

Problem 1 Assume $g(x)$ is a differentiable and integrable function over the real numbers. Let

$$f(x) = 9e^{-2x} + 6\sin(g(x)) - \frac{3}{9x+4}.$$

- a) What is the domain of $y = f(x)$?
- b) Find $f'(x)$ and $f''(x)$.
- c) Find $\int f'(x)dx$.
- d) Find $\int_1^{10} f'(x)dx$. (No need for a calculator; if there are e or \ln or \sin in your answer, leave them as they are. Just simplify the numbers as best you can.)
- e) Let $g(x) = 3x + 2$. Find $\int_3^6 f(x)dx$.

Problem 2 Solve the differential equation $y' = -5y$ for $y = f(x)$ subject to the initial condition $f(0) = 12$.

Problem 3 Assuming the acceleration due to gravity is $-9.8 \frac{\text{m}}{\text{sec}^2}$ on Earth, how high was an object dropped from rest if it took exactly 10 seconds to reach the ground?

Problem 4 Air enters a (spherical) balloon at the rate of 5 cubic feet per minute ($\frac{\text{ft}^3}{\text{min}}$). Recalling that the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere (here, in feet), what is the rate of the radius' growth when the balloon's volume is 400 ft^3 ? (This is a large balloon.)

Problem 5 Minimize the average cost per item of a product with total cost function

$$C(x) = (100 + 5e^{-2x})x + 0.1x^2.$$

Problem 6 Let $f(x) = xh(x^2)$, and $\int h(x)dx = H(x) + C$. What is $\int f(x)dx$?

Problem 7 Find $\int_a^b (cx^n + \frac{5}{x})dx$ for $n = -1$ and $n \neq -1$.

Problem 8

- (a) Approximate $\int_0^4 (x^2 + 6)dx$ by calculating a sum, with right-hand endpoints, for equal box spacing size $\Delta x = 1$ (that is, 4 boxes to check area). Sketch the graph to help you along.
- (b) Calculate the actual integral $\int_0^4 (x^2 + 6)dx$. How far off was your approximation?

Problem 9 Compute $\frac{d}{dx} \tan x$ via the quotient rule (you must show your work).

Problem 10 Find $\int \tan(x)dx$ (you must show the substitution).

Problem 11 Calculate:

- (a) $\frac{d}{dx} \log_3(x)$
- (b) $\int 3^x dx$
- (c) $4^{\log_2 x}$