Syntax

$$
\begin{array}{lcl}
P & ::= & \top \mid p : \tau \, \langle \overline{x : \tau} \rangle \mid P \wedge P \\
\mathbb{T}\,(\mathbb{B}) & ::= & \alpha^{\mathbb{B}} \mid x{:}\mathbb{T}\,(\mathbb{B}) \to \mathbb{T}\,(\mathbb{B}) \ \mid \mathsf{TyCon}^{\mathbb{B}} \ \overline{\mathbb{T}\,(\mathbb{B})} \ \overline{\mathbb{B}} \ \mid \mathsf{Class} \ \overline{\mathbb{T}\,(\mathbb{B})} \\
\mathbb{P}\,(\mathbb{B}) & ::= & \forall P. \mathbb{P}\,(\mathbb{B}) \mid \mathbb{T}\,(\mathbb{B}) \\
\mathbb{S}\,(\mathbb{B}) & ::= & \forall \alpha. \mathbb{S}\,(\mathbb{B}) \mid \mathbb{P}\,(\mathbb{B}) \\
T_P & ::= & \mathbb{S}\,(P) \\
\hat{T} & ::= & \mathbb{S}\,(Q) \\
T & ::= & \mathbb{S}\,(E) \\
\tau & ::= & \mathbb{S}\,(\top) \\
e & ::= & \mathrm{x} \mid \mathrm{C} \mid e \ e \mid \lambda x.e \\
 & \mid & e \ @ \mid \hat{\Lambda} p.e \mid e\,[\tau] \mid \Lambda \alpha.e \\
 & \mid & \mathsf{case} \ x \ \mathsf{of} \ \ |_i \ C_i \ \overline{x_i} \ \to \ e_i \\
 & \mid & \mathsf{let} \ x = e \ \mathsf{in} \ e \mid \mathsf{letrec} \ x = e \ \mathsf{in} \ e
\end{array}
$$

Rules

$$\boxed{\Gamma \vdash_{P,Q} e : T}$$

$$\frac{\Gamma \vdash_{P,Q} e : T_1 \qquad \Gamma \vdash T_1 <: T_2 \qquad \Gamma \vdash T_2}{\Gamma \vdash_{P,Q} e : T_2}$$

$$\frac{\Gamma(x) = \{v : T \mid e\}}{\Gamma \vdash_{P,Q} x : \{v : T \mid e \wedge v = x\}}$$

$$\frac{}{\Gamma \vdash_{P,Q} \mathsf{C} : ty(c)}$$

$$\frac{\Gamma \vdash_{P,Q} e_1 : x{:}T_x \to T \qquad \Gamma \vdash_{P,Q} e_2 : T}{\Gamma \vdash_{P,Q} e_1 \ e_2 : T\,[x \mapsto e_2]}$$

$$\frac{\Gamma, x : \hat{T}_x \vdash_{P,Q} e : \hat{T} \qquad \Gamma \vdash x{:}\hat{T}_x \to \hat{T}}{\Gamma \vdash_{P,Q} \lambda x.e : x{:}\hat{T}_x \to \hat{T}}$$

$$\frac{\Gamma \vdash_{P,Q} e : \forall p.T \qquad e_p = \text{fresh} \ \ (p) \qquad \Gamma, p \vdash e_p}{\Gamma \vdash_{P,Q} e \ @ : T\,[p \mapsto e_p]}$$

define : freshP
do I need the following rule?

$$\frac{\Gamma \vdash_{P,Q} e : T}{\Gamma \vdash_{P,Q} \hat{\Lambda} p.e : \forall p.T}$$

$$\frac{\Gamma \vdash_{P,Q} e : \forall \alpha.T \qquad \hat{T} = \text{fresh} \ \ (\tau)}{\Gamma \vdash_{P,Q} e\,[\tau] : T\left[\alpha \mapsto \hat{T}\right]}$$

define subT, freshT

$$\frac{\Gamma \vdash_{P,Q} e : T \qquad \alpha \notin \Gamma}{\Gamma \vdash_{P,Q} \Lambda\alpha.e : \forall\alpha.T}$$

$$\frac{\begin{array}{c}\Gamma \vdash_{P,Q} e : \mathsf{C}^{e_c}\ \overline{T}\ \overline{e} \\ \forall i.\{\Gamma \vdash_{P,Q} \mathsf{C}_i : \underline{\forall\overline{\alpha}.\forall\overline{p}.y_1{:}T_1 \to \dots y_j{:}T_j} \to \dots y_n{:}T_n \to \mathsf{C}^{e'_c}\ \overline{T'}\ \overline{e'} \\ \Gamma, x_{ij} : T_j\left[\overline{\alpha} \mapsto \overline{T}\right]\left[\overline{p} \mapsto \overline{e}\right] \vdash_{P,Q} e_i : T\}\end{array}}{\Gamma \vdash_{P,Q} \mathsf{case}\ e\ \mathsf{of}\ \ |_i\ C_i\ \overline{x_i}\ \ \to\ \ e_i : T}$$

$$\frac{\Gamma \vdash_{P,Q} e_1 : T_x \qquad \langle P(x), T_x\rangle \models T'_x \qquad \Gamma, x : T'_x \vdash_{P,Q} e_2 : T}{\Gamma \vdash_{P,Q} \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 : T}$$

$$\frac{\Gamma \vdash \hat{T}_x \qquad \Gamma \vdash_{P,Q} e_1 : \hat{T}_x \qquad \left\langle P(x), \hat{T}_x\right\rangle \models \hat{T}'_x \qquad \Gamma, x : \hat{T}'_x \vdash_{P,Q} e_2 : \hat{T}}{\Gamma \vdash_{P,Q} \mathsf{letrec}\ x = e_1\ \mathsf{in}\ e_2 : \hat{T}}$$

$\boxed{\Gamma \vdash T_1 <: T_2}$

$$\frac{[\Gamma] \wedge [e_1] \Rightarrow [e_2]}{\Gamma \vdash \alpha^{e_1} <: \alpha^{e_2}}$$

$$\frac{\Gamma \vdash T_{x_2} <: T_{x_1} \qquad \Gamma, x_2 : T_{x_2} \vdash T_1\left[x_1 \mapsto x_2\right] <: T_2}{\Gamma \vdash x_1{:}T_{x_1} \to T_1 <: x_2{:}T_{x_2} \to T_2}$$

$$\frac{\Gamma \vdash T_1 <: T_2}{\Gamma \vdash \forall\alpha.T_1 <: \forall\alpha.T_2}$$

$$\frac{[\Gamma] \wedge [e_1] \Rightarrow [e_2] \qquad \forall i.\Gamma \vdash T_{1_i} <: T_{2_i} \qquad \forall j.[\Gamma] \wedge [e_{1_j}] \Rightarrow [e_{2_j}]}{\Gamma \vdash \mathsf{C}^{e_1}\ \overline{T_1}\ \overline{e_1} <: \mathsf{C}^{e_2}\ \overline{T_2}\ \overline{e_2}}$$

no subtype for tforallPr!
no subtype for tclass!

$\boxed{\Gamma \vdash T}$

$$\frac{\Gamma, v : \mathrm{shape}\,(T) \vdash e : \mathrm{bool}}{\Gamma \vdash \alpha^{e}}$$

$$\frac{\Gamma \vdash T_x \qquad \Gamma, x : T_x \vdash T}{\Gamma \vdash x{:}T_x \to T}$$

$$\frac{\Gamma \vdash T}{\Gamma \vdash \forall\alpha.T}$$

define tyConP

$$\frac{\Gamma, v : \mathrm{shape}\,(T) \vdash e : \mathrm{bool} \qquad \forall i.\Gamma \vdash T_i \qquad \overline{p} = \mathrm{predicates}\,(\mathsf{C}) \qquad \forall j.\Gamma, p_j \vdash e_j}{\Gamma \vdash \mathsf{C}^{e}\ \overline{T}\ \overline{e}}$$

no subtype for tforallPr!
no subtype for tclass!

$$\boxed{\Gamma, p \vdash e}$$

$$\frac{\Gamma, v:\tau, \overline{x:\tau} \vdash e : \text{bool}}{\Gamma, p:\tau \left\langle \overline{x:\tau} \right\rangle \vdash e}$$

$$\boxed{\langle T_P, T \rangle \models T}$$

$$\frac{\langle T_P, T \rangle \models T'}{\langle T_P, \forall p.T \rangle \models \forall p.T'}$$

$$\frac{\langle T_P, T \rangle \models T'}{\langle \forall p.T_P, T \rangle \models \forall p.T'}$$

$$\frac{\langle T_P\left[\alpha \mapsto \alpha'\right], T \rangle \models T'}{\langle \forall \alpha.T_P, \forall \alpha'.T \rangle \models \forall \alpha'.T'}$$

$$\frac{\langle T_{Px}, T_{x'} \rangle \models T'_x \qquad \langle T_P\left[x \mapsto x'\right], T_{x'} \rangle \models T'}{\langle x{:}T_{Px} \to T_P, x'{:}T_{x'} \to T \rangle \models x'{:}T'_x \to T'}$$

$$\frac{}{\left\langle \mathsf{C}\ \overline{T_P}, \mathsf{C}\ \overline{T} \right\rangle \models \mathsf{C}\ \overline{T}}$$

$$\frac{}{\left\langle \alpha^P, \alpha^e \right\rangle \models \alpha^{e \wedge P(v)}}$$

$$\frac{\forall i. \langle T_{Pi}, T_i \rangle \models T'_i}{\left\langle \mathsf{C}^P\ \overline{T_P}\ \overline{P}, \mathsf{C}^e\ \overline{T}\ \overline{e} \right\rangle \models \mathsf{C}^{e \wedge P(v)}\ \overline{T'}\ \overline{e_i \wedge P_i\left(v\right)}}$$

$$\boxed{\Gamma, p \vdash e}$$

$$\frac{\Gamma, v:\tau, \overline{x:\tau} \vdash e : \text{bool}}{\Gamma, p:\tau \left\langle \overline{x:\tau} \right\rangle \vdash e}$$

$$\frac{\Gamma \vdash e : \text{bool}}{\Gamma, \top \vdash e}$$

$$\frac{\Gamma, P_1 \vdash e_1 \qquad \Gamma, P_2 \vdash e_2}{\Gamma, P_1 \wedge P_2 \vdash e_1 \wedge e_2}$$

$$\boxed{\text{e}\ \left[p \mapsto e\right]}$$

$$\begin{aligned}
\left(e_1 \wedge e_2\right)\left[p \mapsto e\right] &= \left(e_1\left[p \mapsto e\right]\right) \wedge \left(e_1\left[p \mapsto e\right]\right) \\
\left(p\left(v, \left[\overline{x} \mapsto \overline{y}\right]\right)\right)\left[p \mapsto e\right] &= e\left[\overline{x} \mapsto \overline{y}\right] \\
\left(e'\right)\left[p \mapsto e\right] &= e'
\end{aligned}$$

$$\boxed{P\left(v\right)}$$

$$\begin{aligned}
\top\left(v\right) &= \top \\
p:\tau\left\langle \overline{\tau} \right\rangle\left(v\right) &= p(v) \\
P_1 \wedge P_2\left(v\right) &= P_1\left(v\right) P_2\left(v\right)
\end{aligned}$$

$$\boxed{T[p \mapsto e]}$$

$$\begin{aligned}
\alpha^{e'}\,[p \mapsto e] &= \alpha^{e'[p\mapsto e]} \\
x{:}T_x \to T\,[p \mapsto e] &= x{:}T_x\,[p \mapsto e] \to T\,[p \mapsto e] \\
\mathsf{C}^{e'}\ \overline{T}\ \overline{e}\,[p \mapsto e] &= \mathsf{C}^{e'[p\mapsto e]}\ \overline{T\,[p \mapsto e]}\ \overline{e\,[p \mapsto e]} \\
\mathsf{C}\ \overline{T}\,[p \mapsto e] &= \mathsf{C}\ \overline{T\,[p \mapsto e]} \\
\forall P.T\,[p \mapsto e] &= \begin{cases} \forall P.T\,[p \mapsto e] & p \neq P \\ \forall P.T & p = P \end{cases} \\
\forall \alpha.T\,[p \mapsto e] &= \forall \alpha.T\,[p \mapsto e]
\end{aligned}$$

$$\boxed{\Gamma \vdash e : T_P \mid C}$$

$$\frac{T_P = \Gamma(x) \qquad {T_P}' = \text{inst }(T_P)}{\Gamma \vdash x : {T_P}' \mid \emptyset}$$

$$\frac{}{\Gamma \vdash C : ty(C) \mid \emptyset}$$

$$\frac{\Gamma \vdash e_1 : x{:}T_{Px} \to T_{P1} \mid C_1 \qquad \Gamma \vdash e_2 : T_{P2} \mid C_2}{\Gamma \vdash e_1\ e_2 : T_{P1} \mid \{T_{Px} :=: T_{P2}\} \cup C_1 \cup C_2}$$

$$\frac{\Gamma, x : T_{Px} \vdash e : T_P \mid C}{\Gamma \vdash \lambda x.e : x{:}T_{Px} \to T_P \mid C}$$

$$\frac{\Gamma \vdash e : T_P \mid C}{\Gamma \vdash e\ @ : T_P \mid C}$$

$$\frac{\Gamma \vdash \hat{\Lambda} p.e : T_P \mid C}{\Gamma \vdash \hat{\Lambda} p.e : \forall p.T_P \mid C}$$

$$\frac{\Gamma \vdash e : \forall \alpha.T_P \mid C \qquad T_{P\alpha} = \text{fresh }\ (\tau)}{\Gamma \vdash e\,[\tau] : T_P\,[\alpha \mapsto T_{P\alpha}] \mid C}$$

$$\frac{\Gamma \vdash \Lambda \alpha.e : T_P \mid C \qquad \alpha \notin \Gamma}{\Gamma \vdash \Lambda \alpha.e : \forall a.T_P \mid C}$$

$$\frac{\begin{array}{c}\Gamma \vdash e : \mathsf{C}^{e_c}\ \overline{T_P}\ \overline{e} \mid C \\ \forall i.\{\Gamma \vdash \mathsf{C}_i : T_{Pi} \mid C_i \qquad \text{inst }\underline{(T_{Pi}) = \forall \overline{\alpha}.y_1{:}T_{P1} \to \ldots y_j{:}T_{Pj} \to \ldots y_n{:}T_{Pn}} \to \mathsf{C}^{e'_c}\ \overline{T'}\ \overline{e'} \\ \Gamma, \overline{x_{ij}} : T_j\,\left[\overline{\alpha} \mapsto \overline{T_P}\right] \vdash e_i : T_{Pi} \mid C_i\}\end{array}}{\Gamma \vdash \textsf{case}\ e\ \textsf{of}\ \ |_i\ C_i\ \overline{x_i}\ \ \to\ \ e_i : T_P \mid \bigcup_i \{T_{Pi} :=: T_P\} \bigcup_i C_i \cup C}$$

$$\frac{\Gamma \vdash e_1 : T_{Px} \mid C_x \qquad C_x, T_{Px} \vdash T'_{Px} \qquad \Gamma, x : T'_{Px} \vdash e_2 : T_P \mid C}{\Gamma \vdash \textsf{let}\ x = e_1\ \textsf{in}\ e_2 : T_P \mid C}$$

$$\frac{\Gamma \vdash e_1 : T_{Px} \mid C_x \qquad C_x, T_{Px} \vdash T'_{Px} \qquad \Gamma, x : T'_{Px} \vdash e_2 : T_P \mid C}{\Gamma \vdash \textsf{letrec}\ x = e\ \textsf{in}\ e : T_P \mid C}$$

$$\boxed{\text{inst } (T_P)}$$

$$\begin{aligned}
\text{inst } (\forall\alpha.T_P) &= \forall\alpha.\text{inst } (T_P) \\
\text{inst } (\forall\overline{P}.T_P) &= T_P \left[\overline{P} \mapsto \overline{P'}\right] && \overline{P'} \text{ fresh} \\
\text{inst } (T_P) &= T_P
\end{aligned}$$

$$\boxed{\text{gen } (T_P)}$$

$$\begin{aligned}
\text{gen } (\forall\alpha.T_P) &= \forall\alpha.\text{inst } (T_P) \\
\text{gen } (T_P) &= \begin{cases} \forall\overline{P}.T_P & \overline{P} \text{ free in } T_P \\ T & \text{othewise} \end{cases}
\end{aligned}$$

$$\boxed{C, T_P \vdash T_P}$$

$$C, T_P \vdash \text{gen } (T_P[\text{split } (C)])$$

$$\boxed{\text{split } (C)}$$

$$\begin{aligned}
\text{split } (\alpha^p :=: \alpha^q, cs) &= [p \mapsto q] : \text{split } (cs[p \mapsto q]) \\
\text{split } (x_1{:}T_{P x_1} \to T_{P1} :=: x_2{:}T_{P x_2} \to T_{P2}, cs) &= \text{split } ((T_{P x_1} :=: T_{P x_2}) : (T_{P1}\,[x_1 \mapsto x_2] :=: T_{P2}) : cs) \\
\text{split } (\forall\alpha_1.T_{P1} :=: \forall\alpha_2.T_{P2}, cs) &= \text{split } ((T_{P1}[\alpha_1 \mapsto \alpha_2] :=: T_{P2}) : cs) \\
\text{split } (\mathsf{C}\ \overline{T_P} :=: \mathsf{C}\ \overline{T_P}, cs) &= \text{split } (cs) \\
\text{split } (\mathsf{C}^{p_1}\ \overline{T_{P1}}\ \overline{p_1} :=: \mathsf{C}^{p_2}\ \overline{T_{P2}}\ \overline{p_2}, cs) &= [\overline{p_1} \mapsto \overline{p_2}] : [p_1 \mapsto p_2] : \\
&\qquad \text{split } ((\{\overline{T_{P1}} :=: \overline{T_{P2}}\} \cup cs)[p_1 \mapsto p_2]\ o\ [\overline{p_1} \mapsto \overline{p_2}])
\end{aligned}$$