Test Flight Problem Set.

1.

$$(\exists m \in N)(\exists n \in N)(3m + 5n = 12)$$

Prove it is true or false.

Statement is False

~~Proof by Contradiction~~ Proof:

$$3m + 5n = 12 \Rightarrow [n \text{ cannot be } \geq 2]$$

~~Add~~ Subtract $5n$

$$3m + 5n - 5n = 12 - 5n$$

$$3m = 12 - 5n$$

$$m = 4 - \frac{5}{3}n$$

for $n = 1$

$$m = \frac{7}{3}$$

Hence the equation is False. QED

2. Sum of any five consecutive integers is divisible by 5

The statement is true.

Proof:

Let a be any integer

$(\forall a \in Q)$ $(a+(a+1)+(a+2)+(a+3)+(a+4)$ is divisible by 5$)$

$$a+(a+1)+(a+2)+(a+3)+(a+4) = 5a+10$$
$$= 5(a+2)$$

$\Rightarrow$ implies it is divisible by 5

Hence the statement is true QED.

3. For any integer n, n^2+n+1 is odd.

The statement is true.

Proof:

n^2+n+1 can be rewritten as

$n(n+1)+1$

if n is odd $n+1$ is even

if $n+1$ is odd n is even

odd $\times$ even is even

$\Rightarrow \overset{(n(n+1))}{\text{even}} + 1$ is odd

Hence the statement is true. QED

4.

Prove that

$$(\forall a \in \text{odd Natural})(\exists n \in \mathbb{Q})$$

$$[a = 4n+1 \wedge x = 4n+3] \Rightarrow (i)$$

~~The~~ Proof:

$a = 4n+1$ and $x = 4n+3$

is of the form

$a = bq + r$ from the remainder theorem.

Here $b = 4$

Possible values of r are $0 \leq r < 4$

if $r = 0$ or 2 then

$a = 4q + 0$ or $4q + 2$ which is even

for a to be odd r should be 1 or 3

Hence the equation (i) is true. QED

5.

Prove for any integer n, atleast $n, n+2, n+4$ are divisible by 3

Proof

By remainder theorem

$$a = bq + r \quad , 0 \le r \le b$$

for $b = 3$

$$a = 3q + r \quad , 0 \le r \le 3$$

for $q = [n, n+2, n+4]$

~~$a = 3q + r$~~ $a = 3n + r \quad [0 \le r < 3]$

$$a = 3n + 3 + r \quad [0 \le r < 3]$$

$$a = 3n + 6 + r \quad [0 \le r < 3]$$

for $r = 0$ in all cases $3 \mid a$

Hence $(\forall n \in Q)\left[3 \mid n \wedge 3 \mid n+2 \wedge 3 \mid n+4\right]$

QED

6.

Proof by Contradiction

~~Assume (∃n) [n>3]~~

~~that~~

Assume $(\exists n)[n>3 \wedge (n, n+2, n+4)$ are prime$]$

From q5 we know $n, n+2, n+4$ one of them should be divisible by 3

Since 3 is not the prime for $n>3$

Then one of $n, n+2, n+4$ is not a prime

Hence proved. QED.

7. Prove $2+2^2+2^3+\cdots+2^n = 2(2^n-1)$

Proof by Induction.

for $n=1$

$$2^1 = 2(2^1-1)$$

$$2 = 2$$

for $n=k$

$$2+2^2+2^3+\cdots+2^k = 2(2^k-1) \quad -(i)$$

for $n=k+1$

$$2+2^2+2^3+2^4\cdots+2^{k+1} = 2(2^{k+1}-1)$$

from (i) L.H.S

$$\Rightarrow \quad = 2(2^k-1)+2^{k+1}$$

$$= 2^{k+1}-2+2^{k+1}$$

$$= 2(2^{k+1}-1)$$

$$= R.H.S$$

Hence proved by principle of Induction

QED

8. Prove sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$ the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML

Proof:

By convergence theorem

$$|a_n - L| < \frac{\varepsilon}{M} \qquad [\varepsilon > 0]$$

$\Rightarrow$

$$|Ma_n - ML| = M|a_n - L| < M\frac{\varepsilon}{M} = \varepsilon$$

This proves $Ma_n \to ML$

9.

Proof:

Let $A_n = (0, 1/n)$

$A_1 = (0, 1)$

$\Rightarrow A_n \subset A_1$

Let x be $(0, 1)$

we know $(\exists m \in N) [1/m < x]$

$\Rightarrow$ x is not element of A_m

$\Rightarrow x \notin A_n$

$\Rightarrow$ Intersection will be empty set

Hence Proved

QED

10.

Proof:

$A_n = [0, 1/n)$

if $B_n = (0, 1/n)$

then $0 \cup B_n = A_n$

Intersection can be written as $0 \cup (\cap B)$

from Q9 Intersection of B_n is empty set.

$0 \cup \phi = \{0\}$

Hence proved. QED.