Printed Name __________________________ Signature __________________________

Linear Algebra

Exam #2

Show your work and clearly label your answers on this exam.

No scrap paper or notes are allowed, but you may use a scientific or accounting calculator (no phones or computers). Use 6 digits of precision throughout your calculations (and answers), although fractions will likely make for more intelligible answers.

This quiz is scored out of 100 points.

(There are 105 points possible.)

To get credit on a problem, you *must* give a clear, well-written explanation, justifying each step.

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Problem 1 (10+5+10+5 pts) Consider the system $A\vec{x} = b$ given by

$$\begin{aligned} 4x_1 + 21x_2 + 97x_3 - 12x_4 + 19x_5 &= 111 \\ 8x_1 + 35x_2 + 159x_3 - 9x_4 + 45x_5 &= 171 \\ 2x_1 + 7x_2 + 31x_3 + 3x_4 + 13x_5 &= 27. \end{aligned}$$

(a) What is the reduced row echelon form of the coefficient matrix A?

(b) What are the pivot variables? What are the free variables?

(c) What is the full (particular + special) solution set to the system $A\vec{x} = b$?

(d) What is the dimension of the solution set $\{\vec{x} : A\vec{x} = b\}$?
What geometric shape does this set have, inside what larger vector space?

Problem 2 (20+5 pts) A line in $\mathbb{R}^3$ is spanned by the first column of A from Problem 1.

(a) Give the projection matrix P that projects a vector in $\mathbb{R}^3$ onto this line.

(b) What is the projection of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ onto this line?

Problem 3 (20+5+5 pts)

(a) Find the best fit quadratic function $f(x) = a + bx + cx^2$ to the set of points

$$\{(0,5),(-2,8),(2,9),(1,6)\}.$$

(b) Plot the graph and points.

(c) Compute the total squared error from the best fit parabola to the points.

Problem 4 (20 pts) Compute the Gram-Schmidt basis for $\mathbb{R}^3$ generated by the independent columns of A based on the pivot variables you found during Problem 1.

(Yes, you read that right. If you read this first, before starting Problem 1, congratulations, you get a big hint on Problem 1.)

Remember:

$$\begin{aligned} b_1 = a_1 &\implies q_1 = \frac{1}{||b_1||} b_1 \\ b_2 = a_2 - \left(\frac{b_1 \cdot a_2}{b_1 \cdot b_1} \right) b_1 &\implies q_2 = \frac{1}{||b_2||} b_2 ... \end{aligned}$$