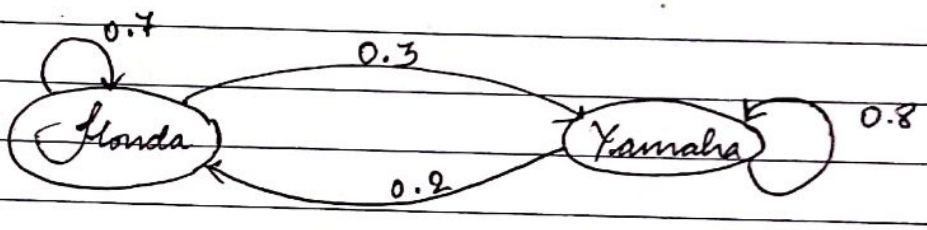
Simulation & Modelling

2073/BCT/17

(ASSIGNMENT I)

6. Given that chance of a Honda bike user to buy Honda at next purchase is 70% and his next purchase will be Yamaha is 30%. The chance of Yamaha bike user to buy Yamaha bike at next purchase is 80% & that his next purchase will be Honda is 20%. What is the probability to buy Yamaha bike after three purchase of a current Honda Bike user?

Soln:



Graph

Transition matrix

	Honda	Yamaha
Honda	0.7	0.3
Yamaha	0.2	0.8

Using Current status distribution matrix,

Current status distribution matrix, Q_0

	Honda	Yamaha
Q_0 =	1	0

Probability distribution matrix / Transition matrix ; P

$$P = \begin{array}{c} \\ \text{Honda} \\ \text{Yamaha} \end{array} \begin{array}{c} \begin{array}{cc} \text{Honda} & \text{Yamaha} \end{array} \\ \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

Since question is asking for three purchases from now,

$$Q_n = Q_0 * P^n$$

$$\boxed{Q_3 = Q_0 * P^3}$$

$$= [1 \quad 0] \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}^3$$

$$= [1 \quad 0] \begin{bmatrix} 0.475 & 0.525 \\ 0.35 & 0.65 \end{bmatrix}$$

$$= \begin{array}{c} [0.475 \quad 0.525] \\ \text{Honda} \quad \text{Yamaha} \end{array}$$

∴ The required probability is 0.525.

8. K-S test is to be performed to test the uniformity of following random numbers with a level of significance of $\alpha = 0.05$.

0.29, 0.75, 0.20, 0.92, 0.15, 0.38, 0.67, 0.88

Solⁿ:

Let H_0 represent the null hypothesis where,

H_0 : the generated numbers are uniform.

Arranging given data in ascending order

i	1	2	3	4	5	6	7
R(i)	0.15	0.20	0.29	0.38	0.75	0.88	0.92

i	R(i)	$\frac{i}{N}$	$\frac{(i-1)}{N}$	$\frac{i}{N} - R(i)$	$R(i) - \frac{(i-1)}{N}$
1	0.15	0.1428	0	-0.00714	0.15
2	0.20	0.2857	0.1428	0.0857	0.05714
3	0.29	0.4285	0.2857	0.13857	0.004285
4	0.38	0.5714	0.4285	0.191428	-0.04857
5	0.75	0.71428	0.5714	-0.0357	0.17857
6	0.88	0.857	0.71428	-0.0228	0.1657
7	0.92	1	0.857	0.06	0.0628

$$D^+ = \max\left\{\frac{i}{N} - R(i)\right\}$$

$$= 0.191428$$

$$D^- = \max\left\{R(i) - \frac{(i-1)}{N}\right\}$$

$$= 0.17857$$

$$D = \max\{D^+, D^-\}$$

$$= 0.191428$$

Critical value of D at $\alpha = 0.05$ & $N = 7$

$$D_\alpha = 0.48343$$

$$\therefore D < D_\alpha$$

Hence, the null hypothesis is accepted i.e. random numbers given are uniform.

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9. In a bank, Custom arrival time is given by Poisson distribution. On the average, a new customer arrives every 420 sec and gets served with an average service time of 300 sec. Find:

a. System Utilization
b. Probability of Zero Customer
c. Probability of 1 customer
d. Probability of 4 or more Customers
e. Average Waiting time
f. Average Number of customers in system
g. Average time customers spends in the system.

Soln:

Average arrival time = 420 sec

Average service time = 300 sec

Avg. arrival rate $(\lambda) = \frac{1}{T} = \frac{1}{420 \text{ sec}} = 0.00238$ /sec

Avg. ~~arrival~~ service rate $(\mu) = \frac{1}{T} = \frac{1}{300} = 0.0033$ /sec

a. System utilization

$$S = \frac{\lambda}{\mu} = \frac{0.00238}{0.0033} = 0.714 \approx \frac{5}{7}$$

Probability of Zero Customer

$$P_0 = 1 - S$$
$$= 1 - 5/7$$
$$= 2/7 = 0.2857$$

c. Probability of 1 customer

$$P_n = s^n P_0$$

$$P_1 = s^1 P_0$$

$$= \left(\frac{5}{7}\right) \times \frac{2}{7}$$

$$= \frac{10}{49}$$

$$= 0.20408$$

d. Probability of 4 or more customer.

$$P_{(n \geq 4)} = 1 - (P_0 + P_1 + P_2 + P_3)$$

$$= 1 - \left(\frac{2}{7} + \frac{10}{49} + \left(\frac{5}{7}\right)^2 \times \frac{2}{7} + \left(\frac{5}{7}\right)^3 \times \frac{2}{7}\right)$$

$$= 1 - \left(\frac{2}{7} + \frac{10}{49} + \frac{50}{343} + \frac{250}{2401}\right)$$

$$= 0.2603$$

e Average waiting time

$$WT = \frac{s}{\mu - \lambda} = \frac{5/7}{0.0033 - 0.00238} = 750 \text{ sec}$$

f. Average no. of customers

$$N = \frac{s}{1-s} = \frac{5/7}{1 - 5/7}$$

$$= 5/2 = 2.5 \approx 3$$

g. Average time customer spends

$$T = \frac{N}{\lambda} = \frac{2.5}{0.00238} = 1050 \text{ sec}$$

1. Differentiate between static physical and dynamic physical model with examples.

Static physical model	Dynamic physical model
> This model describes the relationships / behaviours that do not change with respect to time.	This model describes the time-varying relationships of the object properties.
> It only depicts the object's characteristics at any instance of time, considering that the object's property will not change over time.	It describes the characteristics of the object that changes over time.
> Static model is a scaled down model of a system which does not change with time.	It relies upon the analogy bet^n the system being studied and some other system of a different nature, but have similarity on forces that directs the behaviour of the both systems.
> Example : An architectural model of a house, scale models (wind tunnels, water tanks) etc.	Example : A model of wind tunnel, a model of automobile suspension.

2. Differentiate between static mathematical and dynamic mathematical model with example.

Static mathematical model	Dynamic mathematical model
> It gives relationship bet^n the system attributes when system is in equilibrium.	It allows the changes of system attributes to be derived as function of time.
> It represents the logical view of the system in equilibrium state.	It accounts for the time dependent changes in the logic state of the system.
> Such models are time-invariant.	Such models are time variant.
> It is generally represented by the basic algebric equations	It is generally represented by differential equation or difference equations.
> Example: An eq^n relating the length & weight on each side of a playground variation, supply and demand relationship model of a market and so on.	Eg: the equation of motion of planets around the sun in the solar system.

3. Discuss the advantages and disadvantages of analog computers. Differentiate between analog and digital computer.

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ANALOG COMPUTERS :

a) Advantages :

> Parallel (Real-time) operation - many signal values can be computed simultaneously.

> computation can be done for some applications without the requirement for transducers to convert the inputs/outputs to/from digital electronic form.

> Setup requires the programmer scale the problem for the dynamic range of the computer. This can give insight to the problem and the effects of various errors.

b) Disadvantages.

> Computation elements have a limited useful dynamic range, usually not much more than 120 dB, about 6 significant digits of accuracy.

> Useful solution of problem any size can take an inordinate amount of setup time.

> For a given size and power consumption, digital computers can solve larger problems.

> Solution appear in real time, and may be difficult to record for later use or analysis.

Analog computer	Digital computer
The analog computer works on a continuous signal. The output is a voltage signal they are not exactly values and are in graphical form.	The digital computer works on a discrete signal. The outputs are in numbers, exact values are seen on displays.

> Analog computer uses a network of resistors & capacitors.	It uses large number of logic gates, microprocessors and on/off switches.
> The analog computer measure the analog quantities like voltage, temperature, etc.	The digital computers calculate mathematical operations, complex calculations, media streaming etc.
> The data storing in analog computers are quite difficult as they use continuous signals which are difficult to store.	Storing data in digital computers are quite easy as they just store either 0 or 1 which can be easily stored.

5. Write short notes on Monte - Carlo simulation.

→ A Monte - Carlo simulation is a model used to predict the probability of different outcomes when the invention of random variables is present. Monte - Carlo simulations help to explain the impact of risk and uncertainty in prediction and forecasting models. A variety of fields utilize Monte Carlo simulations, including finance engineering, supply chain and science. The basic of a Monte - Carlo simulation involves assigning multiple values to an uncertain variable to achieve multiple results and then to average the results to obtain an estimate. Monte Carlo simulations assume perfectly efficient markets.

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For instance Monte-Carlo simulation can be used to compute the value at risk of a portfolio. It tries to predict the worst return expected from a portfolio given a certain confidence interval for a specified time period.

7. Explain M/M/4/20/2000/FCFS and D/M/2/LIFO/18

i) M/M/4/20/2000/FCFS

- This is a Kendall Notation / Queuing Notation for a Queuing System.
- Here the notation indicates that the system has 4 parallel servers with exponential interarrival time distribution and exponential service time distribution.
- It has a maximum queue size of 20 customers & the queue follows FCFS (FIFO) discipline. The total population size that are being served in the system is 2000.

ii) D/M/2/LIFO/18

- This Kendall notation indicates the system has 2 parallel servers with deterministic interarrival time distribution and exponential service time distribution.
- It has a maximum queue size of 18 customers, ~~28 lots~~ & the queue follows LIFO discipline. Unlimited customer population can be served.

4. Write atleast four significance of differential equations with examples.

→ Significance

$y' = f(x, y)$ (unknown function)

- Most physical and chemical process occurring in the nature involves rate of change, which requires differential equations to provide mathematical model.

- It can be used to understand general effects of growth trends as differential equations can represent a growth rate.

eg: $L\ddot{q} + R\dot{q} + \frac{q}{C} = \frac{E(t)}{C}$

$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$ {rate of change of velocity}

- Partial differential equation (PDEs) are used to formulate problems involving functions of several variables & are either solved in closed form or used to create a relevant computer model. They can be used to describe phenomena (sound, heat, electrostatic, etc).

- Differential equations are important because their solutions are useful functions, which may be difficult to acquire by other means.
- It is important for the study of behavior of the systems.