and fit the slope. If the slope is significantly different from 1, the model is not calibrated. In calzone, Cox's calibration slope/intercept can be computed as follows:

```python
from calzone.metrics import cox_regression_analysis

cox_slope, cox_intercept, cox_slope_ci, cox_intercept_ci = cox_regression_analysis(
    wellcal_dataloader.labels,
    wellcal_dataloader.probs,
    class_to_calculate=1,
    print_results=True,
```

```
    fix_slope=True
)
```

The values of the slope and intercept can represent miscalibration throughout the range of probabiliy outputs. The integrated calibration index (ICI) is very similar to Expected calibration error (ECE). It also tries to measure the average deviation between predicted probability and true probability. However, ICI does not use binning to estimate the true probability of a group of samples with similar predicted probability. Instead, ICI uses curve smoothing techniques to fit the regression curve and uses the regression result as the true probability (Austin & Steyerberg, 2019). The ICI is then calculated using the following formula:

$$\text{ICI} = \frac{1}{n}\sum_{i=1}^{n} |f(p_i) - p_i|$$

where f is the fitting function and p is the predicted probability. The curve fitting is usually done with loess regression. However, it is possible to use any curve fitting method to calculate the ICI. In calzone, we provide Cox's ICI and loess ICI support while the user can also use any curve fitting method to calculate the ICI using functions in calzone.

```
from calzone.metrics import (
    cox_regression_analysis,
    lowess_regression_analysis,
    cal_ICI_cox
)

### calculating cox ICI
cox_ici = cal_ICI_cox(
    cox_slope,
    cox_intercept,
    wellcal_dataloader.probs,
    class_to_calculate=1
)

### calculating loess ICI
loess_ici, lowess_fit_p, lowess_fit_p_correct = lowess_regression_analysis(
    wellcal_dataloader.labels,
    wellcal_dataloader.probs,
    class_to_calculate=1,
    span=0.5,
    delta=0.001,
    it=0
)
```

Notice that flexible curve fitting methods such as Loess regression are very sensitive to the choice of span and delta parameters. The user can visualize the fitting result to avoid overfitting or underfitting.

Spiegelhalter's Z-test

Spiegelhalter's Z-test is a test of calibration proposed by Spiegelhalter in 1986 (Spiegelhalter, 1986). It uses the fact that the Brier score can be decomposed into:

$$B = \frac{1}{N}\sum_{i=1}^{N}(x_i - p_i)^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - p_i)(1 - 2p_i) + \frac{1}{N}\sum_{i=1}^{N} p_i(1 - p_i)$$

And the TS of Z test is defined as:

$$Z = \frac{B - E(B)}{\sqrt{\text{Var}(B)}} = \frac{\sum_{i=1}^{N}(x_i - p_i)(1 - 2p_i)}{\sum_{i=1}^{N}(1 - 2p_i)^2 p_i(1 - p_i)}$$

and it is asymptotically distributed as a standard normal distribution. In calzone, it can be calculated using:

```
from calzone.metrics import spiegelhalter_z_test

z, p_value = spiegelhalter_z_test(
    wellcal_dataloader.labels,
    wellcal_dataloader.probs,
    class_to_calculate=1
)
```

Metrics class

calzone also provides a class called CalibrationMetrics() to calculate all the metrics mentioned above. The user can also use this class to calculate the metrics.

```
from calzone.metrics import CalibrationMetrics

metrics = CalibrationMetrics(class_to_calculate=1)

CalibrationMetrics.calculate_metrics(
    wellcal_dataloader.labels,
    wellcal_dataloader.probs,
    metrics='all'
)
```

Other features

Confidence intervals

In addition to point estimates of calibration performance, calzone also provides bootstrapping to calculate the confidence intervals of the metrics. The user can specify the number of bootstrap samples and the confidence level.

```
from calzone.metrics import CalibrationMetrics

metrics = CalibrationMetrics(class_to_calculate=1)

CalibrationMetrics.bootstrap(
    wellcal_dataloader.labels,
    wellcal_dataloader.probs,
    metrics='all',
    n_samples=1000
)
```

and a structured numpy array will be returned.

Subgroup analysis

calzone will perform subgroup analysis by default in the command line user interface. If the user input CSV file contains a subgroup column, the program will compute metrics for the entire dataset and for each subgroup.

Prevalence adjustment

calzone also provides prevalence adjustment to account for prevalence changes between training data and testing data. Since calibration is defined using posterior probability, a mere shift in the prevalence of the testing data will result in miscalibration. It can be fixed by searching for the optimal derived original prevalence such that the adjusted probability minimizes a proper scoring rule such as cross-entropy loss. The formula of prevalence adjusted probability is:

$$P'(D=1|\hat{p}=p) = \frac{\eta'/(1-\eta')}{(1/p-1)(\eta/(1-\eta))} = p'$$

where η is the prevalence of the testing data, η' is the prevalence of the training data, and p is the predicted probability (Chen et al., 2018; Gu & Pepe, 2010; Horsch et al., 2008; Tian et al., 2020). We search for the optimal η' that minimizes the cross-entropy loss.

Multiclass extension

calzone also provides multiclass extension to calculate the metrics for multiclass classification. The user can specify the class to calculate the metrics using a 1-vs-rest approach and test the calibration of each class. Alternatively, the user can transform the data and make the problem become a top-class calibration problem. The top-class calibration has a similar format to binary classification, but the class 0 probability is defined as 1 minus the probability of the class with the highest probability, and the class 1 probability is defined as the probability of the class with the highest probability. The labels are transformed into whether the predicted class equals the true class, 0 if not and 1 if yes. Notice that the interpretation of some metrics may change in the top-class transformation.

Command line interface

calzone also provides a command line interface to calculate the metrics. The user can visualize the calibration curve, calculate the metrics and their confidence intervals using the command line interface. To use the command line interface, the user can run `python cal_metrics.py -h` to see the help message.

Acknowledgements

The authors acknowledge the Research Participation Program at the Center for Devices and Radiological Health administered by the Oak Ridge Institute for Science and Education through an interagency agreement between the U.S. Department of Energy and the U.S. Food and Drug Administration (FDA).

Conflicts of interest

The authors declare no conflicts of interest.

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The Journal of Open Source Software

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