Report on the manuscript

Generalized Rybicki Press Algorithm

by S. Ambikasaran

In the manuscript *Generalized Rybicki Press Algorithm* the author describes a fast ($\mathcal{O}(N)$) direct solver for semi-separable matrices with fixed ($\mathcal{O}(1)$) semi-separable rank p. The main idea is to embed the semiseparable matrix into a larger banded matrix of size $\mathcal{O}(pN)$ and band $\mathcal{O}(p)$, for which fast direct solver are avaialable. The main application of the method described in the manuscript is in the context of Continuous time AutoRegressive-Moving-Average (CARMA) models, for the solution of linear system and the computation of determinants of dense covariance matrices of the form

$$A_{ij} = \sum_{l=1}^{p} \exp(\beta_l |t_i - t_j|).$$

I would be happy to reccomend this manuscript for publication with minor revisions.

1. In equation (4.1) the notation I_1 and I_2 may be misleading since I usually denotes the identity matrix, while I_1 and I_2 are indeed triangular matrices.

2. In formula (4.1), is the diagonal matrix D such that $D_{ii} = a_{ii}$? If so please make it more explicit, if not please provide the exact formula.

3. In equation (4.2) the Schur complement matrix should read

$$D - \begin{bmatrix} V_b & U_b \end{bmatrix} \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}^{-1} \begin{bmatrix} U_a \\ V_a \end{bmatrix}.$$

4. In Claim 4.1, since the author wrote that the Schur complement of A_{ex} is equal to A, should not be $\det(A_{ex}) = \det(A)$ without ambiguity on the signum? Otherwise, if the Schur Complement of A_{ex} is $-A$, then we have $\det(A_{ex}) = \det(A)$ if the size of A is even, and $\det(A_{ex}) = -\det(A)$ if the size of A is odd.

5. In section 7.1, please provide more details regarding the algorithm used to perform the sparse LU factorization of A_{ex}. Is A_{ex} stored in CSR format or skyline format (i.e. the banded matrix format in LAPACK)? Is pivoting allowed in this sparse factorization? If so, does it preserve the banded structure? If not, is the factorization backward stable?

6. In Table 7.1, it would be interesting to compare the determinats computed using the *usual* and the *fast* method.

7. In table 7.1, and also below formula (7.1), I would suggest to write *infinity norm of the residual* instead of *absolute error*.

8. Please add a reference in the text for Fig. 7.1 and Fig. 7.2.

END of REPORT