Calculus II Sample Final Exam

Problem 1 Compute the equation of the line tangent to the *inverse* of the function $f(x) = 4^{3x-1}$ at the point $(a, f^{-1}(a)) = (16, 1)$.

Problem 2 Compute

$$\frac{d}{dx}\left(\cos(\arcsin(x^2))\right)$$

and evaluate the derivative at $x = 1$.

Problem 3

(a) Sketch the graph of the region contained by the curves

$$y = x^3,\ x = 0,\ y = 0,\ y = 8.$$

(b) Compute the volume of the solid obtained by revolving the curve in (a) around the x-axis.

Problem 4

$$\int_0^{\pi} \sin^3(3x)\cos^2(3x)dx =$$

Problem 5

$$\int \sqrt{100 - x^2}dx =$$

Problem 6

(a)

$$\lim_{x\to 0} \frac{\sqrt{25 - x^2} - 5}{x^2} =$$

(b)

$$\lim_{x\to \frac{\pi}{2}} \frac{\cos(5x)}{\cos(3x)} =$$

Problem 7

$$\int_1^{\infty} \frac{\ln(x)}{x^2}dx =$$

For each sequence in Problems 8 and 9,

(a) say if the sequence converges (and to what limit) or diverges (and how);

(b) say if the sequence is: bounded/unbounded, strictly/monotonic increasing/decreasing.

Problem 8 $b_n = \frac{7n^2}{n^2+1}$, $n \geq 1$

Problem 9 $c_1 = 0.1$, $c_2 = 3$, $c_{n+2} = c_n c_{n+1}$, $n \geq 1$

Problem 10

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+2}{n+1}\right) =$$

Problem 11

(a) Rewrite $6.\overline{24}$ as a geometric series.

(b) Rewrite $6.\overline{24}$ as a ratio of whole numbers (in lowest terms).

Problem 12 Test the series for convergence. If it converges, compute the limit or give bounds within 0.25 of the correct value.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} =$$

Problem 13 For

$$f(x) = 2e^{3x-1},$$

give:

(a) the fourth degree MacLaurin polynomial;

(b) the fourth degree Taylor polynomial around $a = 2$;

(c) the interval of convergence for the Taylor series around $a = 2$.