Calculus II Test #1

There are a total number of 120 points available (100 are needed for a grade of 100%).
Show your work and clearly label your answers. *No scrap paper, calculators, or notes are allowed.*
To get credit on a problem, you *must* give a *clear, well-written* explanation. An answer alone will not suffice.
You have 90 minutes to complete this test.

READ, AND PRINT AND SIGN YOUR NAME BEFORE BEGINNING THE TEST.
I will neither give nor receive unauthorized assistance on this test.
Printed Name _____ Signature _____

Problem 1 ((2+3+5)x3 pts) Give the domain, range, and inverse of each of the following functions:

(a) $f(x) = \ln(x^3 + 2)$

(b) $g(x) = \sqrt{9 - x^2}$

(c) $y = \arctan\left(\frac{x}{2}\right)$

Problem 2 (5+5 pts)

(a) State and prove the Inverse Derivative Theorem.

(b) Use (a) to find the slope of the line tangent to the inverse of $y = x^2 - 5$ at the point $(a, f^{-1}(a)) = (-4, 1)$.

Problem 3 (10 pts)

Compute the equation of the line tangent to the *inverse* of the function

$$f(x) = 3\arcsin(x-1)$$

at the point $(a, f^{-1}(a)) = (\frac{\pi}{2}, \frac{3}{2})$.

Problem 4 (10+5 pts)

(a) Compute

$$\frac{d}{dx}\left(\sin(\arctan(x^2))\right)$$

and write your answer with *no trigonometric functions* in it.

(b) Evaluate the derivative found in (a) at $x = \frac{1}{2}$.

Problem 5 (10 pts) Compute

$$\int_0^1 \frac{x^2}{x^6+6} dx.$$

Problem 6 (5+5 pts)

(a) Sketch the graph of the region completely enclosed by the curves

$$f(x) = 13x + 12 \text{ and } g(x) = x^3.$$

(Hint: Guess one of the intersection points, and use that to help find the others, *before* sketching. How many intersections are there?)

(b) Compute the area of the region sketched in (a).

Problem 7 (5+10+10+10 pts)

(a) Sketch the graph of the region contained by the curves

$$y = 0,\ y = (x-4)^2 + 1,\ x = 2,\ x = 6.$$

(b) Compute the volume of the solid obtained by revolving the region in (a) around the x-axis *using the washer method*.

(c) Compute the volume of this solid from (a) *using the shell method*.
(Hint: compute the volume of *half* of the solid, and mulitply your answer by 2.)

(d) Set up, but do not compute, the surface area of this solid.