Implementation of large scale integral operators with modern HPC solutions

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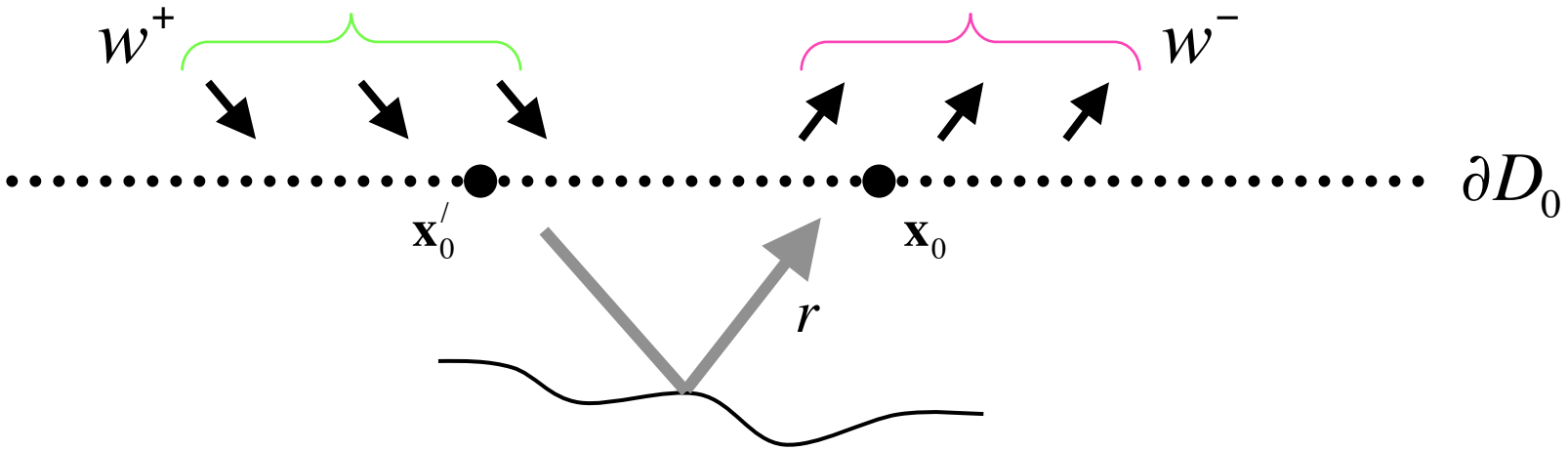
EAGEOnline2020

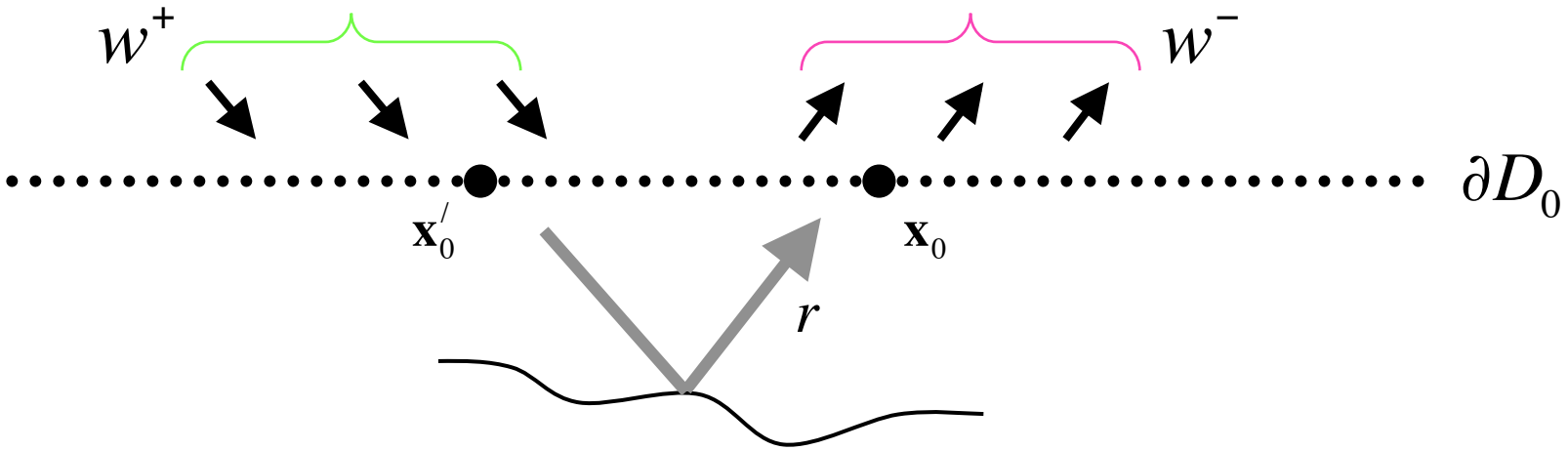
A common building block for...



Multidimensional convolution (MDC)

$$w^-(\mathbf{x}_0,\mathbf{x}_S,t)=\int_{\partial D_0}\int w^+(\mathbf{x}_0^{/},\mathbf{x}_S,t-\tau)r(\mathbf{x}_0,\mathbf{x}_0^{/},\tau)\,d\tau\,d\mathbf{x}_0^{/}$$

A common building block for...



Multidimensional convolution (MDC)

$$w^-(\mathbf{x}_0,\mathbf{x}_S,t) = F^{-1}\{\int_{\partial D_0} w^+(\mathbf{x}_0^/,\mathbf{x}_S,\omega)F\left\{r(\mathbf{x}_0,\mathbf{x}_0^/,t)\right\}d\mathbf{x}_0^/\}$$

...many problems in geophysics

- Surface multiple attenuation: SRME
- Internal multiple attenuation: Jakubovic method, ISS

} **1 step**

- FWM and JMI
- Multi-dimensional deconvolution (MDD)

} **Fully or partially separable in sources** **(repeated)**

- EPSI and closed-loop SRME
- Marchenko-based methods

} **All data at once (repeated)**

MDC as a linear operator

$$\mathbf{w}^{-} = \mathbf{W}^{+}\mathbf{r}$$

$$\mathbf{F}^{\mathbf{H}} \quad \mathbf{I}^{H}_{\omega_{\max}} \quad \hat{\mathbf{W}}^{+} \quad \mathbf{I}_{\omega_{\max}} \quad \mathbf{F}$$

Real IFFT

Padding

Batch matrix multiplication

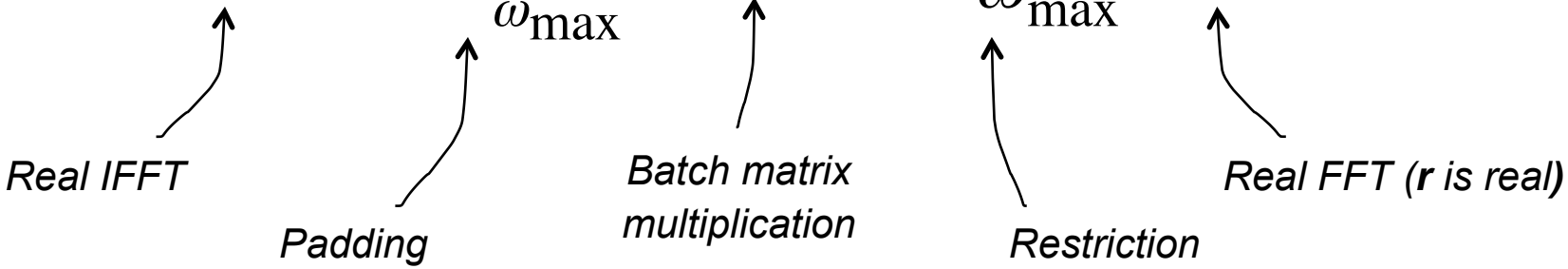
Restriction

*Real FFT (**r** is real)*

MDC as a linear operator

$$\mathbf{r} \approx \mathbf{W}^{+\,H}\,\mathbf{w}^{-}$$

$$\mathbf{F}^{\mathbf{H}}\quad \mathbf{I}^{H}_{\omega_{\max}}\quad \hat{\mathbf{W}}^{+\,H}\quad \mathbf{I}_{\omega_{\max}}\quad \mathbf{F}$$



MDC as a linear operator

$$r(\mathbf{x}_0, \mathbf{x}_0^{/}, \tau) \approx \int_{\partial D_S} \int w^{+}(\mathbf{x}_0^{/}, \mathbf{x}_S, t+\tau) w^{-}(\mathbf{x}_0, \mathbf{x}_S, t)\, dt\, d\mathbf{x}_S$$

$$\mathbf{F}^{\mathbf{H}} \quad \mathbf{I}^{H}_{\omega_{\max}} \quad \hat{\mathbf{W}}^{+H} \quad \mathbf{I}_{\omega_{\max}} \quad \mathbf{F}$$

Real IFFT

Padding

Batch matrix multiplication

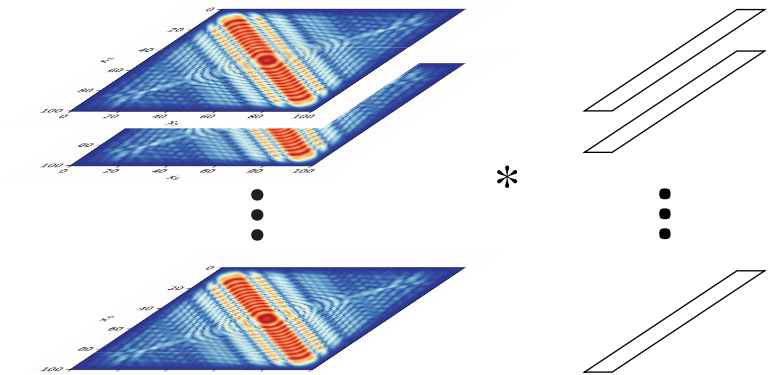
Restriction

*Real FFT (**r** is real)*

MDC as a linear operator

$\hat{\mathbf{W}}^{+}$

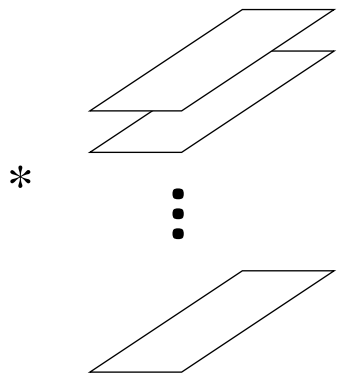
$N_{\omega_{\max}} \; x \; N_S \; x \; N_R$ $\quad$ $N_{\omega_{\max}} \; x \; N_R \; x \; 1$



Batch matrix-vector multiplication

OR

$N_{\omega_{\max}} \; x \; N_R \; x \; N_{VS}$

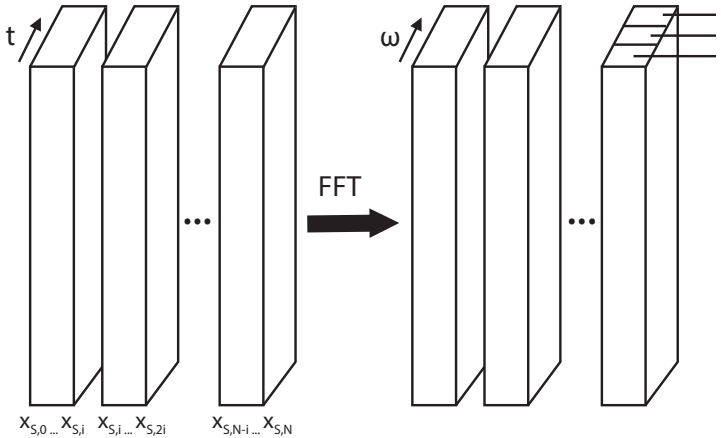


Batch matrix-matrix mult.

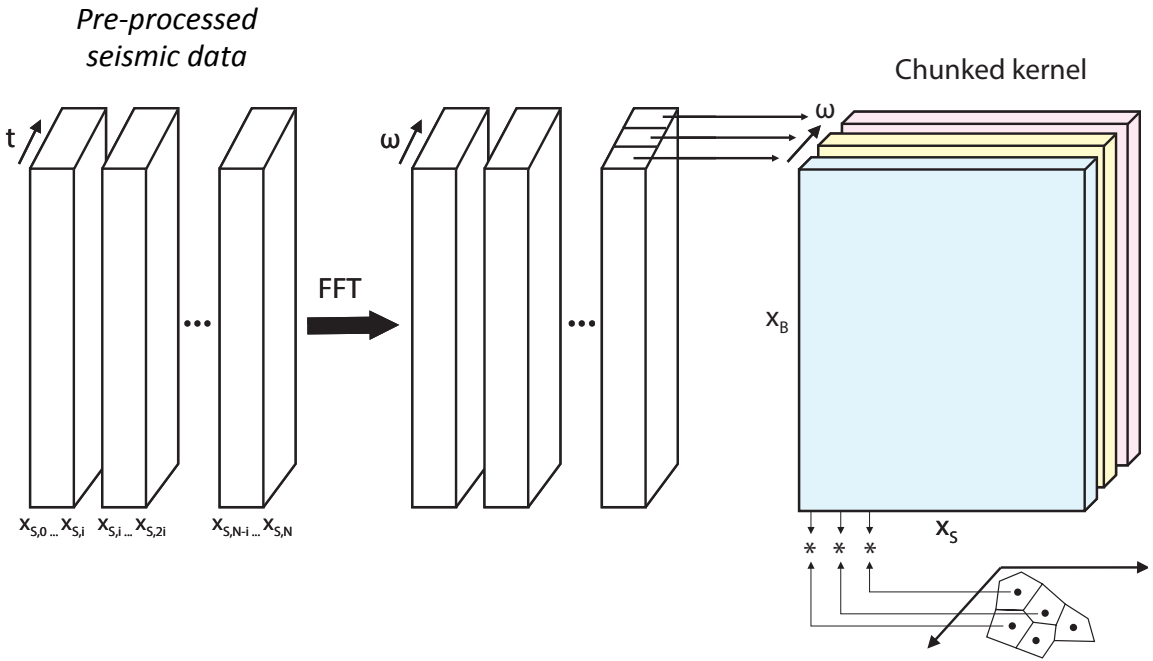
Fully decoupled computations along on frequency axis → parallel computations…

MDC as a distributed operator

Pre-processed seismic data

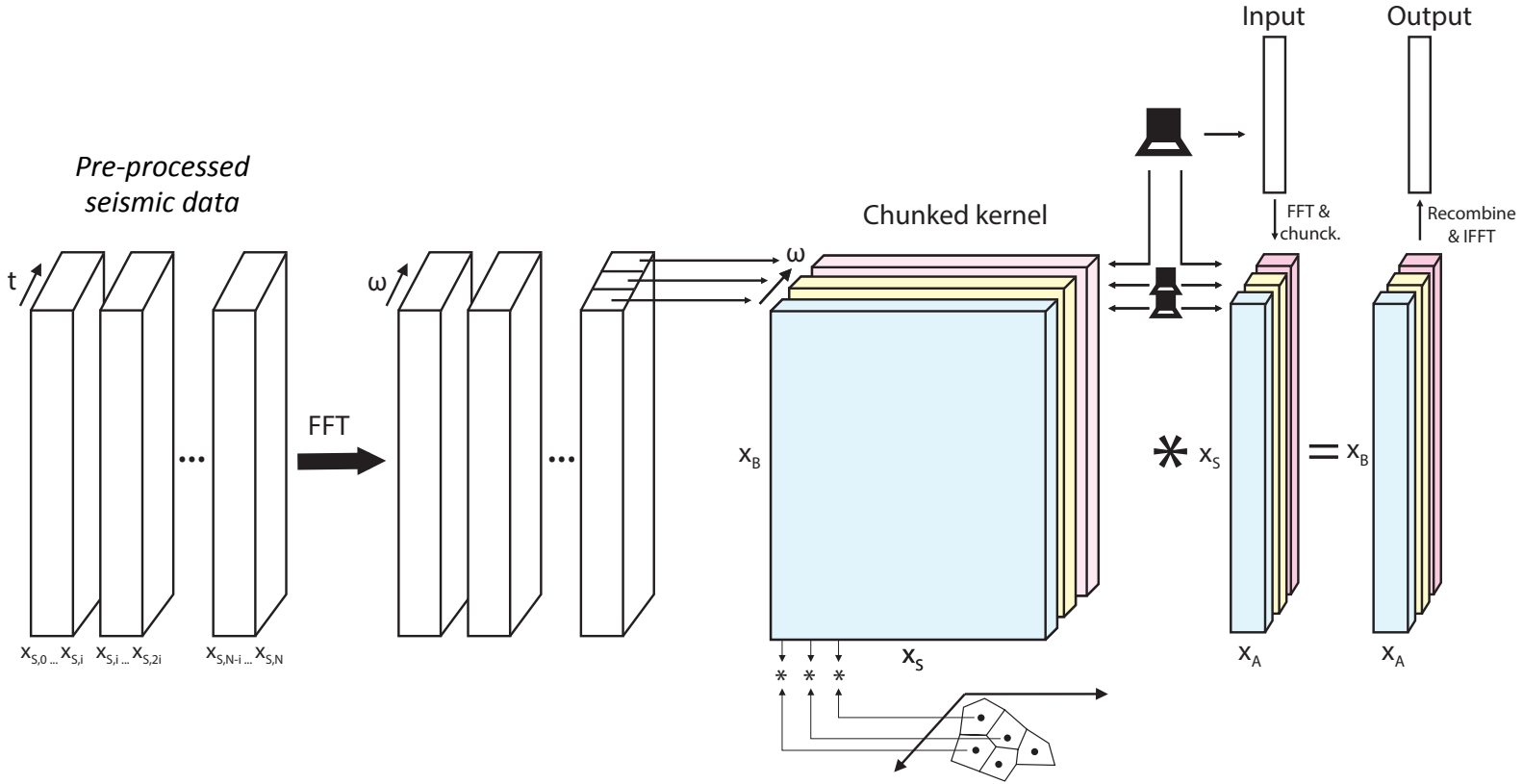


MDC as a distributed operator



$$\int_{\partial D_0} \int w^{+}(\mathbf{x}_0^{/}, \mathbf{x}_S, t-\tau) \cdot d\tau\, d\mathbf{x}_0$$ → Upfront data scaling (sampling): $\cdot \Delta t \cdot \Delta x_0$

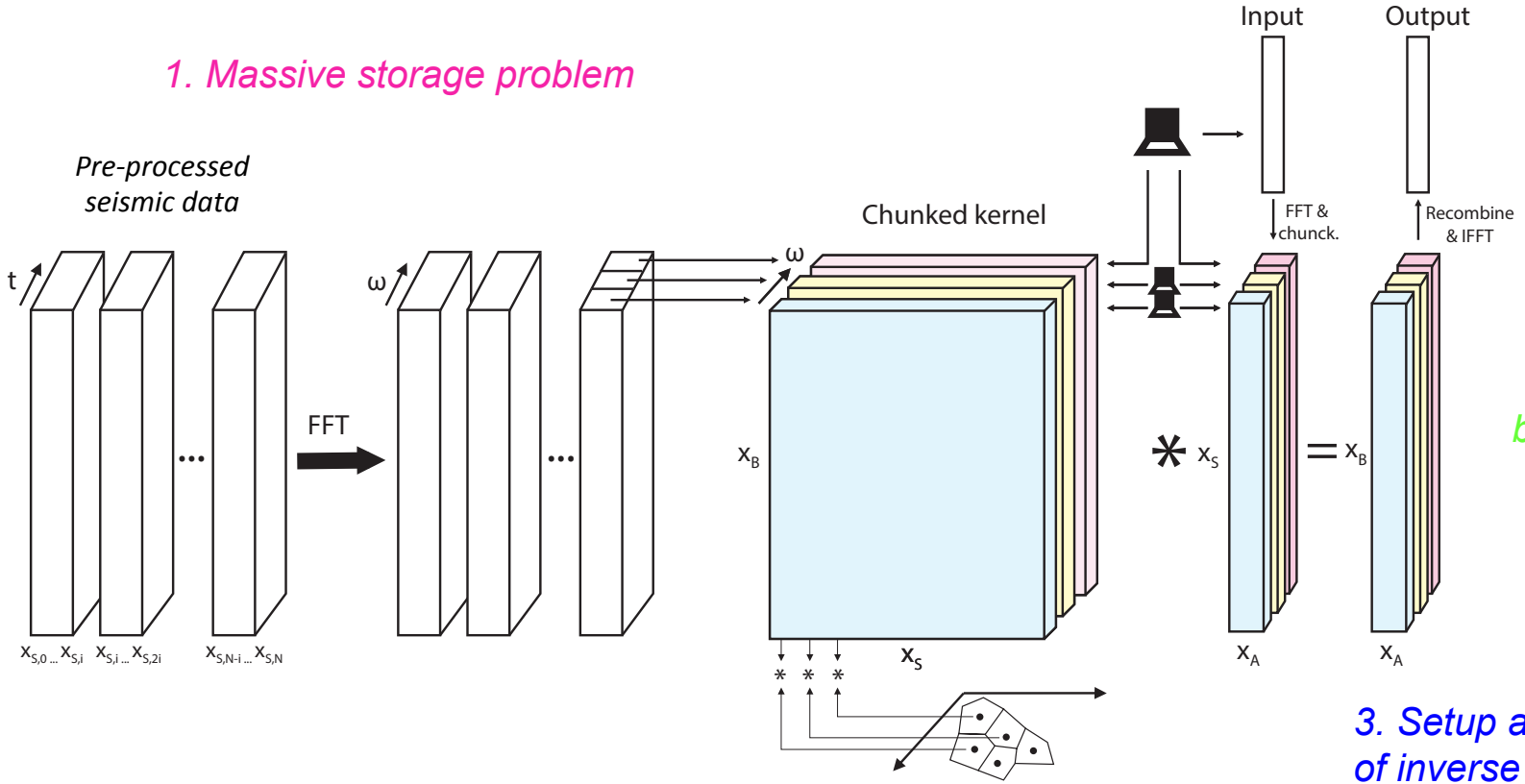
MDC as a distributed operator



$$\int_{\partial D_0} \int w^{+}(\mathbf{x}_0^{/}, \mathbf{x}_S, t-\tau) \cdot d\tau\, d\mathbf{x}_0$$ → Upfront data scaling (sampling): $\cdot \Delta t \cdot \Delta x_0$

MDC as a distributed operator

1. Massive storage problem



2. Out-of-core batched matrix multiplication

3. Setup and solution of inverse problem

$$\int_{\partial D_0} \int w^+(\mathbf{x}_0', \mathbf{x}_S, t-\tau) \cdot d\tau\, d\mathbf{x}_0$$ → Upfront data scaling (sampling): $\cdot \Delta t \cdot \Delta x_0$

MDC as a distributed operator

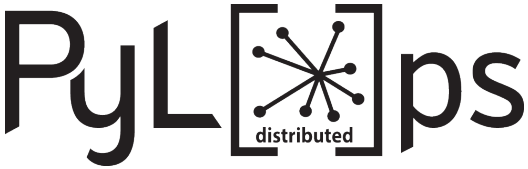
1. Massive storage problem

*"**Chunked, compressed, N-dimensional arrays** to be written and read concurrently from multiple threads or processes."*

2. Out-of-core batched matrix multiplication

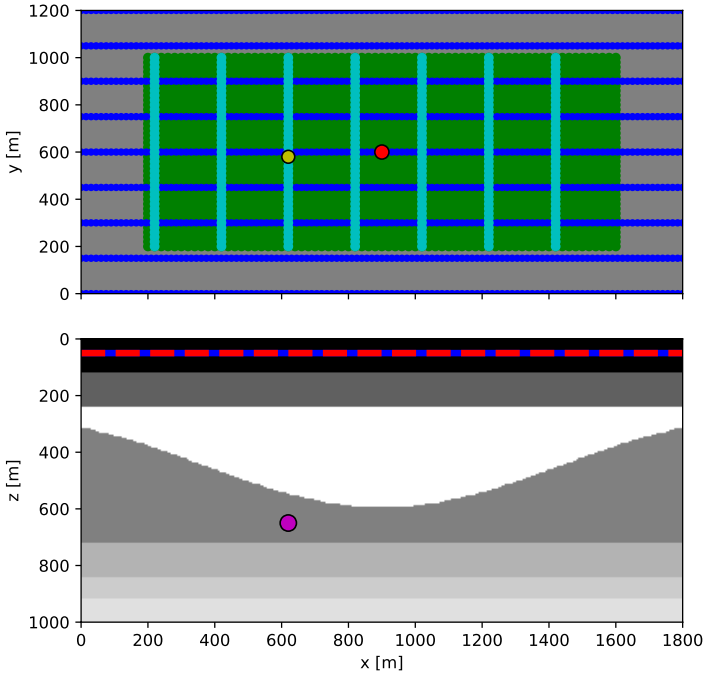
*"**Distributed linear algebra operations** in python, with* directed acyclic graph of tasks with data dependencies *"*

3. Setup and solution of inverse problem



*"Expressive inverse problem setup and solution with **distributed linear operators**"*

Example setup



Redatuming carpet @ 650m

Marchenko lines in displays

MDD lines in displays

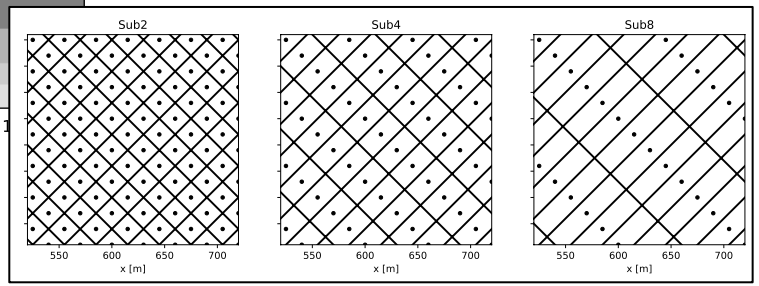
- *Constant velocity, variable density model of 1.8km x 1.2km x 1km size*

- *Regular carpet of sources and receivers*

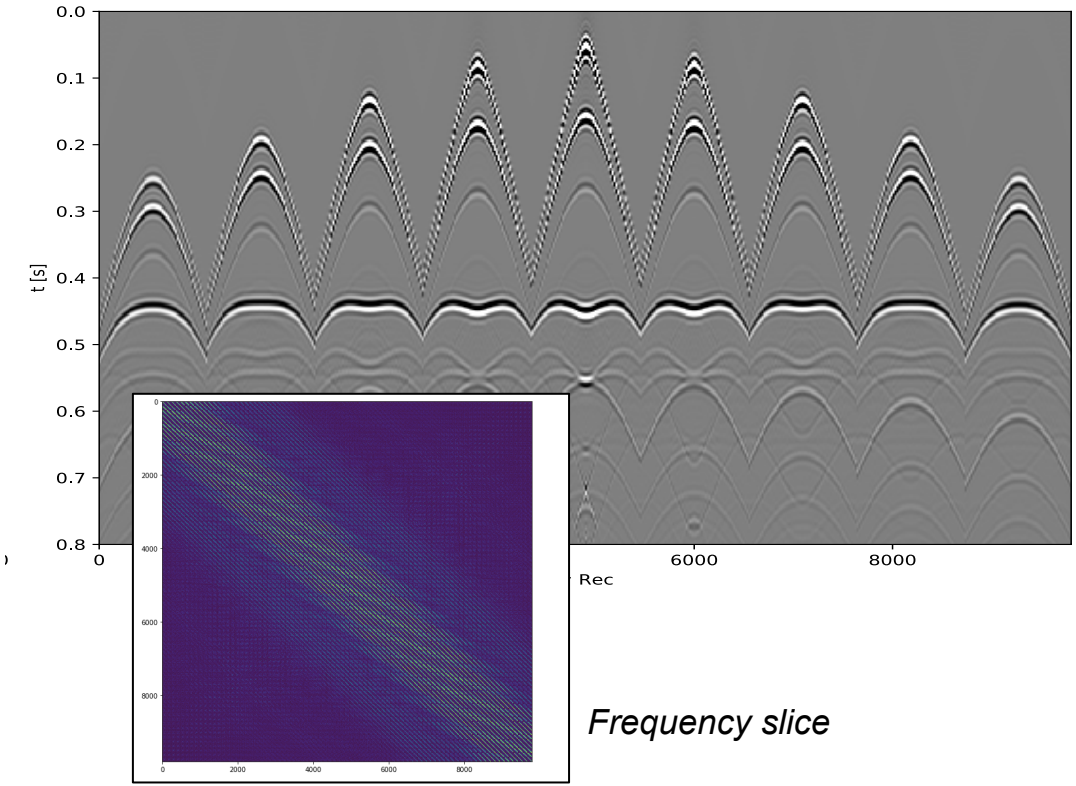
  $N_S = N_R = 9801$

  $dx_S = dx_R = 15m$

- *Subsampled geometries by factor of 2,4, and 8.*



Example setup



Frequency slice

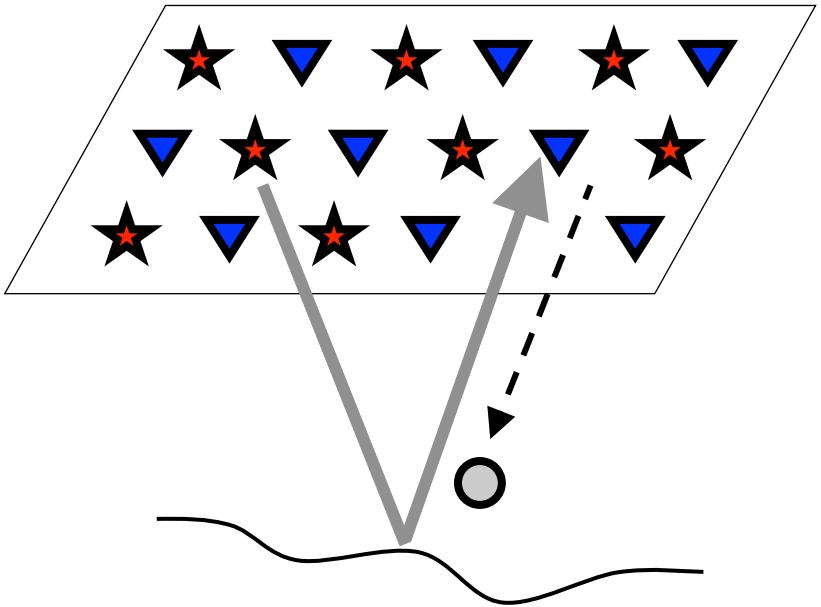
- Broadband reflection response (MDC kernel)

- 20 Hz Ricker wavelet input wavefields

- $N_{\omega,max}$ = 250 (f<60 Hz)

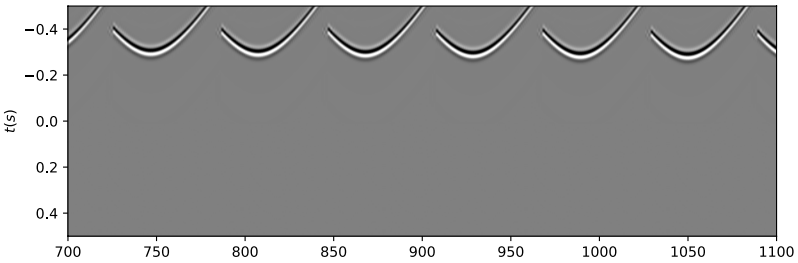
- Data stored in Zarr format with chunking along frequency axis

MDC example

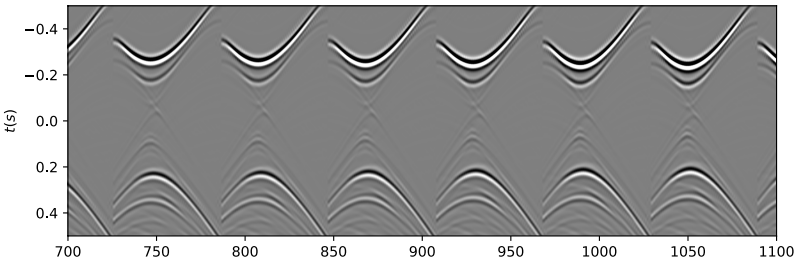


$$\mathbf{g}_0^- \approx \mathbf{R}\mathbf{g}_\mathbf{d}^{-1}$$

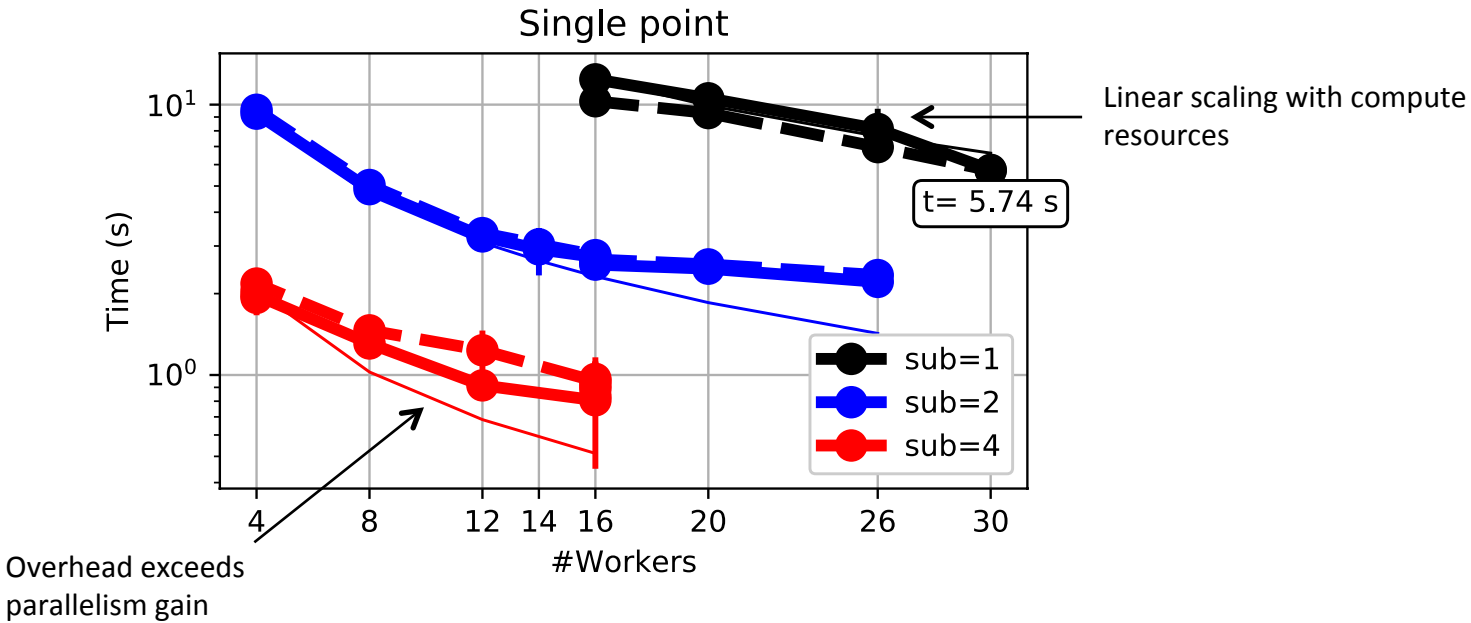
$\mathbf{g}_\mathbf{d}^{-1}$



$\mathbf{g}^-$

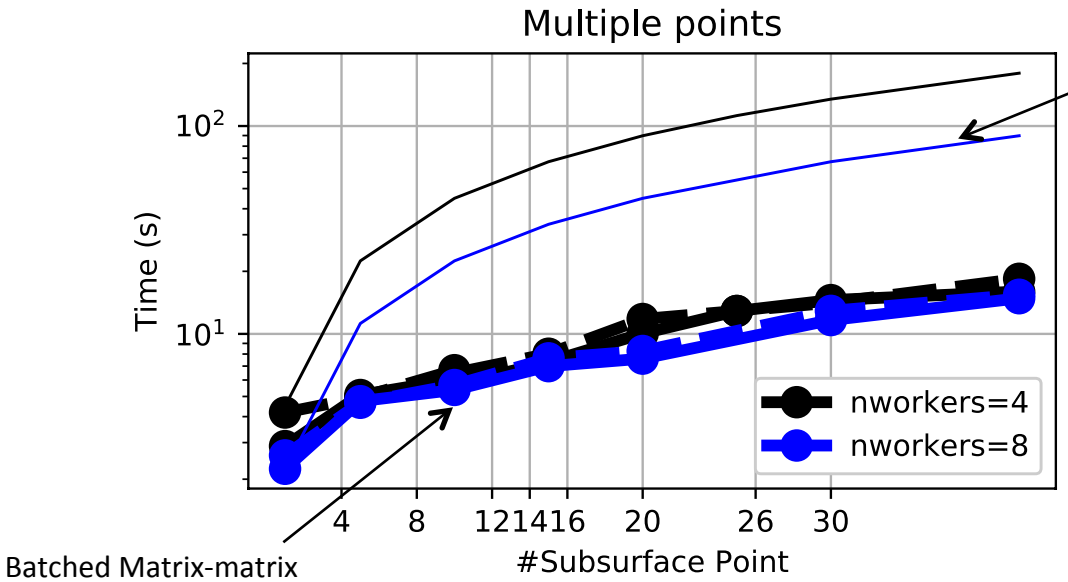


MDC example



Geometry	Data dims.	Kernel size (GB)
Full	300 x 9801 x 9801	230
Sub2	300 x 4901x 4901	57
Sub4	300 x 2451x 2451	14

MDC example



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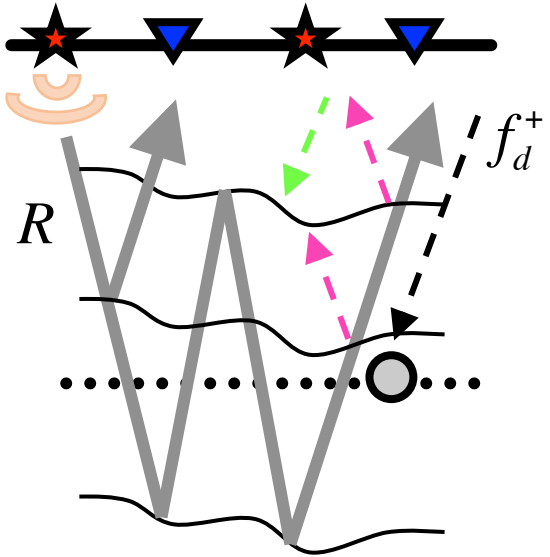
Marchenko example

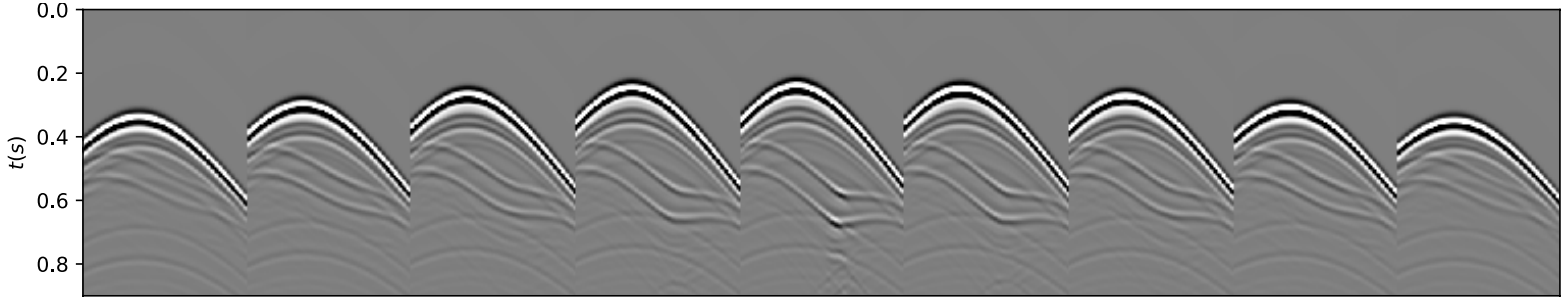


$$\begin{bmatrix} \Theta \mathbf{R} \mathbf{f}_\mathbf{d}^+ \\ \mathbf{0} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & -\Theta \mathbf{R} \\ \Theta \mathbf{R}^* & \mathbf{I} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} \mathbf{f}^- \\ \mathbf{f}_\mathbf{m}^+ \end{bmatrix}$$

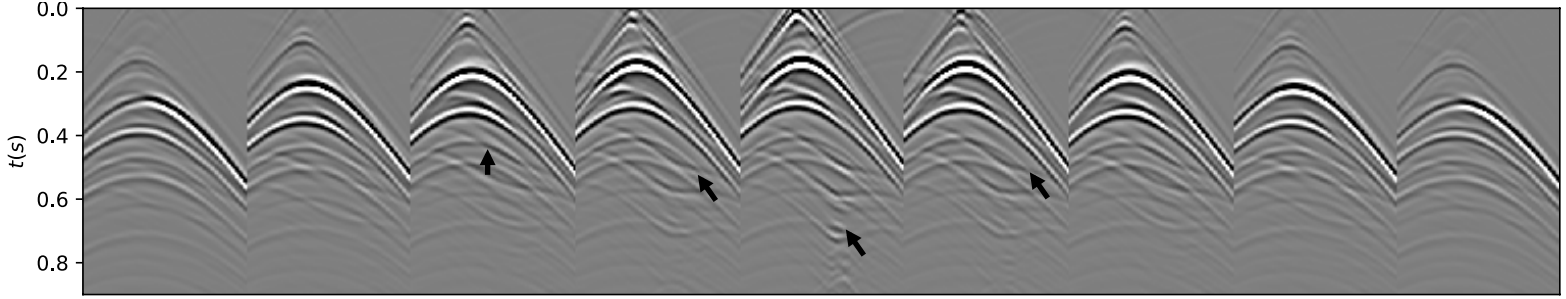
Forward: 2 MDC, Adjoint: 2 MDC
CGLS (10 iterations): 40 MDC

Marchenko example

FD modelling reference



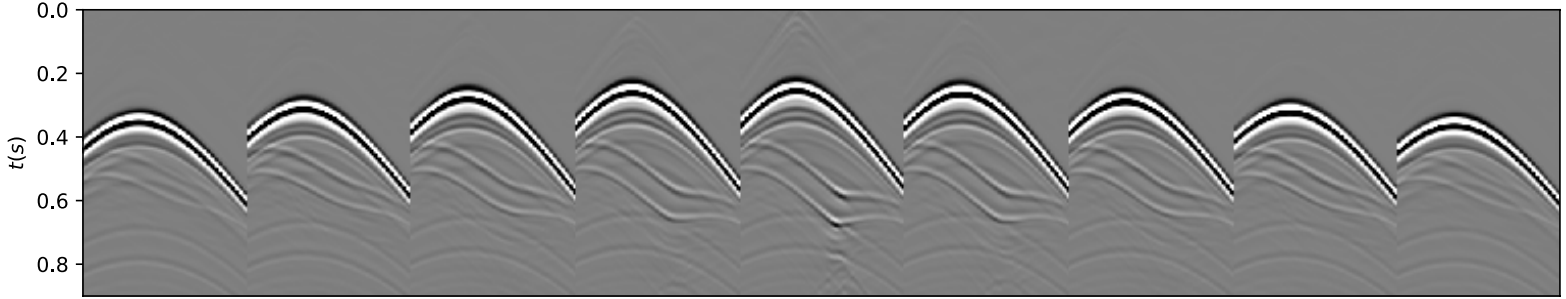
Standard redatuming Upgoing Green's function



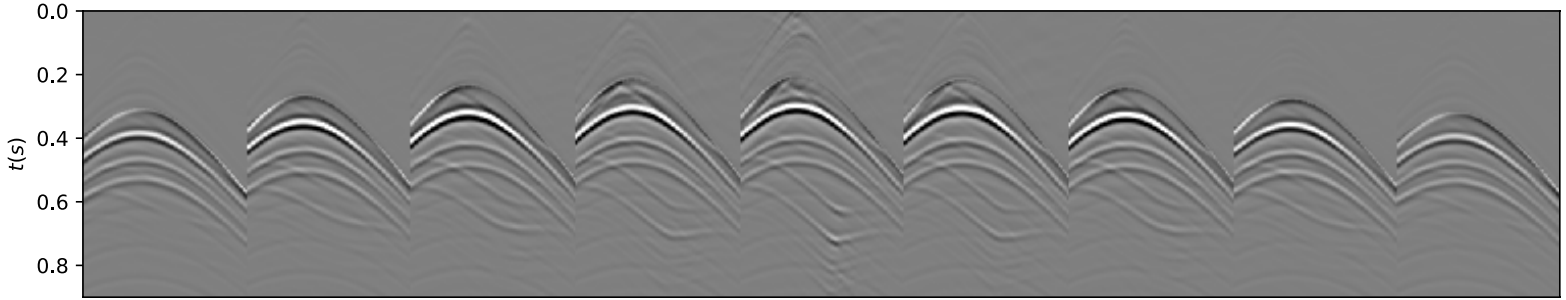
Marchenko example

Full − 15m x 15m

Marchenko Full Green's function



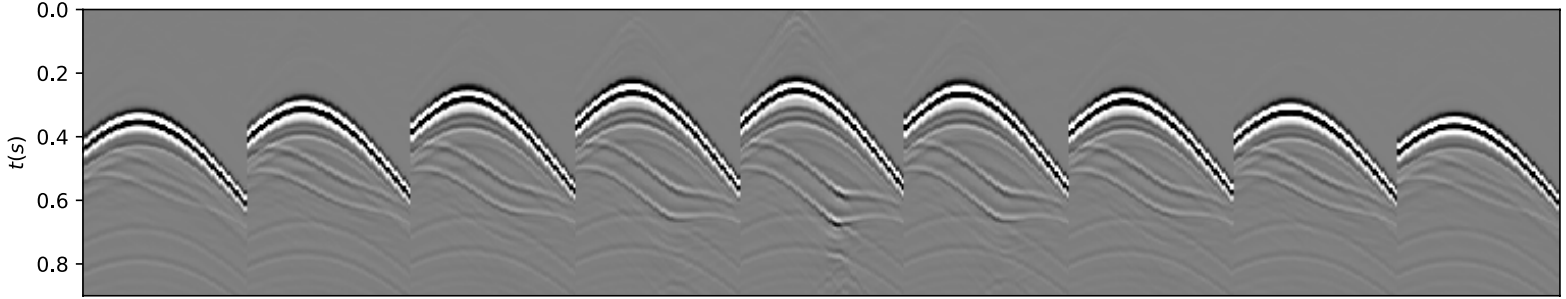
Marchenko Upgoing Green's function



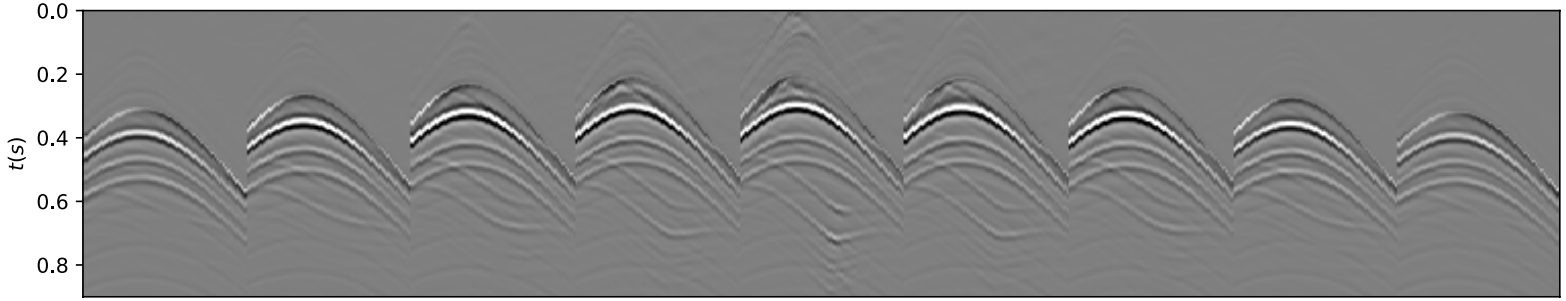
Marchenko example

Sub2– 21m x 21m

Marchenko Full Green's function



Marchenko Upgoing Green's function

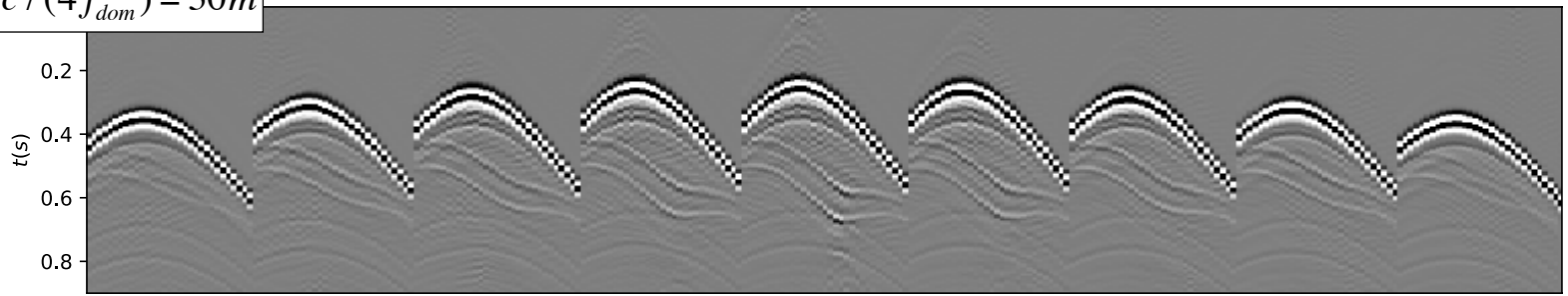


Marchenko example

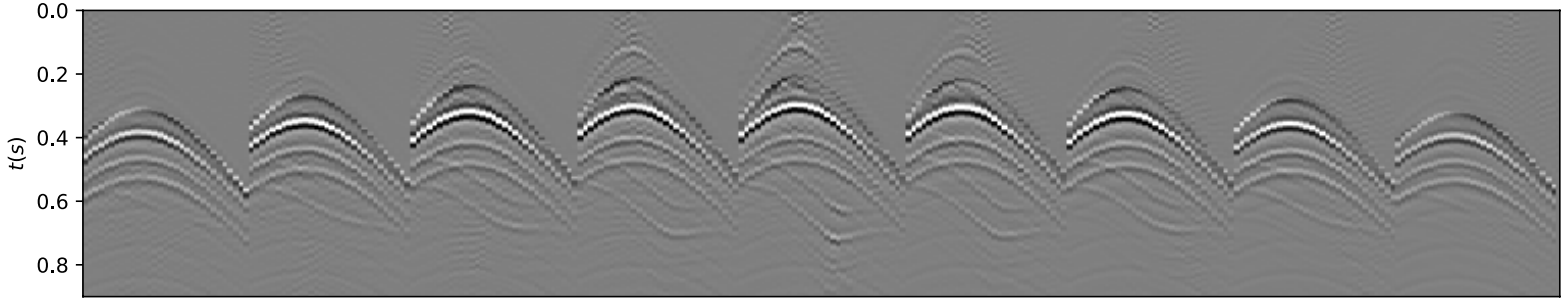
Sub4– 21m x 42m

$$\lambda_{dom} / 4 = c / (4 f_{dom}) = 30m$$

Marchenko Full Green's function



Marchenko Upgoing Green's function

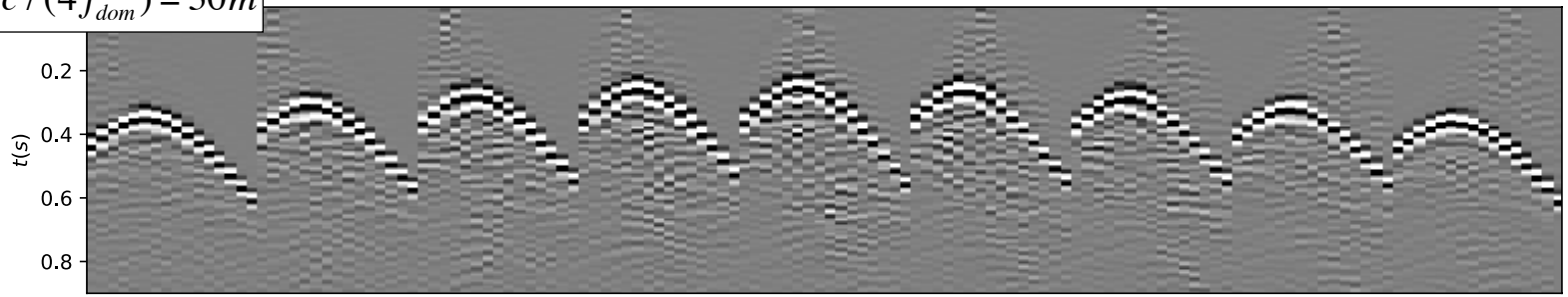


Marchenko example

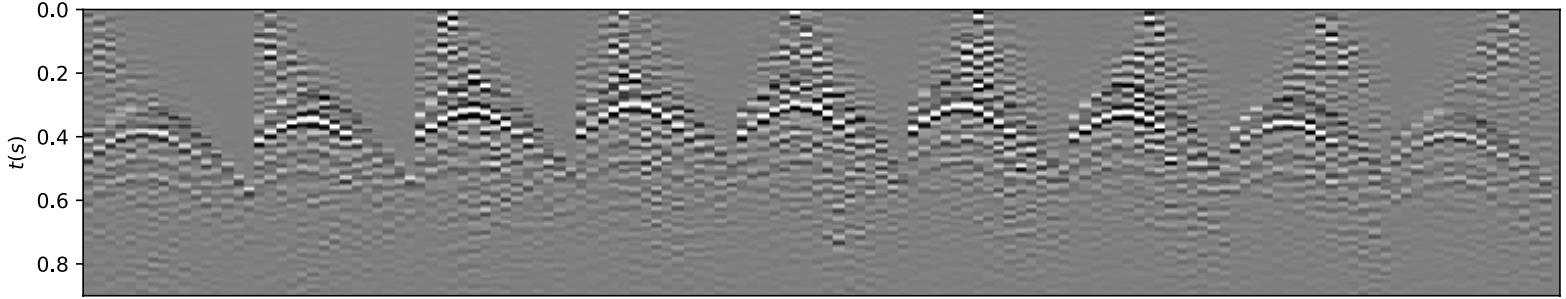
Sub8– 21m x 84m

$$\lambda_{dom} / 4 = c / (4 f_{dom}) = 30m$$

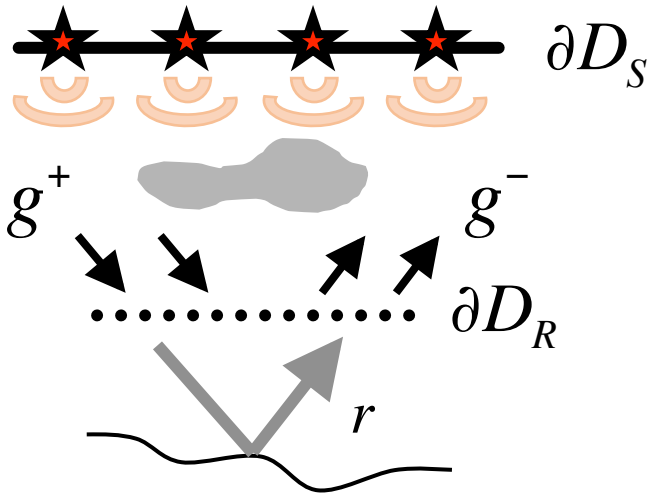
Marchenko Full Green's function



Marchenko Upgoing Green's function



MDD example

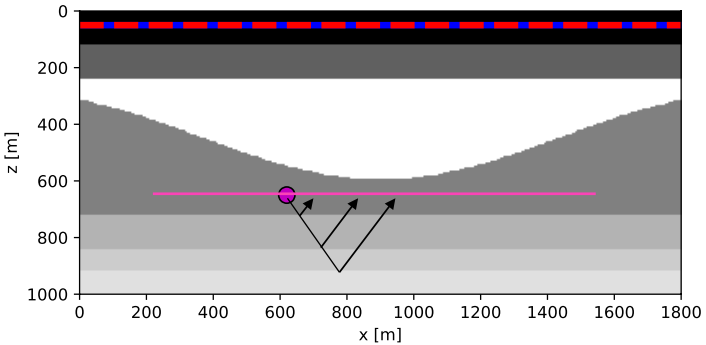


$$\mathbf{g}^- = \mathbf{G}^+\mathbf{r}$$

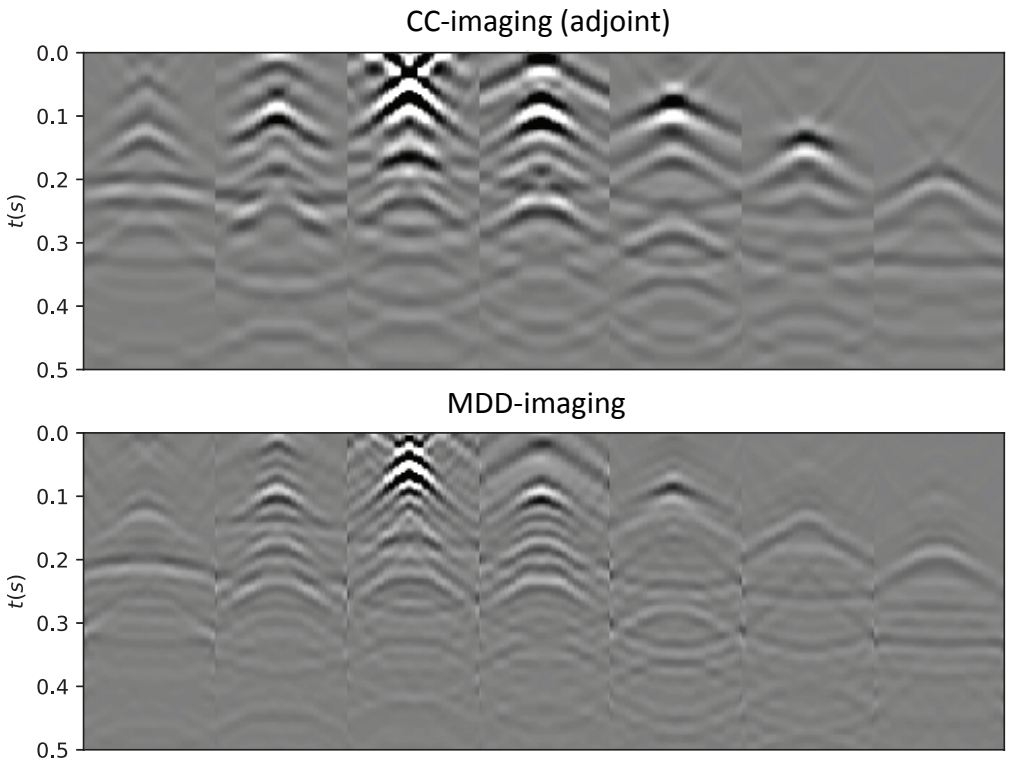
Forward: 1 MDC, Adjoint: 1 MDC
CGLS (10 iterations): 20 MDC

MDD example

Full – 15m x 15m

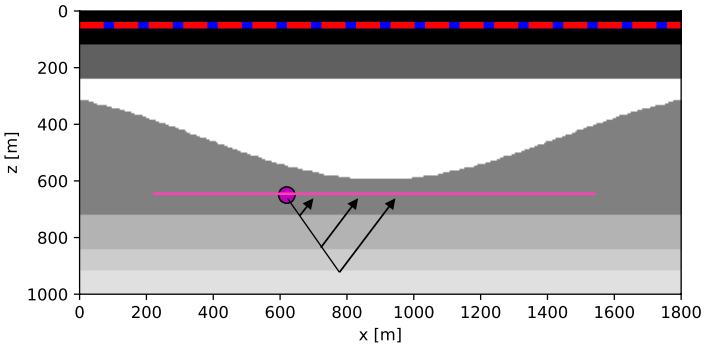


Standard redatuming input fields

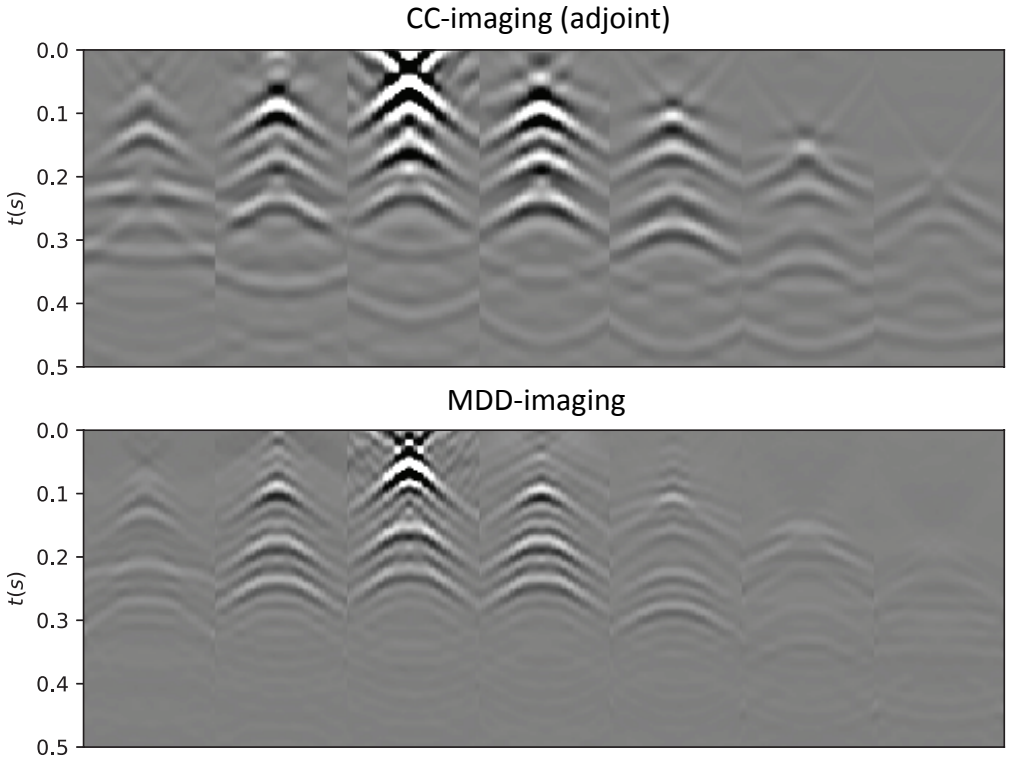


MDD example

Full – 15m x 15m

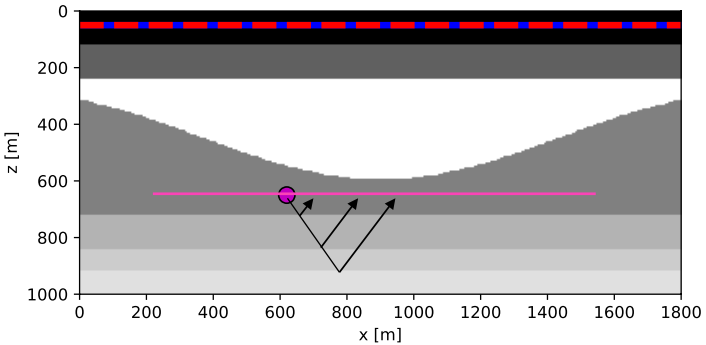


Marchenko redatuming input fields



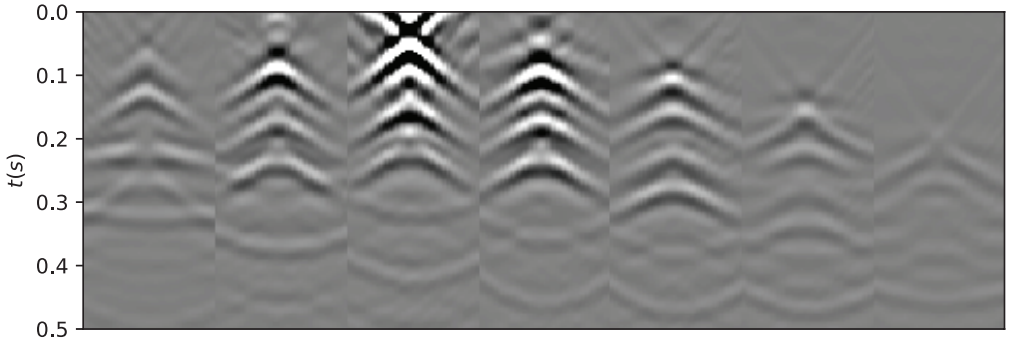
MDD example

Sub4– 21m x 42m

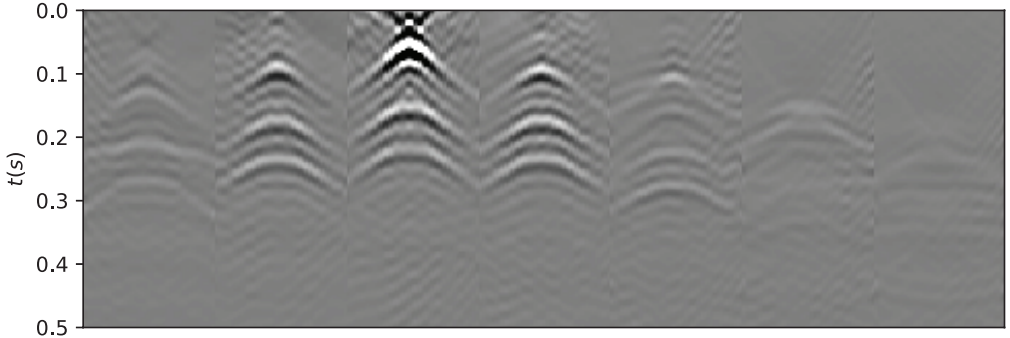


Marchenko redatuming input fields

CC-imaging (adjoint)

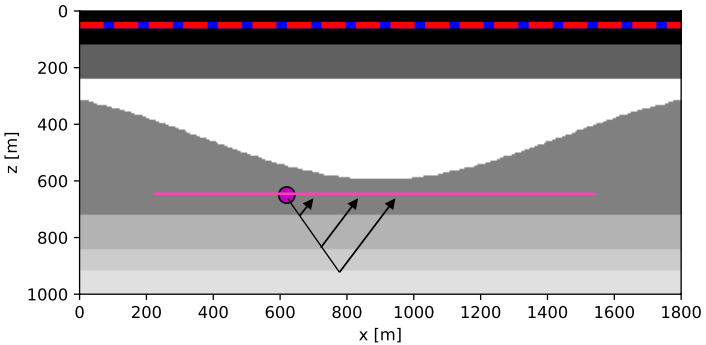


MDD-imaging

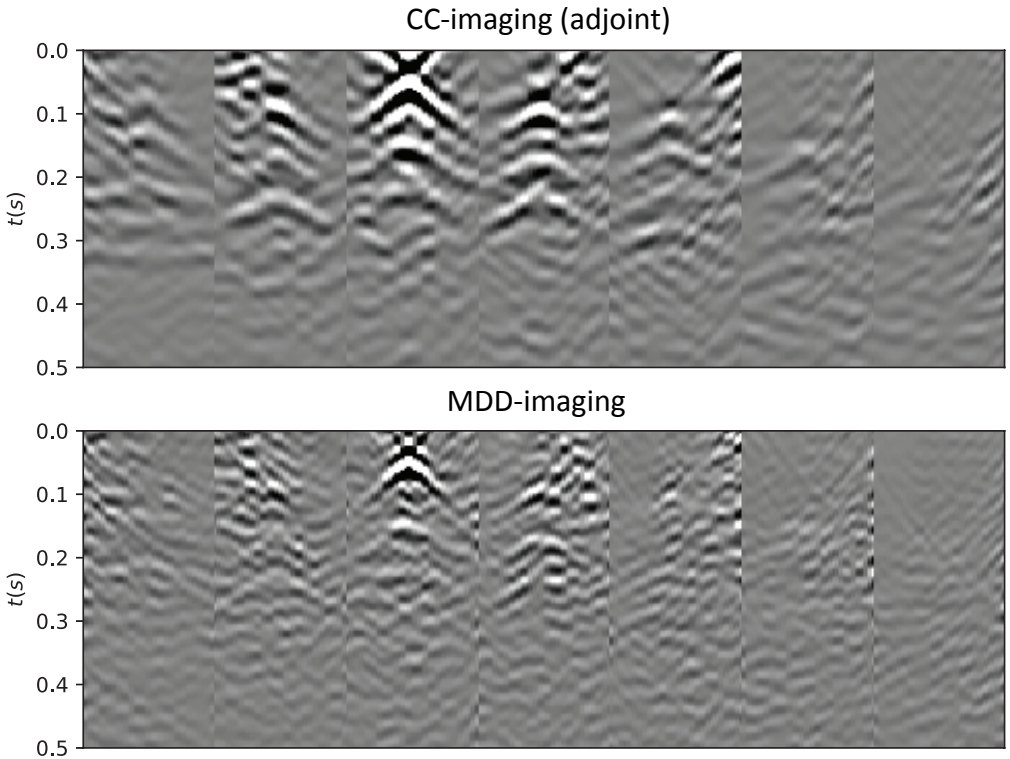


MDD example

Sub8– 21m x 84m

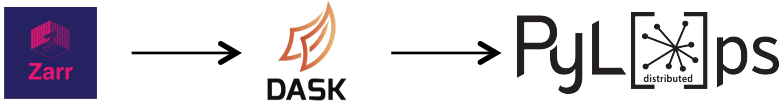


Marchenko redatuming input fields



Conclusions

Multi-dimensional convolution: **theoretical** and **computational** challenges

A **distributed Python framework** 

Sensitivity study on **spatial sampling** shows **robustness** beyond spatial Nyquist at the cost of **strong incoherent noise.**

Future:

- **Multi-node GPUs** and **low rank compression** to speed up computations
- Need for **data interpolation** (upfront or on-the-fly as done in SRME)

Links

https://zarr.readthedocs.io/en/stable/

https://dask.org

https://pylops-distributed.readthedocs.io

Equinor open-source projects: https://github.com/equinor

Reproducible notebooks: https://github.com/mrava87/EAGE_MDCHPC_2020

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