14. Surface waves

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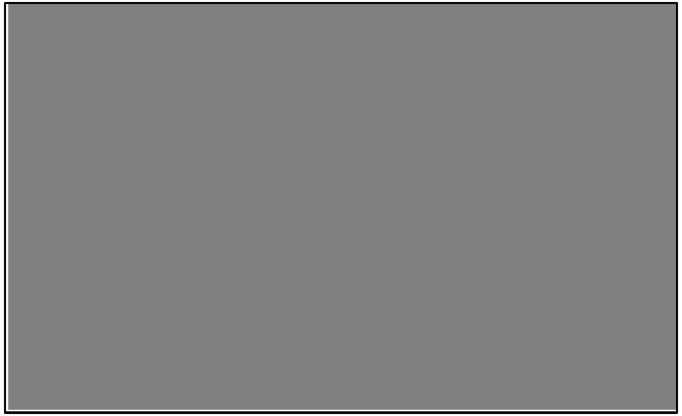
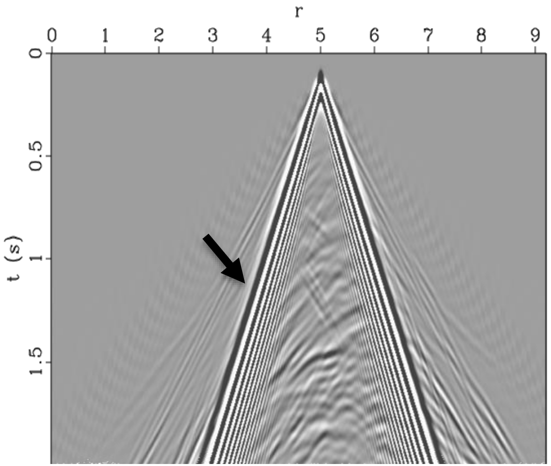
ERSE 210 Seismology

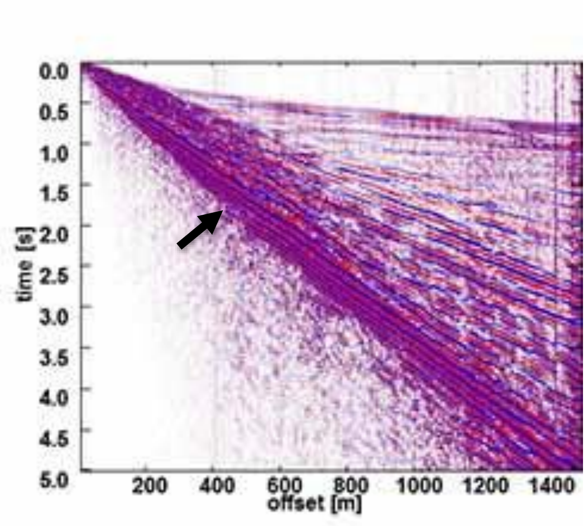
Surface waves

Synthetic

Field







Surface waves

Additional waves (to body waves) generated in the presence of **free-surface** (i.e., air-solid interface → vanishing vertical traction $\mathbf{T}_z = (\tau_{xz}, \tau_{yz}, \tau_{zz}) = 0$)

- **Love waves:** transversely polarized (requires velocity gradient) – like SH-waves, measured by Transverse component
- **Rayleigh waves:** radially polarized (exists always) – like P-SV-waves, measured by Vertical and Radial component
- **Stoneley and Sholte waves:** special type of surface waves at solid-solid interface and solid-fluid interface

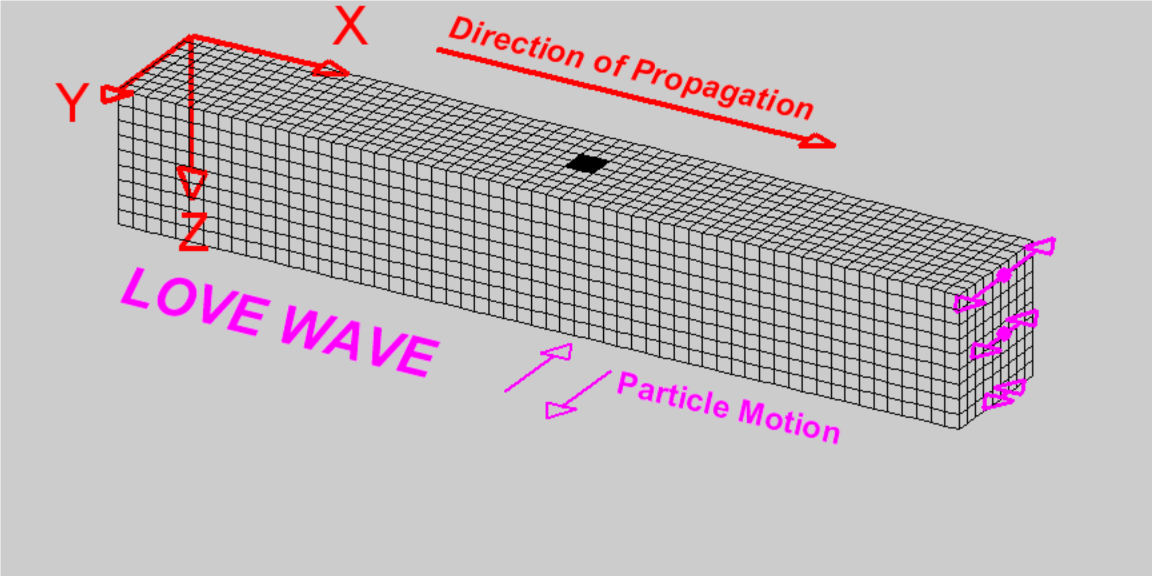
Surface waves

Additional waves (to body waves) generated in the presence of **free-surface** (i.e., air-solid interface → vanishing vertical traction $\mathbf{T}_z = (\tau_{xz}, \tau_{yz}, \tau_{zz}) = 0$)

Differences with body waves

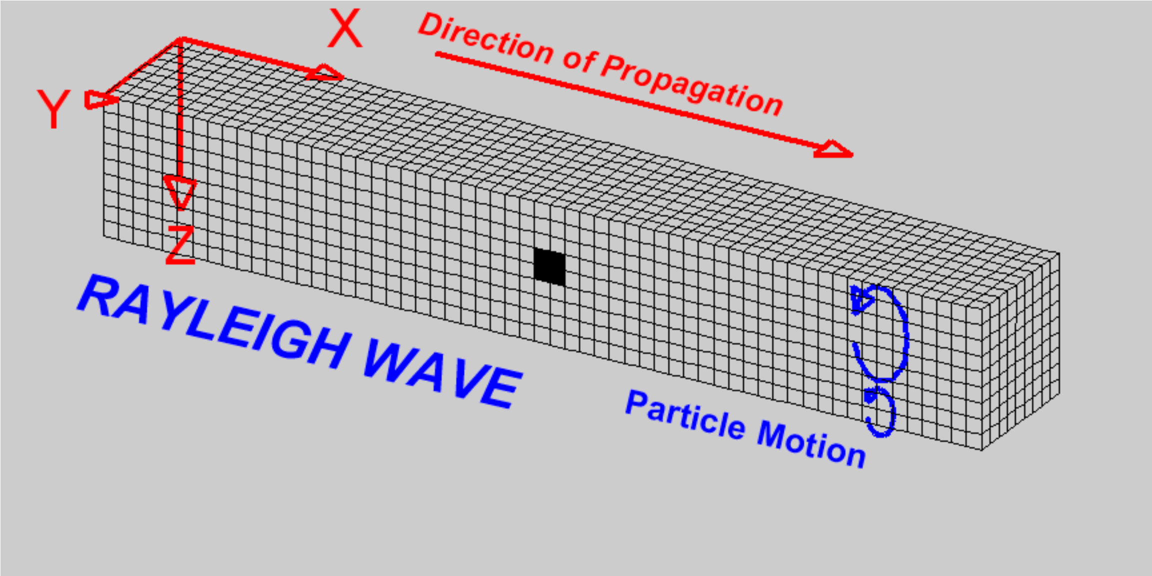
- Strongest arrivals in seismic recordings as they travel along the surface (instead of in a volume)
- Travel slower than body waves
- Strong frequency dependance (dispersive behavior)

Love Waves



Source: https://web.ics.purdue.edu/~braile/edumod/waves/Lwave.htm

Rayleigh Waves

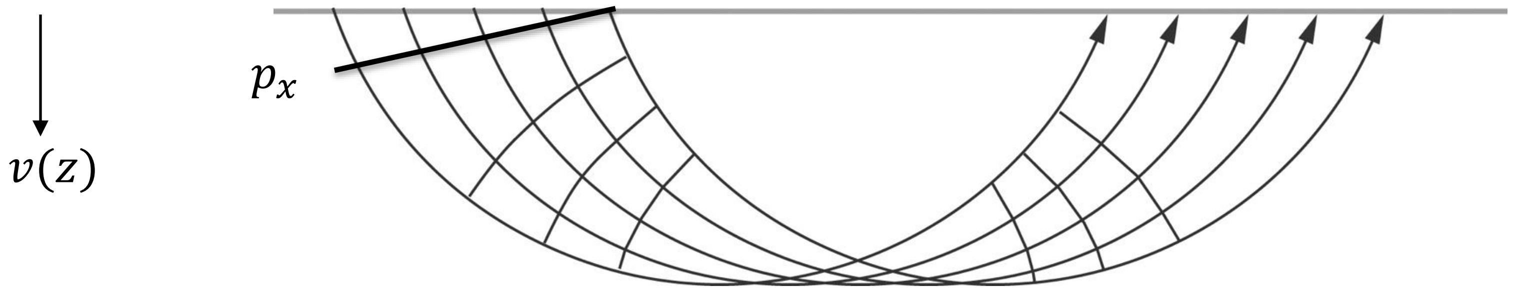


Source: https://web.ics.purdue.edu/~braile/edumod/waves/Rwave.htm

Turning waves

Before we can understand how Love waves originate, we need to introduce a fundamental concept, **phase advance at turning point**

Plane wave with velocity gradient



Turning point

The wavefront start as plane and as the wave propagate it bends (to remain perpendicular to the rays). However, locally the fronts are still plane.

Turning waves

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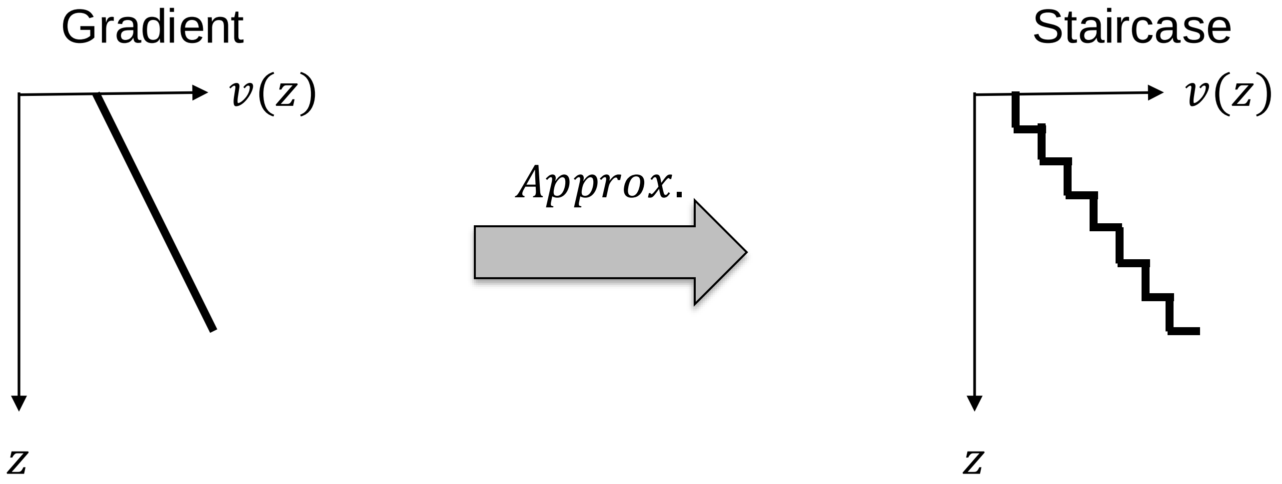
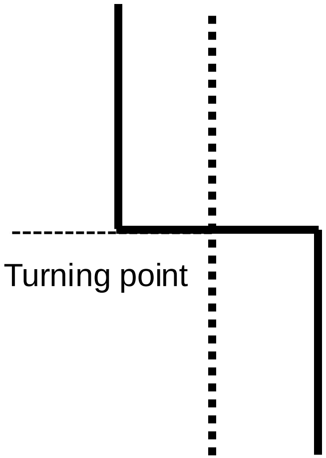


Let's use the theory developed to propagate a plane wave across and interface to compute the overall amplitude at the turning point

Turning waves

Before we can understand how Love waves originate, we need to introduce a fundamental concept, **phase advance at turning point**



$$\beta^- = \beta - \delta\beta$$

$$\rho^- = \rho - \delta\rho$$

$$\beta^+ = \beta + \delta\beta$$

$$\rho^+ = \rho + \delta\rho$$

$$\grave{S}\acute{S} = \frac{\rho^-\beta^- cos\theta_1 - \rho^+\beta^+ cos\theta_2}{\rho^-\beta^- cos\theta_1 + \rho^+\beta^+ cos\theta_2}$$

$$p_x = \frac{1}{\beta_{turning}} = \frac{1}{\beta}$$

Turning waves

Before we can understand how Love waves originate, we need to introduce a fundamental concept, **phase advance at turning point**

$$cos\theta_i = \sqrt{1 - sin\theta_i^2} = \sqrt{1 - p_x^2\beta_i^2} = \sqrt{1 - \frac{1}{\beta^2}(\beta - \delta\beta)^2} = \sqrt{1 - \frac{1}{\beta^2}(\beta^2 - 2\beta\delta\beta + \cancel{\delta\beta^2})} = \sqrt{\frac{2}{\beta}\delta\beta}$$

$$= \sqrt{1 - \frac{1}{\beta^2}(\beta + \delta\beta)^2} = \sqrt{1 - \frac{1}{\beta^2}(\beta^2 + 2\beta\delta\beta + \cancel{\delta\beta^2})} = \sqrt{-\frac{2}{\beta}\delta\beta}$$

$$\grave{\boldsymbol{S}}\acute{\boldsymbol{S}} = \frac{\rho^-\beta^-\sqrt{\frac{2}{\beta}\delta\beta} - \rho^+\beta^+\sqrt{-\frac{2}{\beta}\delta\beta}}{\rho^-\beta^-\sqrt{\frac{2}{\beta}\delta\beta} + \rho^+\beta^+\sqrt{-\frac{2}{\beta}\delta\beta}} \approx \frac{\rho\beta\sqrt{\frac{2}{\beta}\delta\beta} - \rho\beta\sqrt{-\frac{2}{\beta}\delta\beta}}{\rho\beta\sqrt{\frac{2}{\beta}\delta\beta} + \rho\beta\sqrt{-\frac{2}{\beta}\delta\beta}} = \frac{1-j}{1+j} = \boldsymbol{-j} = \boldsymbol{e^{-j\pi/2}}$$

Turning waves

Before we can understand how Love waves originate, we need to introduce a fundamental concept, **phase advance at turning point**

$$\cos\theta_i = \sqrt{1-\sin\theta_i^2} = \sqrt{1-p_x^2\beta_i^2} = \sqrt{1-\frac{1}{\beta^2}(\beta-\delta\beta)^2} = \sqrt{1-\frac{1}{\beta^2}(\beta^2-2\beta\delta\beta+\cancel{\delta\beta^2})} = \sqrt{\frac{2}{\beta}\delta\beta}$$

Turning point = 90deg phase advance of wave = Hilbert transform of downgoing is equal to upgoing, $u(t) = \varkappa(d(t))$

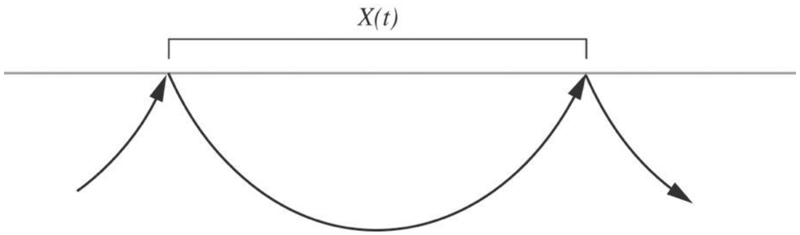
→ high-freq approximation valid away from turning point

$$\grave{S}\acute{S} = \frac{\rho^-\beta^-\sqrt{\frac{2}{\beta}\delta\beta} - \rho^+\beta^+\sqrt{-\frac{2}{\beta}\delta\beta}}{\rho^-\beta^-\sqrt{\frac{2}{\beta}\delta\beta} + \rho^+\beta^+\sqrt{-\frac{2}{\beta}\delta\beta}} \approx \frac{\rho\beta\sqrt{\frac{2}{\beta}\delta\beta} - \rho\beta\sqrt{-\frac{2}{\beta}\delta\beta}}{\rho\beta\sqrt{\frac{2}{\beta}\delta\beta} + \rho\beta\sqrt{-\frac{2}{\beta}\delta\beta}} = \frac{1-j}{1+j} = \boldsymbol{-j = e^{-j\pi/2}}$$

Love waves

Surface waves created by interference of high-order SH-waves (SS, SSS, SSSS, …) and free-surface (r=-1)

→ We model Love waves as superposition of SH-body waves. Given vertical velocity gradient and a monochromatic plane wave with $p_x = \frac{1}{\beta}$



Traveltime along surface: $T_{surface}(p_x) = p_x X(p_x)$

Traveltime along path (as computed in lecture 7)

Phase delay between two arrivals:

$$\omega T_{surface}(p_x) = \omega p_x X(p_x)$$

$\omega T(p_x) - \pi/2$ ← Due to turning point

Love waves

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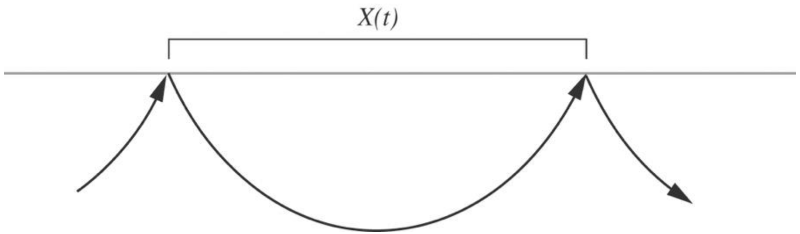


Constructive interference at:

$$\omega p_x X(p_x) = \omega T(p_x) - \pi/2 - 2\pi n$$

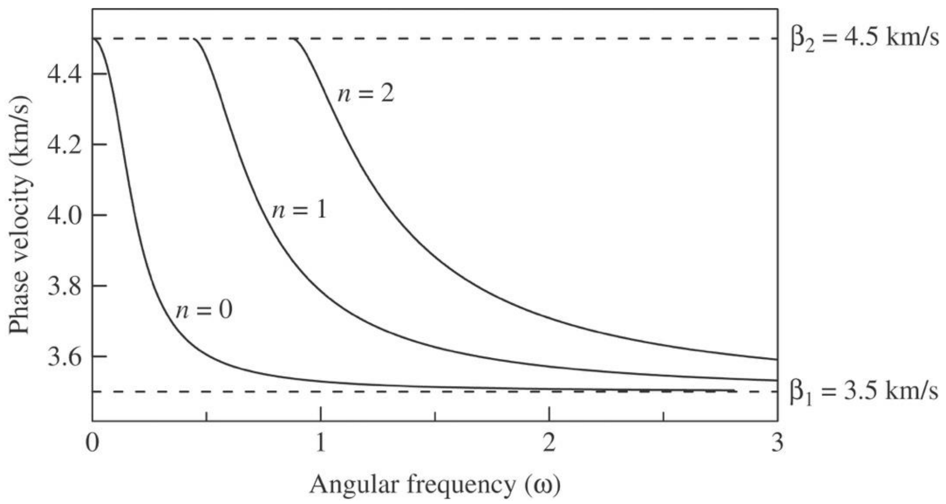
$$\boxed{\omega = \frac{2\pi n + \frac{\pi}{2}}{T(p_x) - p_x X(p_x)}}$$

Dispersion curves

Given a certain $v(z)$ and p_x we can find a series of frequencies ω where there is constructive interference.

n=0 Fundamental mode

n > 0 Higher modes



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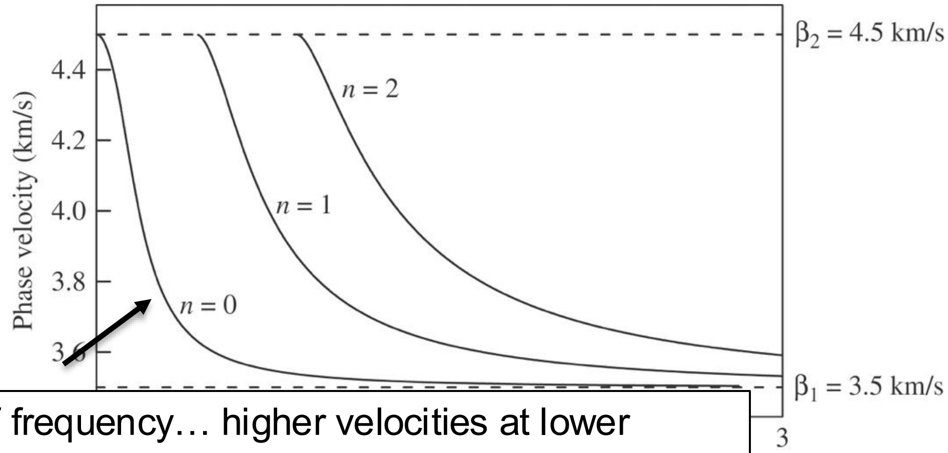

Dispersion curve: velocity as function of frequency… higher velocities at lower frequencies as they travel deeper, lower velocities at higher frequencies as they travel shallower. This dispersion is due to WAVE INTERFERENCE and has nothing to do with the medium itself being dispersive.

Surface waves in global seismology