8. Traveltime inversion

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ERSE 210 Seismology

Raytracing vs. traveltime inversion

Forward problem:

Velocity v $\longrightarrow$ Raytracing $\longrightarrow$ Traveltime t

Inverse problem:

Traveltime t $\longrightarrow$ Trav. Inversion or tomography $\longrightarrow$ Velocity v

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Traveltime inversion

2 steps approach:

- Find a simple (possibly 1D parametric, e.g. gradient) velocity field to be used as **background model**

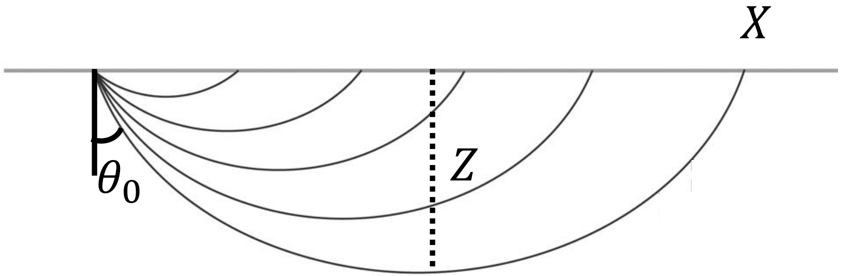
$$t \rightarrow v_0: \; t = f(v_0) \qquad e.g. \; v_0(x,z) = a(x)z + b(x)$$

- Refine the background velocity by inverting the **residual traveltimes**

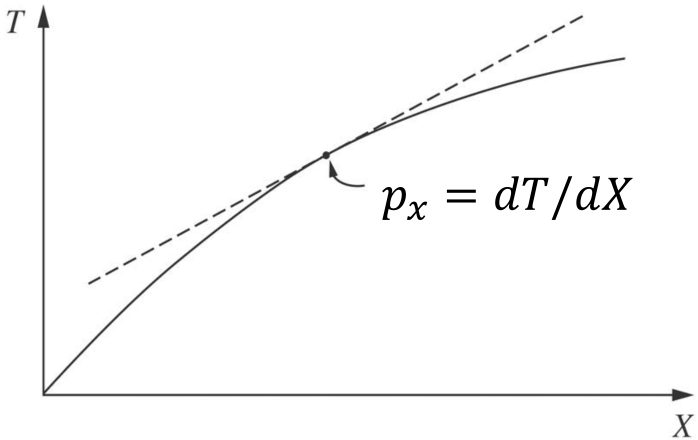
$$\Delta t = t^{obs} - t_0 \rightarrow \Delta v \qquad v = v_0 + \Delta v$$

1D Velocity inversion

Let's start from the traveltime curve of a turning wave



$$p_x = p_{t_p} = \frac{1}{v_{t_p}} = \frac{1}{v(Z)}$$



→ Given a traveltime curve, if we estimate the slope at a given offset ($p_x = dT/dX$), we know the velocity at the turning point (v(Z)). If we start from the top, we can bootstrap a 1D velocity model.

1D Velocity inversion

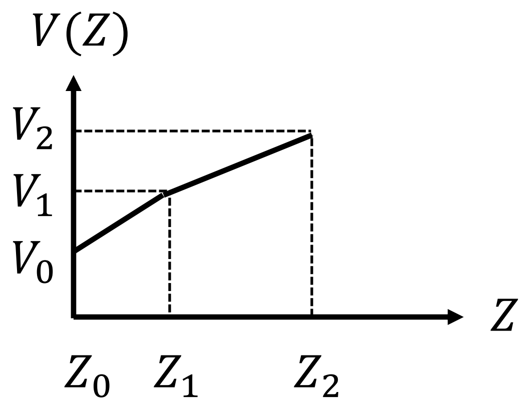
We first compute velocities for a set of offsets where receivers are located (X_0, X_1, X_2, X_n):

$$v_0 = v(Z_0 = 0) = \frac{1}{dT/dX|X_0 = 0}$$

$$v_1 = v(Z_1) = \frac{1}{dT/dX|X_1}$$

...

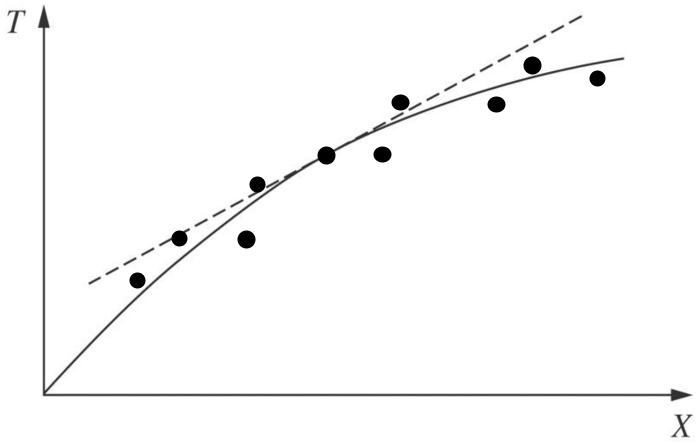
Then given a measure of $T(X_1)$, we find Z_1 such that the profile $V = \{v(Z_0), v(Z_1)\}$ matches the observed time. Repeat for $T(X_2)$ and so on.



Using: $X(p_x) = 2p_x \int_0^{Z_1} \frac{dz}{\sqrt{p^2(z) - p_x^2}}$ $\quad T(p_x) = 2\int_0^{Z_1} \frac{p^2(z)dz}{\sqrt{p^2(z) - p_x^2}}$

1D Velocity inversion

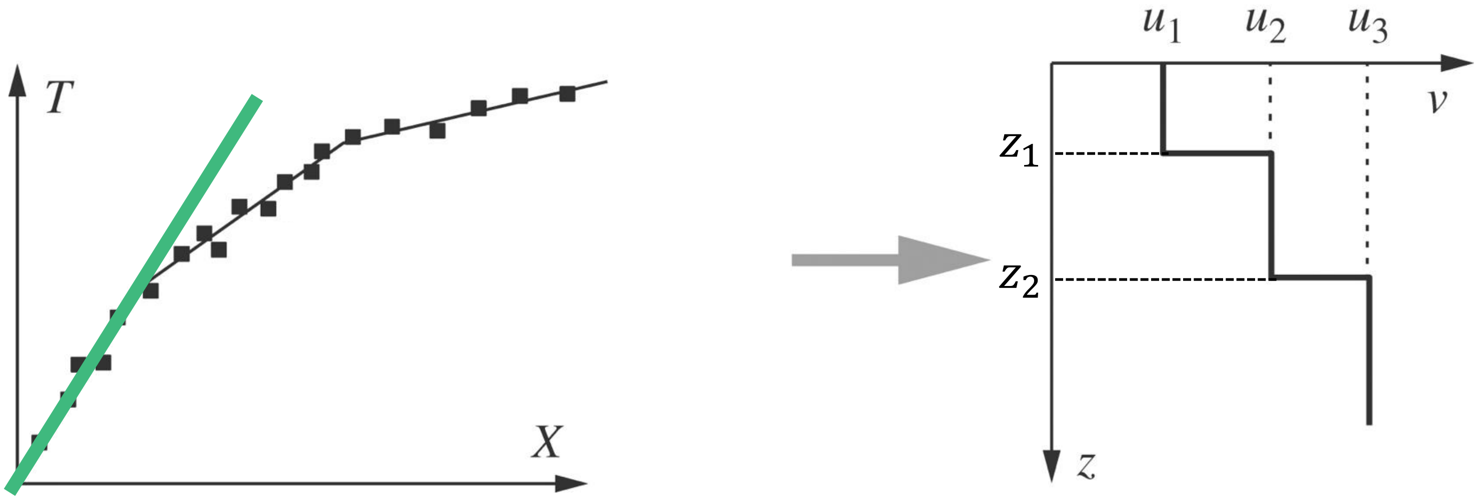
In practice we do not have exact traveltimes



- Noisy measurements

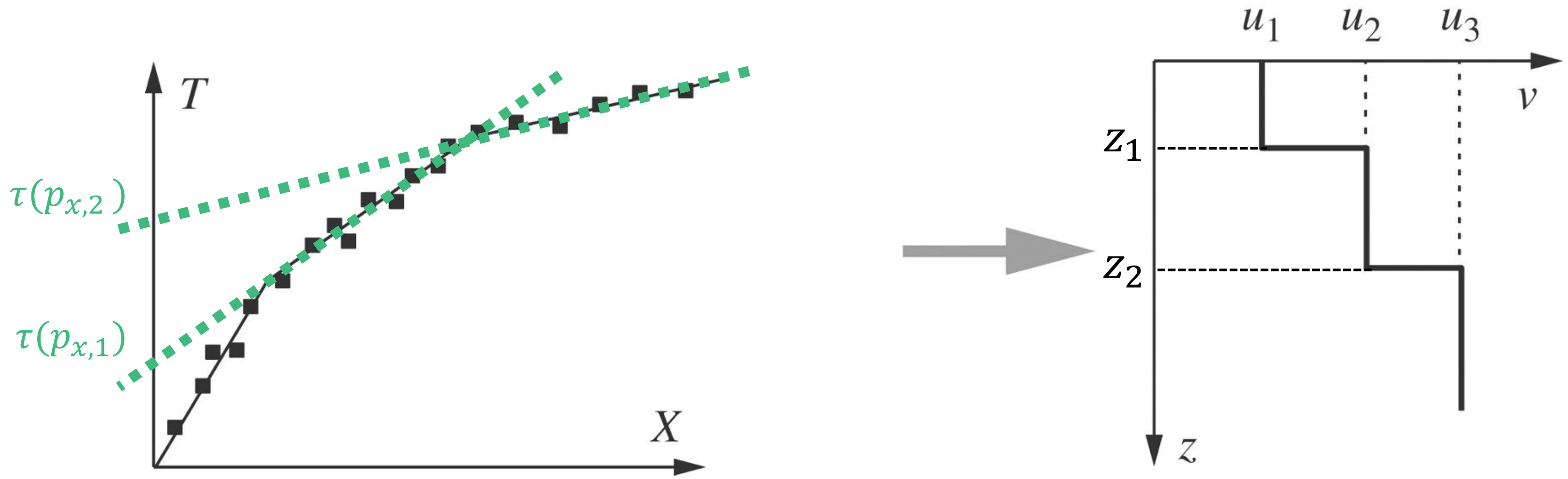
The previous approach can be somehow unstable!

1D Velocity inversion – straight line approach



For the first layer: $t = X/V_0$ → Find slop of the straight line

1D Velocity inversion – straight line approach



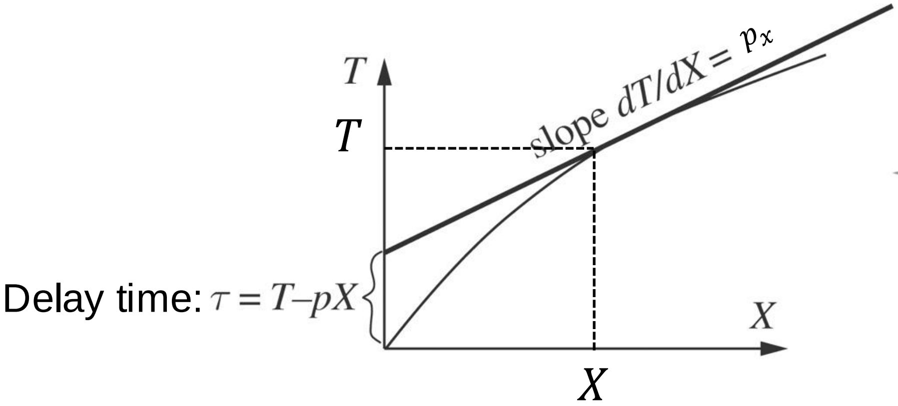
For all other layers, the slope is proportional to $1/V_i$

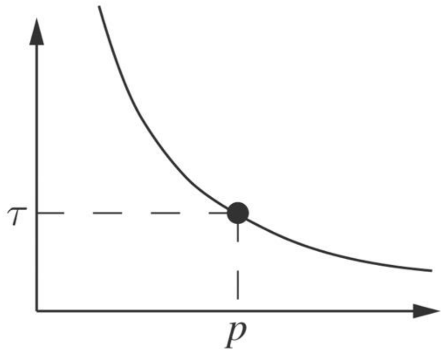
Tau-P function

The curve $T(X)$ is not always well behaved (i.e., caustics). However, $X(p)$ is well behaved in that there is only one value of X for any value of p.

Let's define the $\tau(p_x)$ curve:

$$\boxed{\tau(p_x) = T(p_x) - p_x X(p_X)}$$





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This can be derived from the Taylor expansion of $t(x)$ at $x = X$:

$$t = T + p(x - X) + \ldots$$

$$at\ x = 0\colon\ t(x = 0) = \tau = T - pX$$

Tau-P function

Since we know the expressions for $X(p_x)$ and $T(p_x)$, we can write:

$$\tau(p_x) = 2\int_0^{Z_p}\left[\frac{p^2(z)}{\sqrt{p^2(z)-p_x^2}} - \frac{p_x^2}{\sqrt{p^2(z)-p_x^2}}\right]dz$$
$$= 2\int_0^{Z_p}\sqrt{p^2(z)-p_x^2}\,dz$$

Which in a 'discretized' Earth:

$$\boxed{\tau(p_{x,i}) = 2\sum_j \sqrt{p_j^2 - p_{x,i}^2}\,\Delta z_j\,, \qquad p_j > p_{x,i}}$$

1D Inversion with Tau-P function

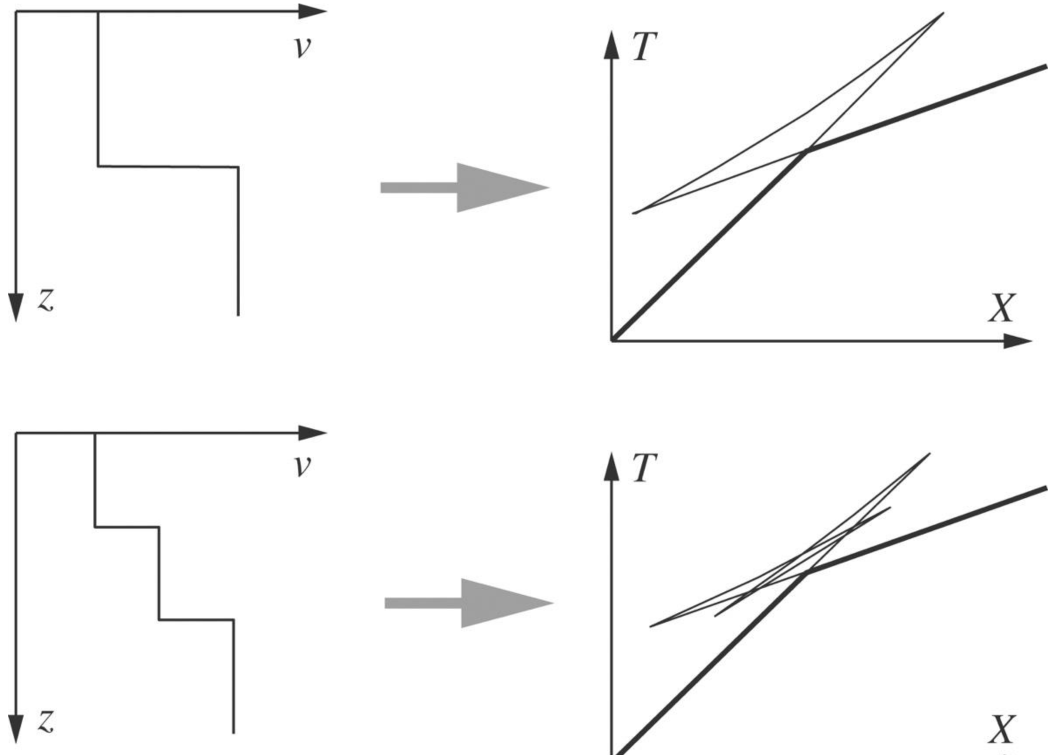
This leads to a simple bootstrapping method:

$$v_0 = \frac{1}{p_{x,0}} \qquad \Delta z_0 = \frac{\tau_1}{2\sqrt{p_0^2 - p_{x,1}^2}}$$

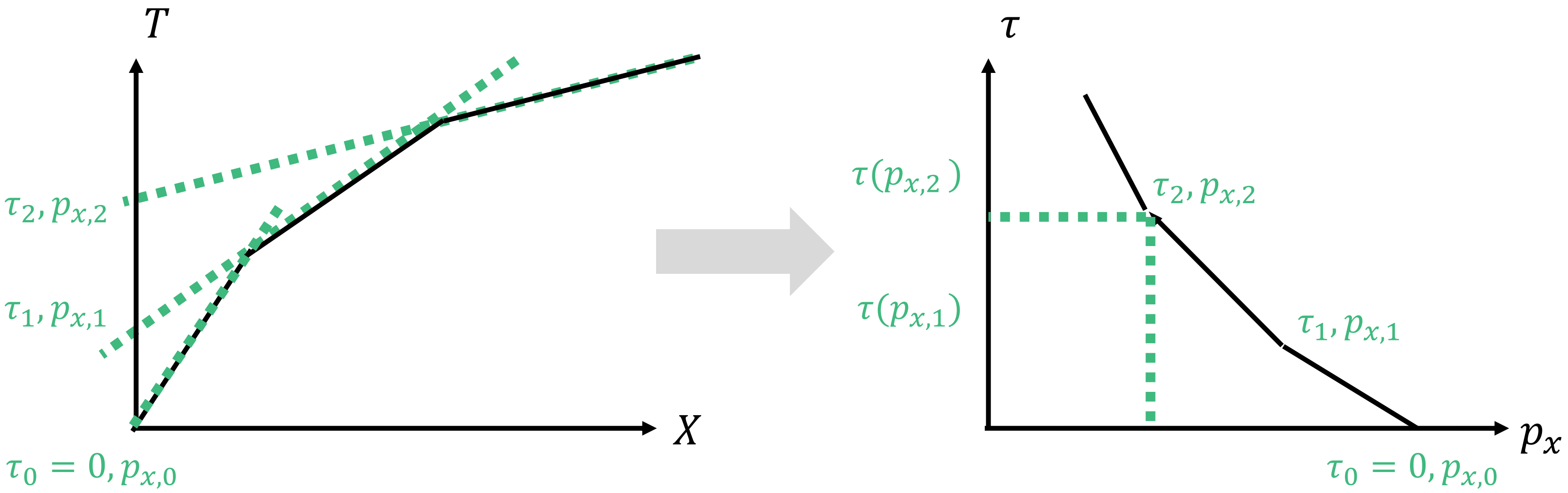
$$v_1 = \frac{1}{p_{x,1}} \qquad \Delta z_1 = \frac{\tau_2 - 2\sqrt{p_0^2 - p_{x,2}^2}\Delta z_0}{2\sqrt{p_1^2 - p_{x,2}^2}}$$

$$v_i = \frac{1}{p_{x,i}} \qquad \Delta z_i = \frac{\tau_{i+1} - 2\sum_{j=1}^{i}\sqrt{p_j^2 - p_{x,i+1}^2}\Delta z_0}{2\sqrt{p_i^2 - p_{x,i+1}^2}}$$

1D Non-uniqueness



Slant stack / Radon transform



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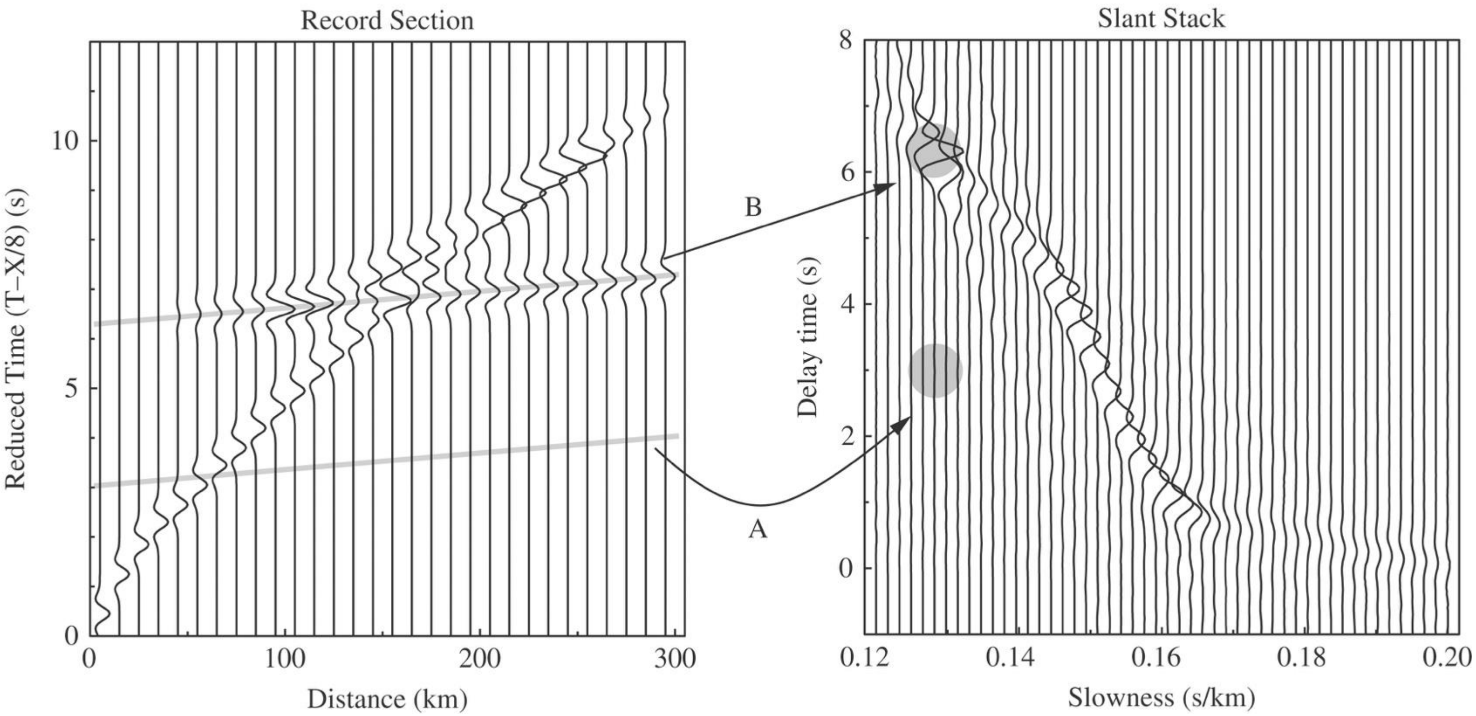
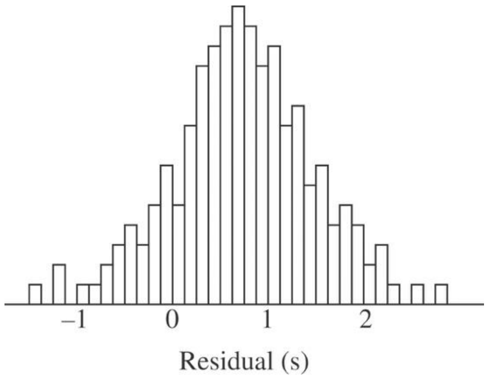


From 1D to 3D inversion

Once a simple 1D velocity model $v(z)$ is available, one can compute

$$\Delta t = t^{obs} - t(v(z))$$



$\Delta t < 0$ slower than correct prediction

$\Delta t > 0$ faster than correct prediction

$\sigma(\Delta t)$ is due to noise in data and should not be overfitted

$\mathrm{E}[\Delta t] \neq 0$ → systematic error, may require lateral changes to explain it

Δt as input for a residual 3D inversion (aka **seismic tomography**).

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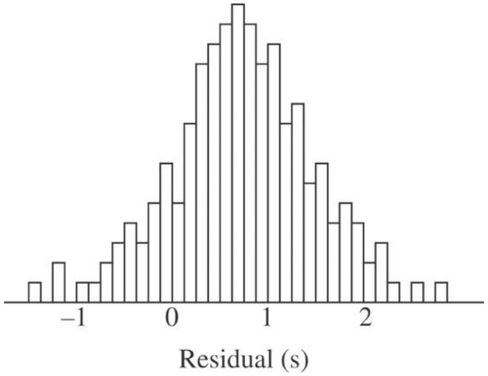

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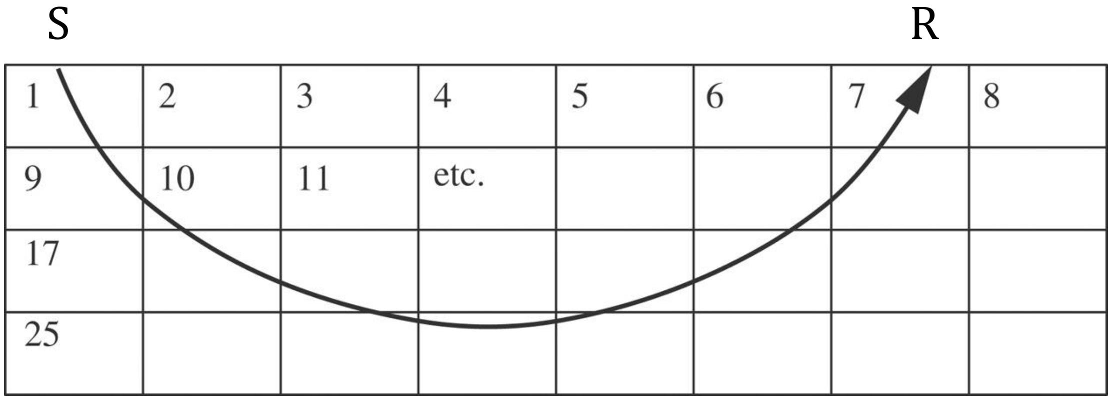
Seismic tomography

For each source-receiver pair:



$$t_{SR} = \int_{l_{SR}} s \, dl$$

Seismic tomography

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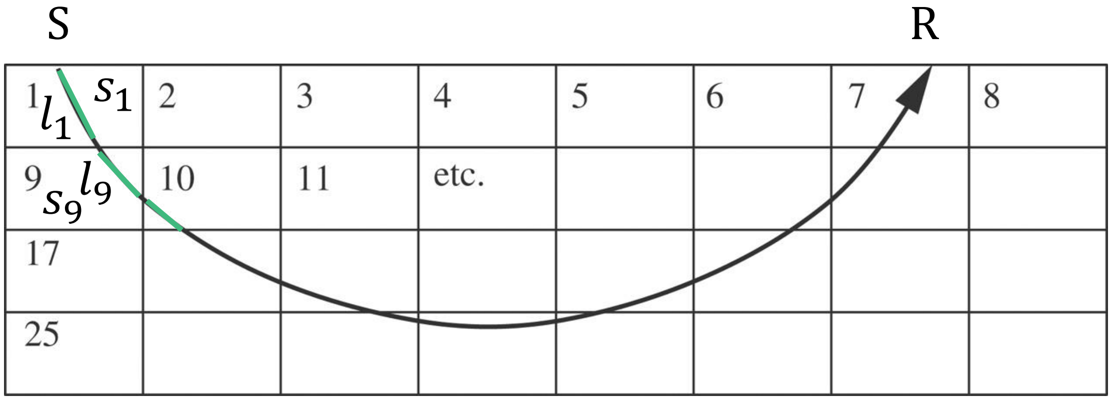


$$\mathrm{t_{SR}} = \int_{l_{SR}} s\, dl \rightarrow \mathrm{t_{SR}} = \sum_i s_i l_i$$

Seismic tomography

In practice, the first problem is to find a ray that goes from S to R:

- **Ray shooting:** shoot a fan of rays from the source with different take-off angles and choose the one that reaches the surface closer to R

- **Ray bending**: S and R are fixed and the ray is moved to fit Fermat's principle

Seismic tomography

The second problem is to discretize the rays (as shown in previous slides) and set up a linear system of eqs:

$$\begin{bmatrix} t_{S_1R_1} \\ t_{S_1R_2} \\ \cdots \\ t_{S_1R_{N_R}} \\ t_{S_2R_1} \\ \cdots \\ t_{S_{N_S}R_{N_R}} \end{bmatrix} = \begin{bmatrix} l_{1-S_1R_1} & l_{2-S_1R_1} & \cdots & \cdots & l_{N_xN_z-S_1R_1} \\ l_{1-S_1R_1} & l_{2-S_1R_2} & \cdots & \cdots & l_{N_xN_z-S_1R_2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ l_{1-S_{N_S}R_{N_R}} & l_{2-S_{N_S}R_{N_R}} & \cdots & \cdots & l_{N_xN_z-S_{N_S}R_{N_R}} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ s_{N_xN_z} \end{bmatrix}$$

$$\downarrow$$

$$\mathbf{t} = \mathbf{L}\mathbf{s}$$

After removing the traveltime of the background model

$$\downarrow$$

$$\Delta\mathbf{t} = \mathbf{L}\Delta\mathbf{s}$$

Note: $\mathbf{L}$ is usually a very sparse matrix

Seismic tomography

The tomographic problem is usually over-determined ($N_S N_R > N_x N_z$)

$$\widehat{\boldsymbol{\Delta s}} = \left(\mathbf{L}^{\mathbf{T}}\mathbf{L}\right)^{-\mathbf{1}} \ \mathbf{L}^{\mathbf{T}}\boldsymbol{\Delta}\mathbf{t}$$

However, $\mathbf{L}^{\mathbf{T}}\mathbf{L}$ is hard to invert since:

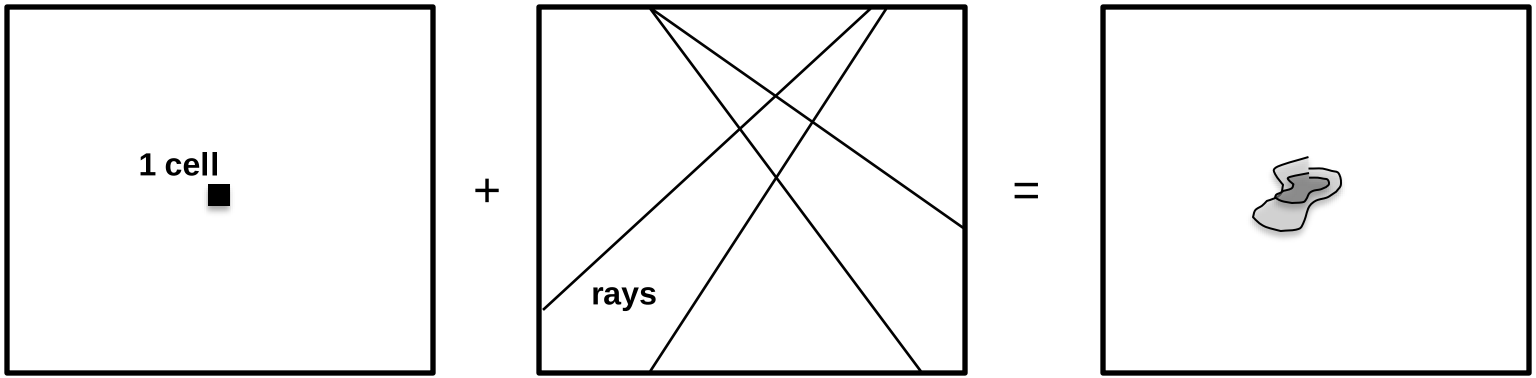
- multiple rays can pass through a single cell and give contrasting info (for noisy data)
- some cells may be undersampled (or not sampled at all)

Solution: add prior information (aka regularization)

Seismic tomography

An initial assessment of the ‘expected’ quality of the inversion (purely based on the acquisition geometry and background velocity model):

<u>Impulse response</u>



Seismic tomography

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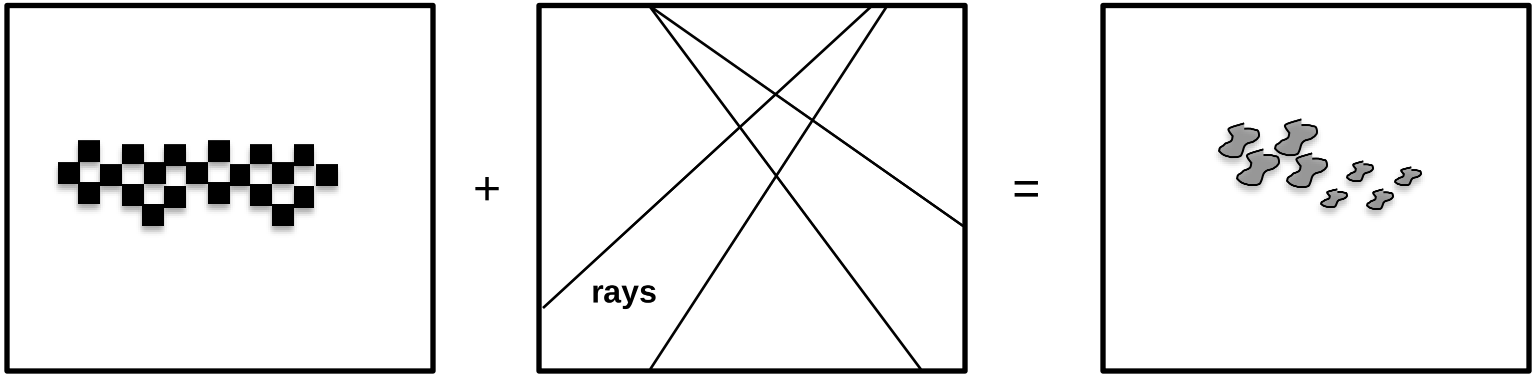
Checkerboard test



Finite-frequency tomography