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# HOMework 3

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## Group Members

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## Solutions

1. (25 points) Suppose each edge of an undirected graph  $G$  is associated with a cost function  $f_e(t) = a_e t^2 + b_e t + c_e$  where  $a_e > 0$  (think of  $t$  as a time parameter and  $f_e(t)$  as the length of  $e$  at time  $t$ ). Given the graph  $G$  and the values  $\{a_e, b_e, c_e\}$  as input, give an algorithm that returns a value  $t$  at which the minimum spanning tree has a minimum cost (over all time  $t$ ). Assume that the arithmetic operations on  $\{a_e, b_e, c_e\}$  can be done in constant time per operation. Any correct algorithm with time complexity polynomial in  $|V|$  and  $|E|$  gets full credits; i.e.,  $|V|^a \cdot |E|^b$  for constant  $a, b$ .

### Answer:

Our objective is to find the time  $t$  at which the minimum spanning tree of an undirected graph  $G$  has the lowest possible cost over time. The cost of edge will be change respect to time  $t$ , this means the structure of MST can be different with different  $t$  value. Recalling the interesting properties we discuss in class, when the weight of edges changes, if the sorted order of edges weight remains, the MST remains the same. This means rather than computing the cost of MST for every possible  $t$ , we only need to compute the cost whenever the relative order of the edge weight changes.

One possible solution is to first identify the critical points where the order of edges might change due to their cost function. Therefore, for each pair of edges  $(e, e')$  in graph  $G$ , we find the critical point at  $f_e(t) = f_{e'}(t)$  where the order of edge weight could change. Once we have all the critical point we can sort them in ascending order.

After we have sort the critical point, for each critical point  $t$ , we sort all the edges according to their cost function  $f_e(t)$ , then compute the MST using Kruskal's algorithm at current  $t$ . Let the total cost of the MST at time  $t$  be  $C_t(t)$ . Notice that in a time interval  $(t_{i-1}, t_i)$ , the structure of MST remains the same until time  $t > t_i$ , so we define The MST cost function within each interval as:

$$C_{t_i}(t) = \sum_{e \in MST} (a_e t^2 + b_e t + c_e)$$

which is also a quadratic function.

In addition, in each time interval, the structure of MST remains the same, but the total cost might changes, so for each  $t$ , we calculate the local minimum of each time interval

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by differentiating the quadratic equation  $C_{t_i}(t)$  respect to  $t$  and solve  $C'_{t_i}(t) = 0$ . If such local minimum exist at some time  $t'$ , calculate the cost  $C_{t_i}(t')$ . Additionally, we also evaluate  $C_{t_i}(t)$  at each critical points. Finally, after we tracked all the minimum MST cost across each each interval and at the interval boundaries, we sort all the total cost  $C_{t_i}(t)$  in order, and the smallest value is the global minimum MST cost, return the corresponding time  $t$ .

**Proof of Correctness:**

**Claim 1:** The only times at which the MST structure may change are at the critical points, where the order of edge weights may change. Within each interval  $(t_{i-1}, t)$ , the MST structure remains stable and can be determined by the edge order at  $t_i$

**Proof:** Giving the properties in lecture, the MST structure will change only when the order of edge weights changes. Consider any two edges  $e$  and  $e'$  in graph  $G$  with time-dependent cost function  $f_e(t) = a_e t^2 + b_e t + c_e$ , the order of weight change only happens when  $f_e(t) = f_{e'}(t)$ , solving this equation for  $t$  gives us the critical points where a reordering might occur.

The algorithm finds all critical points by solving  $f_e(t) = f_{e'}(t)$  for each pair  $(e, e')$  and sorting them in ascending order. This ensures that we capture every possible change in the order of edge weights. Hence, every potential change in the MST structure is being evaluate.

**Claim 2:** Within each interval  $(t_{i-1}, t)$ , the MST structure remains stable and can be determined by the edge order at  $t_i$ .

**Proof:** Using similar properties, Kruskal's algorithm depends solely on the relative order of edge weights, not their exact values. Since the order of edge weights is stable within each interval, the MST structure remains constant in each interval as well.

By computing the MST structure at the right endpoint  $t_i$  of each interval  $(t_{i-1}, t)$ , the algorithm ensures that this structure is valid for the entire interval.

**Claim 3:** The algorithm correctly finds the minimum MST cost within each interval  $(t_{i-1}, t)$

**Proof:** Within each interval  $(t_{i-1}, t)$  the MST structure is stable, so the total cost of the MST is a quadratic function of  $t$  is given by:

$$C_{t_i}(t) = \sum_{e \in MST} (a_e t^2 + b_e t + c_e)$$

Since  $C_{t_i}(t)$  will also be a quadratic function, it is continuous and differentiable. The minimum of a quadratic function on an interval occurs either at a local minimum within the interval or at the endpoints of the interval.

The algorithm differentiates  $C_{t_i}(t)$  with respect to  $t$  and compute  $C'_{t_i}(t) = 0$  to find the any local minimum within the interval  $(t_{i-1}, t)$ . If a local minimum  $t'$ , exists within

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the interval, it is added to the list of candidate times. The algorithm also considers the endpoints  $t_{i-1}$  and  $t$ , ensuring that the minimum MST cost within the interval  $(t_{i-1}, t)$  is correctly evaluated.

**Claim 4:** The global minimum MST cost occurs at one of the evaluated candidate times, either at a critical point or a local minimum within an interval.

**Proof:** Since we have identified all intervals  $(t_{i-1}, t)$  and calculated the MST cost function  $C_{t_i}(t)$  within each interval, we have accounted for every possible configuration of the MST across all times  $t$ . By evaluating the cost function  $C_{t_i}(t)$  at every local minimum and at every critical point, the algorithm ensures that the global minimum MST cost is included among the evaluated values.

The final step of the algorithm compares all calculated MST costs and selects the smallest one and return the corresponding time  $t$ . Therefore, the algorithm correctly identifies the global minimum MST cost over time, as it evaluates every possible configuration that could yield a lower cost.

#### Time Complexit analysis:

Let  $E$  be the number of edges in graph  $G$

For each pair of edges  $(e, e')$ , we solve  $f_e(t) = f_{e'}(t)$ , to find the critical point, there are  $O(|E|^2)$  pairs of edges, solving the equation takes constant time  $O(1)$ , thus finding critical point takes  $O(|E|^2)$  time. Sorting this critical point takes  $O(|E|^2 \log |E|)$ .

For each critical point, we calculate the cost  $f(e)$  for each edge, which is  $O(|E|)$ , sorting these edges takes  $O(|E| \log |E|)$ , running Kruskal's algorithm takes  $O(|E| \log |E|)$ . Since we repeat this for each critical point and there are  $O(|E|^2)$  critical points, the total time for this step is  $O(|E|^3 \log |E|)$ .

There are  $O(|E|^2)$  time interval, finding local minimum of each takes  $O(E)$  since we need to define the MST cost function, which is a sum of quadratic function for each edge in MST. So finding local minimum takes total of  $O(|E|^3)$ . For the Final calculation of all MST cost at all local minima and interval boundaries, there are  $O(|E|^2)$  possible time, finding the minimum value takes  $O(|E|^2)$  by iterating through the list of MST cost.

Hence the final time complexity is  $O(|E|^3 \log |E|)$ .

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2. (25 points) **Periodic scheduling.** There are  $n$  tasks. Each task takes one unit of time to perform, and the machine can process at most 1 task in each time slot. The requirement is that the task  $i$  should be scheduled once in each time period  $p_i$ . For example, if task  $i$  has a period  $p_i$ , we must schedule it in each time-window of the form  $[p_i j + 1, \dots, p_i(j + 1)]$ . Let  $L = LCM(p_1, p_2, \dots, p_n)$  (LCM stands for least common multiple). Answer the following questions.

- (a) (10 points) Prove that if  $\sum_i \frac{1}{p_i} > 1$ , there is no periodic schedule (i.e., you can not find a schedule for the first  $L$  time slots).

First, consider the expression  $\sum_i \frac{1}{p_i} > 1$ , we multiply both sides by  $L$ :

$$\sum_i \frac{L}{p_i} > L$$

Then, consider the term  $\frac{L}{p_i}$ , it presents the total time slots that  $task_i$  needs.

By summing over all  $p_i$ ,  $\sum_i \frac{L}{p_i}$  means that total time slots that all tasks need.

If  $\sum_i \frac{1}{p_i} > 1$ , which means  $\sum_i \frac{L}{p_i} > L$ , then the total demand of time for all tasks exceeds the capacity  $L$ , then there will be no periodic schedule.

- (b) (15 points) Suppose  $\sum_i \frac{1}{p_i} \leq 1$ , then prove the following greedy algorithm can find a feasible schedule for the first  $L$  time slots: At each time slot, schedule the unfulfilled task with the nearest deadline (if there are several, do it in an arbitrary order).

*Hint:* consider the first task that can not be fulfilled by the algorithm, and prove this case can not happen if  $\sum_i 1/p_i \leq 1$ .

According to the greedy algorithm, at each time slot, schedule the unfulfilled task with nearest deadline, then there will be at most  $L$  time slots needed since it at most schedule one task at each time slot.

Since  $\sum_i \frac{1}{p_i} \leq 1$ , then  $\sum_i \frac{L}{p_i} \leq L$  which means that there will be enough time slots to schedule all the tasks.

**Case 1:**  $\sum_i \frac{1}{p_i} = 1$

$\sum_i \frac{1}{p_i} = 1$ , then  $\sum_i \frac{L}{p_i} = L$  which implies that the total demand over  $L$  is equal to  $L$ . Every slot is occupied with no idle time.

Let  $t_k$  be the first task that can not be fulfilled by the algorithm.  
Let  $d_k$  be the deadline of  $t_k$ . ( $d_k < L$ )

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**Contradiction:**

Since  $t_k$  missed its deadline, it means that from the beginning(slot 0) to  $d_k$ , there are not enough time slots to schedule the tasks since slot 0 to  $d_k$  are all occupied and  $t_k$  is still not scheduled. In other words, the total demand of time slots from slot 0 to  $d_k$  exceeds to system capacity from slot 0 to  $d_k$ .

However, given  $\sum_i \frac{1}{p_i} = 1$ , the system capacity should exactly matches the total demand of all tasks. Otherwords, the total demand of time slots from slot 0 to  $d_k$  exactly matches the system capacity from slot 0 to  $d_k$ .

Therefore, the assumption that  $t_k$  is the first task that can not be fulfilled by the algorithm contradicts with the condition that  $\sum_i \frac{1}{p_i} = 1$ . It means that there should be enough space to schedule all tasks without missing any deadlines.

**Case 2:**  $\sum_i \frac{1}{p_i} < 1$ 

$\sum_i \frac{1}{p_i} < 1$ , then  $\sum_i \frac{L}{p_i} < L$  which means that the total demand time of all tasks is less than L. There will be some idle slots.

Let  $t_k$  be the first task that can not be fulfilled by the algorithm.

Let  $d_k$  be the deadline of  $t_k$ . ( $d_k < L$ )

**Contradiction:**

Since  $t_k$  missed its deadline, it means that from the beginning(slot 0) to  $d_k$ , there are not enough time slots to schedule the tasks that have deadlines in this interval. In other words, the total demand of time from slot 0 to  $d_k$  for those tasks exceeds the system capacity from slot 0 to  $d_k$ .

However, given  $\sum_i \frac{1}{p_i} < 1$ , the system capacity is greater than the total demand of all tasks. In other words, the system capacity from slot 0 to  $d_k$  either exactly matches the total demand of time from slot 0 to  $d_k$  or exceeds it. It is impossible for the total demand within this interval to exceed the system's capacity.

Therefore, the assumption that  $t_k$  is the first task that can not be fulfilled by the algorithm contradicts with the condition that  $\sum_i \frac{1}{p_i} < 1$ . It means that there should be enough space to schedule all tasks without missing any deadlines.

**Conclusion:**

Under the condition  $\sum_i \frac{1}{p_i} \leq 1$ , the greedy algorithm successfully schedules all tasks without missing any deadlines, which confirms that a feasible schedule exists for the first L time slots.

3. (20 points) Given a list of  $n$  natural (i.e., positive integers) numbers  $d_1, \dots, d_n$ , show how to decide in polynomial time<sup>1</sup> whether there exists an undirected graph whose degrees are precisely the number  $d_1, \dots, d_n$  ( $G$  should not contain multiple edges between the same pair of vertices or “loop” edges with both endpoints equal to the same node). *Hint.* Assume  $d_1 \geq d_2 \geq \dots \geq d_n$ , and try to obtain an undirected graph  $G'$  whose nodes have degrees  $d_2 - 1, d_3 - 1, \dots, d_{d_1+1} - 1, d_{d_1+2}, \dots, d_n$ ; the problem of deciding whether  $G$  exists reduces to whether  $G'$  exists. From there, obtain your greedy algorithm.

**Answer:**

Here is the algorithm to decide whether such graph exists.

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**Algorithm 1** Algorithm for Graph Realizability

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**Require:** A degree sequence  $D = [d_1, d_2, \dots, d_n]$  where  $d_i \in \mathbb{N}$  for all  $i$

```

1: Sort  $D$  in non-increasing order initially
2: while  $D$  is not empty do
3:   Remove any zeros from  $D$ 
4:   if  $D$  is empty then
5:     return True {If all degrees are zero, the sequence is realizable}
6:   end if
7:    $d \leftarrow D[1]$  {Take the largest degree}
8:   Remove  $D[1]$  from  $D$ 
9:   if  $d > |D|$  then
10:    return False {Not enough vertices to connect to}
11:  end if
12:  for  $i = 1$  to  $d$  do
13:     $D[i] \leftarrow D[i] - 1$  {Decrement the degree of the next  $d$  vertices}
14:    if  $D[i] < 0$  then
15:      return False {Negative degree means it's not realizable}
16:    end if
17:  end for
18: end while
19: return True

```

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The initial sort is  $O(n \log n)$ . The while loop iterates up to  $n$  times. Inside the while loop, the inner loop only decrements without sorting, achieving an amortized  $O(n^2)$  complexity overall.

We will prove the correctness of the algorithm using mathematical induction.

## Base Case

For  $n = 1$ , the only possible degree sequence is  $[0]$ , which corresponds to a single isolated node without any edges. This sequence is trivially realizable as a simple

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<sup>1</sup>For this problem, you only need to give an  $O(n^2)$  algorithm. With some data structures, you can speed it up to quasi-linear.

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graph (a single vertex with no edges), and the algorithm correctly terminates, returning **True**.

## Inductive Hypothesis

Assume that for any degree sequence of length less than  $n$ , the algorithm correctly determines whether there exists a simple graph with the specified degree sequence.

## Inductive Step

Now, consider a degree sequence  $D = [d_1, d_2, \dots, d_n]$ , sorted in non-increasing order such that  $d_1 \geq d_2 \geq \dots \geq d_n$ .

1. **Checking Validity:** If  $d_1 \geq n$ , then it is impossible to construct a simple graph with this degree sequence, since the maximum degree of a vertex cannot exceed the total number of other vertices in the graph. In this case, the algorithm correctly returns **False**.
2. **Reduction Step:** If  $d_1 < n$ , we take the largest degree  $d_1$ , remove it from the sequence, and decrement the degrees of the next  $d_1$  vertices by 1. This corresponds to connecting the vertex with degree  $d_1$  to the next  $d_1$  vertices, thereby reducing their degrees.
3. **Reordering and Checking Negatives:** After decrementing the degrees of these  $d_1$  vertices, we record the greatest  $d$  in the rest of the sequence. If any degree becomes negative, it indicates that the original degree sequence is not realizable as a simple graph, and the algorithm returns **False**.
4. **Applying the Inductive Hypothesis:** By removing the degree  $d_1$  and reducing the next  $d_1$  degrees, we have effectively reduced the length of the sequence by one. According to our inductive hypothesis, the reduced degree sequence can be checked by the algorithm. If the reduced sequence is realizable, then the original sequence is also realizable; otherwise, it is not.
5. **Termination Condition:** The algorithm repeats the above steps until the sequence becomes empty or all degrees are zero. If all degrees are reduced to zero, the algorithm returns **True**, indicating that the original sequence can be realized as a simple graph. This is because a degree sequence with all zeros corresponds to an empty graph, which is trivially realizable.

## Conclusion

Thus, by mathematical induction, such algorithm correctly determines whether a given degree sequence can be realized as a simple undirected graph.

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4. (10 points) Given two arrays  $A[1 \dots n]$  and  $B[1 \dots n]$  whose entries are between 0 to  $n$ , design an algorithm to output  $s_k$  for every  $k$  between 0 and  $2n$ .  $s_k$  is defined as the number of distinct  $(i, j)$  pairs such that  $A[i] + B[j] = k$ . Your algorithm should have a quasi-linear time complexity, i.e., at most  $\mathcal{O}(n \log n)$ .

**Proof:**

Let  $a_{\text{freq}}$  and  $b_{\text{freq}}$  be two frequency arrays, where:

$$\begin{aligned} a_{\text{freq}}[i] &= \text{the number of occurrences of } i \text{ in } A, \\ b_{\text{freq}}[j] &= \text{the number of occurrences of } j \text{ in } B. \end{aligned}$$

Each entry  $a_{\text{freq}}[i]$  counts how many times the value  $i$  appears in array  $A$ , and similarly, each entry  $b_{\text{freq}}[j]$  counts occurrences in array  $B$ .

To compute  $s_k$ , which is the number of distinct  $(i, j)$  pairs such that  $A[i] + B[j] = k$ , we use the convolution which is defined as:

$$s_k = \sum_{i+j \equiv k \pmod{2n}} a_{\text{freq}}[i] \cdot b_{\text{freq}}[j].$$

Here,  $s_k$  represents the total number of ways to select elements  $A[i]$  and  $B[j]$  from arrays  $A$  and  $B$ , respectively, such that  $A[i] + B[j] = k \pmod{2n+1}$ . This is because  $a_{\text{freq}}[i] \cdot b_{\text{freq}}[j]$  gives the number of ways to select elements with values  $i$  and  $j$  that add up to  $k$ .

One last thing to prove is that the modulo operation in the convolution formula does not affect the correctness of our result. The modulo operation implies that any sums exceeding the range  $[0, 2n]$  would wrap back into this range. However, since each entry in the arrays  $A$  and  $B$  ranges from 1 to  $n$ , the maximum possible sum  $A[i] + B[j]$  is  $n + n = 2n$ . This means no sums will exceed  $2n$ , so no wrap-around occurs.

Additionally, we padded each frequency array to length  $2n + 1$ , covering the full range from 0 to  $2n$ . Values in the range  $[n + 1, 2n]$  of each frequency array are effectively zero because no element in  $A$  or  $B$  can produce values in this range by itself. As a result, any products involving these padded zeros contribute nothing to the final sum for any index  $s_k$  in  $s$ . Therefore, even with the modulo operation, the convolution result remains unaffected and correctly represents the number of ways to achieve each sum  $k$  from 0 to  $2n$ .

By the convolution theorem, we can compute  $s_k$  by transforming both frequency arrays to the frequency domain using the Fast Fourier Transform (FFT), performing an element-wise multiplication, and then transforming back to the time domain with the Inverse FFT:

$$s = \text{IFFT}(\text{FFT}(a_{\text{freq}}) \cdot \text{FFT}(b_{\text{freq}})).$$

This allows us to derive the psuedo code below:

**Runtime Analysis:**

Initializing the frequency arrays and counting occurrences for each element in  $A$  and  $B$  takes  $\mathcal{O}(n)$  time.



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**Algorithm 2** Count Pairs with Given Sum Using Convolution

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**Require:** Arrays  $A$  and  $B$  of length  $n$

**Ensure:** Array  $c$  of length  $2n + 1$  where  $c[k]$  is the number of pairs  $(i, j)$  such that  $A[i] + B[j] = k$

- 1: Initialize  $a\_freq$  and  $b\_freq$  of size  $2n + 1$  to all zeros
  - 2: **for** each element  $x$  in  $A$  **do**
  - 3:    $a\_freq[x] \leftarrow a\_freq[x] + 1$
  - 4: **end for**
  - 5: **for** each element  $y$  in  $B$  **do**
  - 6:    $b\_freq[y] \leftarrow b\_freq[y] + 1$
  - 7: **end for**
  - 8: Use FFT to obtain convolution  $s \leftarrow \text{IFFT}(\text{FFT}(a\_freq) \cdot \text{FFT}(b\_freq))$
  - 9: **return**  $s$
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Performing the FFT on  $a_{\text{freq}}$  and  $b_{\text{freq}}$ , the element-wise multiplication, and the Inverse FFT each take  $\mathcal{O}(n \log n)$  time.

Summing up the values in the resulting array  $c$  also takes  $\mathcal{O}(n)$  time.

Since the FFT operations dominate the overall complexity, the total time complexity is  $\mathcal{O}(n \log n)$ , which satisfies the quasi-linear time requirement.

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5. (20 points) Given an array  $A[1 \dots n]$  of  $n$  distinct numbers between 1 to  $n$ , compute the number of inversions for every  $A[1 \dots i]$ . Two array elements  $A[i]$  and  $A[j]$  form an inversion if  $A[i] > A[j]$  and  $i < j$ . You should output  $n$  different answers, one for each  $A[1 \dots i]$ . The expected time complexity is  $\mathcal{O}(n \log n)$ .

**Answer:**

To compute the number inversions with the expected time complexity, through the course material, we can use two methods: divide and conquer(merge sort) and segment tree. We choose the second one because we need to output  $n$  different answers, one for each  $A[1 \dots i]$ .

The main idea is to process each element of the array sequentially and, for each element  $A[i]$ , count the number of elements that are less than  $A[i]$  and have already been processed. This count, *inversions\_with\_A<sub>i</sub>*, corresponds to the number of inversions that  $A[i]$  forms with previous elements.

It can be easily seen that the results we want, which correspond to  $n$  different answers, one for each  $A[1 \dots i]$ , can be computed by adding both *inversions\_with\_A<sub>i</sub>* and already accumulated result *inversions\_count*[ $i - 1$ ]. As a result, when we iterate the original array A, we can sequentially get what we want.

With the help of segment tree, we can easily do the counting operation in  $\mathcal{O}(\log n)$  time complexity and for every update/modification, we also have  $\mathcal{O}(\log n)$  time complexity, which can result the expected time complexity. More specifically, we can use a segment tree to:

- (a) **Query** the number of elements less than  $A[i]$  that have already been inserted into the segment tree.
- (b) **Update** the segment tree by inserting  $A[i]$  into it.

The algorithm is shown as follows:

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**Algorithm 5** Counting Inversions Using Segment Tree

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1: procedure COUNTINVERSIONS( $n, A$ )
2:   Initialize segment tree  $st$  of size  $n$ 
3:   Initialize inversion count array  $inversion\_count$  of size  $n$  with all values 0
4:   for  $i \leftarrow 0$  to  $n - 1$  do
5:     Decrement  $A[i]$  by 1 to make it 0-based
6:   end for
7:   for  $i \leftarrow 0$  to  $n - 1$  do
8:      $inversions\_with\_A_i \leftarrow st.query(A[i] + 1, n - 1)$ 
9:      $inversion\_count[i] \leftarrow (i > 0 ? inversion\_count[i - 1] : 0) + inversions\_with\_A_i$ 
10:     $st.update(A[i])$ 
11:  end for
12:  for  $i \leftarrow 0$  to  $n - 1$  do
13:    Output  $inversion\_count[i]$ 
14:  end for
15: end procedure
16: procedure UPDATE( $node, l, r, index$ )
17:   if  $l = r$  then
18:     Increment  $tree[node]$  by 1
19:   return
20:   end if
21:    $mid \leftarrow \frac{l+r}{2}$ 
22:   if  $index \leq mid$  then
23:     Update left child  $2 \times node$ 
24:   else
25:     Update right child  $2 \times node + 1$ 
26:   end if
27:    $tree[node] \leftarrow tree[2 \times node] + tree[2 \times node + 1]$ 
28: end procedure
29: procedure QUERY( $node, l, r, ql, qr$ )
30:   if  $ql > r$  or  $qr < l$  then
31:     return 0
32:   end if
33:   if  $ql \leq l$  and  $qr \geq r$  then
34:     return  $tree[node]$ 
35:   end if
36:    $mid \leftarrow \frac{l+r}{2}$ 
37:   return Query left child + Query right child
38: end procedure
```

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Since we process each element of the array (total  $n$  elements), the time complexity for processing all elements is:

$$O(n \log n)$$

This is due to performing  $O(\log n)$  operations for each of the  $n$  elements.

# Coding Problem Record

Yuchen:

## 3334. Find the Maximum Factor Score of Array Solved

Medium Topics Companies Hint

You are given an integer array `nums`.

The **factor score** of an array is defined as the *product* of the LCM and GCD of all elements of that array.

Return the **maximum factor score** of `nums` after removing **at most** one element from it.

**Note** that both the **LCM** and **GCD** of a single number are the number itself, and the **factor score** of an **empty** array is 0.

**Example 1:**

Input: `nums = [2,4,8,16]`

65 8 10

Submissions

Status	Language	Runtime	Memory	Notes
Accepted	Python3	47 ms	16.8 MB	
Oct 31, 2024				

## 621. Task Scheduler Solved

Medium Topics Companies Hint

You are given an array of CPU `tasks`, each labeled with a letter from A to Z, and a number `n`. Each CPU interval can be idle or allow the completion of one task. Tasks can be completed in any order, but there's a constraint: there has to be a gap of **at least** `n` intervals between two tasks with the same label.

Return the **minimum** number of CPU intervals required to complete all tasks.

**Example 1:**

Input: `tasks = ["A","A","A","B","B","B"], n = 2`

Output: 8

10.7K 144 10

Submissions

Status	Language	Runtime	Memory	Notes
Accepted	Python3	432 ms	18.5 MB	
Oct 25, 2024				

## 295. Find Median from Data Stream Solved

Hard Topics Companies

The **median** is the middle value in an ordered integer list. If the size of the list is even, there is no middle value, and the median is the mean of the two middle values.

- For example, for `arr = [2,3,4]`, the median is 3.
- For example, for `arr = [2,3]`, the median is  $(2 + 3) / 2 = 2.5$ .

Implement the MedianFinder class:

- `MedianFinder()` initializes the `MedianFinder` object.
- `void addNum(int num)` adds the integer `num` from the data stream to the data structure.
- `double findMedian()` returns the median of all elements so far. Answers within  $10^{-5}$  of the actual answer will be accepted.

12.1K 80 10

Submissions

Status	Language	Runtime	Memory	Notes
Accepted	Python3	128 ms	38.7 MB	
Oct 25, 2024				

## 654. Maximum Binary Tree Solved

Medium Topics Companies

You are given an integer array `nums` with no duplicates. A **maximum binary tree** can be built recursively from `nums` using the following algorithm:

- Create a root node whose value is the maximum value in `nums`.
- Recursively build the left subtree on the **subarray prefix** to the **left** of the maximum value.
- Recursively build the right subtree on the **subarray suffix** to the **right** of the maximum value.

Return the **maximum binary tree** built from `nums`.

**Example 1:**

5.3K 26 10

Submissions

Status	Language	Runtime	Memory	Notes
Accepted	Python3	23 ms	17 MB	
Oct 24, 2024				

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---

### Description

## 200. Number of Islands

[View Question](#) | [Topics](#) | [Comments](#)

Given an  $n \times m$ , 2D binary grid ( $\text{grid}$ ) which represents a map of  $'1'$ s (land) and  $'0'$ s (water). Return the number of islands.

An island is surrounded by water and is formed by connecting adjacent lands horizontally or vertically. You may assume all four edges of the grid are all surrounded by water.

```
Example 1:
Input: grid = [
  ["1","1","1","1","0"],
  ["1","1","0","1","0"],
  ["1","1","0","1","0"],
  ["1","1","1","1","0"],
  ["1","1","0","1","0"]
]
Output: 1

Example 2:
Input: grid = [
  ["1","1","0","0","0"],
  ["1","1","0","0","0"],
  ["0","0","0","0","0"]
]
```

Solved ✔
Python ▶ Auto

```

1 class Solution(object):
2     def numIslands(self, grid):
3         """
4         :type grid: List[List[str]]
5         :rtype: int
6         """
7         if not grid:
8             return 0
9
10        def dfs(i, j):
11            # If the current cell is not of lands or water ("0"), skip the DFS
12            if i <= 0 or i >= len(grid) or j <= 0 or j >= len(grid[0]) or grid[i][j] == "0":
13                return
14            # Mark the current cell as visited by setting it to "0"
15            grid[i][j] = "0"
16            # Visit all adjacent cells (up, down, left, right)
17            dfs(i - 1, j)
18            dfs(i + 1, j)
19            dfs(i, j - 1)
20            dfs(i, j + 1)
21
22        count = 0
23        # Traverse the entire grid
24        for i in range(len(grid)):
25            for j in range(len(grid[0])):
26                # Count the number of islands by checking whether there is land at the current location

```

[Editorial](#) | [Solutions](#)

>> Test Results
Testcase
Submissions

Status ▼
Language ▶ Runtime
Memoiry
Notes

Accepted
Python
217 ms
27 MB

**Problem List < |> |>**

### Description

#### 796. Rotate String

[View](#) [Topics](#) [Companies](#)

Given two strings `s` and `t`, return `true` if `s` is equal to `t` after some number of shifts on `s`.

A shift on `s` consists of moving the leftmost character of `s` to the rightmost position.

- For example, if `s = "abcde"`, then it will be "`bcdea`" after one shift.

**Example 1:**

```
Input: s = "abcde", goal = "cdeab"
Output: true
```

**Example 2:**

```
Input: s = "abcde", goal = "abced"
Output: false
```

**Constraints:**

- `1 <= s.length <= 100`
- `t.length == s.length`
- `s` and `t` consist of lowercase English letters only.

---

**Solution**

3. Test Result: [Testcase](#) [Submissions](#)

Status: Language: Runtime Memory Notes

Accepted  
Nov 10, 2024 Python 11.6 MB

```
Python 3 - Auto
1 class Solution(object):
2     def rotateString(self, s, goal):
3         """
4         :type s: str
5         :type goal: str
6         :rtype: bool
7         """
8         if len(s) != len(goal):
9             return False
10        # concatenate s with itself
11        double_s = s + s
12
13        # Check if goal is a substring
14        return goal in double_s
```

13

Yuheng:

Description

Accepted

Editorial

Solutions

Submissions

164. Maximum Gap

Solved

Medium

Topics

Companies

Given an integer array `nums`, return the *maximum difference between two successive elements in its sorted form*. If the array contains less than two elements, return `0`.

You must write an algorithm that runs in linear time and uses linear extra space.

Example 1:

274. H-Index

Solved

Medium

Topics

Companies

Hint

Given an array of integers `citations` where `citations[i]` is the number of citations a researcher received for their  $i^{\text{th}}$  paper, return the *researcher's h-index*.

According to the [definition of h-index on Wikipedia](#): The h-index is defined as the maximum value of  $h$  such that the given researcher has published at least  $h$  papers that have each been cited at least  $h$  times.

Example 1:

Input: citations = [3,0,6,1,5]

Output: 3

Explanation: [3,0,6,1,5] means the researcher has 5 papers in total and each of them had received 3, 0,

## 264. Ugly Number II

Medium Topics Companies Hint

An **ugly number** is a positive integer whose prime factors are limited to `2`, `3`, and `5`.

Given an integer `n`, return the  $n^{\text{th}}$  *ugly number*.

Example 1:

## 410. Split Array Largest Sum

Hard Topics Companies

Given an integer array `nums` and an integer `k`, split `nums` into  $k$  non-empty **minimized**.

Return the *minimized largest sum of the split*.

A **subarray** is a contiguous part of the array.

Example 1:

Input: nums = [7,2,5,10,8], k = 2  
Output: 18

Peng Yao:

218. The Skyline Problem

Solved

Hard Topics Companies

A city's **skyline** is the outer contour of the silhouette formed by all the buildings in that city when viewed from a distance. Given the locations and heights of all the buildings, return the **skyline** formed by these buildings collectively.

The geometric information of each building is given in the array `buildings` where `buildings[i] = [lefti, righti, heighti]`:

- `lefti` is the x coordinate of the left edge of the *i*<sup>th</sup> building.
- `righti` is the x coordinate of the right edge of the *i*<sup>th</sup> building.
- `heighti` is the height of the *i*<sup>th</sup> building.

6K 30 19.7 MB

Submissions

Status Language Runtime Memory Notes

Accepted C++ 10 ms 19.7 MB

a few seconds ago

315. Count of Smaller Numbers After Self

Hard Topics Companies

Given an integer array `nums`, return an integer array `counts` where `counts[i]` is the number of smaller elements to the right of `nums[i]`.

8.9K 35 248.5 MB

Submissions

Status Language Runtime Memory Notes

Accepted C++ 368 ms 248.5 MB

a few seconds ago

327. Count of Range Sum

Solved

Hard Topics Companies

Given an integer array `nums` and two integers `lower` and `upper`, return the number of range sums that lie in `[lower, upper]` inclusive.

Range sum `S(i, j)` is defined as the sum of the elements in `nums` between indices `i` and `j` inclusive, where `i <= j`.

2.3K 18 206.1 MB

Submissions

Status Language Runtime Memory Notes

Accepted C++ 313 ms 206.1 MB

a few seconds ago

493. Reverse Pairs

Solved

Hard Topics Companies Hint

Given an integer array `nums`, return the number of reverse pairs in the array.

A reverse pair is a pair `(i, j)` where:

- `0 <= i < j < nums.length` and
- `nums[i] > 2 * nums[j]`.

6.3K 59 109.5 MB

Submissions

Status Language Runtime Memory Notes

Accepted C++ 236 ms 109.5 MB

a few seconds ago

Bowen Yu:

**76. Minimum Window Substring** Solved

Given two strings  $s$  and  $t$  of lengths  $m$  and  $n$  respectively, return the **minimum window substring** of  $s$  such that every character in  $t$  (including duplicates) is included in the window. If there is no such substring, return the empty string  $""$ .

The testcases will be generated such that the answer is **unique**.

**Example 1:**

Input:  $s = "ADOBECODEBANC"$ ,  $t = "ABC"$

Accepted 18.2K 208 45.1 MB

**Submissions**

Status	Language	Runtime	Memory	Notes
Accepted	Java	99 ms	45.1 MB	

**Code**

```
1 class Solution {
2     public String minWindow(String s, String t) {
3         int[] mapT = new int[128];
4         int[] mapS = new int[128];
5         StringBuilder sb = new StringBuilder();
6         String ans = "";
7         char[] cs = s.toCharArray();
8         char[] ct = t.toCharArray();
9         for(char c:ct){
10             mapT[c]++;
11         }
12         int cntT = 0;
13         for(int i=0; i<cs.length; i++){
14             if(mapS[cs[i]]<mapT[cs[i]]){
15                 cntT++;
16             }
17             SetCharacter set = new HashSet<>();
18             boolean bl = false;
19             for(int right = 0; right<cs.length; right++){
20                 char cright = cs[right];
21                 str.append(cright);
22                 mapS[cright]++;
23                 if(mapT[cright]<=mapS[cright]){mapT[cright]}{
```

**1838. Frequency of the Most Frequent Element** Solved

The **frequency** of an element is the number of times it occurs in an array.

You are given an integer array  $nums$  and an integer  $k$ . In one operation, you can choose an index of  $nums$  and increment the element at that index by 1.

Return the **maximum possible frequency** of an element after performing at **most**  $k$  operations.

Accepted 4.8K 124 56.3 MB

**Submissions**

Status	Language	Runtime	Memory	Notes
Accepted	Java	28 ms	56.3 MB	

**Code**

```
1 class Solution {
2     public int maxFrequency(int[] nums, int k) {
3         Arrays.sort(nums);
4         int left = 0;
5         int ans = 0;
6         long sum = 0;
7         for(int right = 0; right<nums.length; right++){
8             sum += (long)nums[right]*nums[right-1];
9             while(sum>k){
10                 sum -= (long)nums[left]*nums[left];
11                 left++;
12             }
13             ans = Math.max(ans, right-left+1);
14         }
15         return ans;
16     }
17 }
```

**2516. Take K of Each Character From Left and Right** Solved

You are given a string  $s$  consisting of the characters  $'a'$ ,  $'b'$ , and  $'c'$  and a non-negative integer  $k$ . Each minute, you may take either the **leftmost** character of  $s$ , or the **rightmost** character of  $s$ .

Return the **minimum number of minutes** needed for you to take **at least**  $k$  of each character, or return  $-1$  if it is not possible to take  $k$  of each character.

**Example 1:**

Input:  $s = "aabbaaacaabc"$ ,  $k = 3$

Accepted 721 9 ms 45.4 MB

**Submissions**

Status	Language	Runtime	Memory	Notes
Accepted	Java	9 ms	45.4 MB	

**Code**

```
1 int[] has = new int[3];
2 int left = 0;
3 int ans = 0;
4 char[] cs = s.toCharArray();
5 for(int right=cs.length-1; right>=0; right--){
6     has[cs[right]-'a']++;
7     if(max[0]<has[0]||max[1]<has[1]||max[2]<has[2]){
8         return -1;
9     }
10     for(int right=cs.length-1; right>=0; right--){
11         int curIndex = cs[right]-'a';
12         has[curIndex]++;
13         while(has[curIndex]>max[curIndex]){
14             int leftIndex = cs[left]-'a';
15             has[leftIndex]--;
16             left++;
17         }
18         ans = Math.max(ans, right-left+1);
19     }
20     return cs.length-ans;
21 }
```

**2831. Find the Longest Equal Subarray** Solved

You are given a **0-indexed** integer array  $nums$  and an integer  $k$ .

A subarray is called **equal** if all of its elements are equal. Note that the empty subarray is an equal subarray.

Return the length of the **longest possible equal subarray** after deleting at **most**  $k$  elements from  $nums$ .

A **subarray** is a contiguous, possibly empty sequence of elements within an array.

Accepted 686 11 65.1 MB

**Submissions**

Status	Language	Runtime	Memory	Notes
Accepted	Java	54 ms	65.1 MB	

**Code**

```
1 for(int i = 0; i<nums.length; i++){
2     posList.add(new ArrayList<>());
3 }
4 for(int i = 0; i<nums.length; i++){
5     int x = nums.get(i);
6     posList.get(x).add(i);
7 }
8 int ans = 0;
9 for(List<Integer> posList: posList){
10     if(posList.size()>ans){
11         continue;
12     }
13     int left = 0;
14     for(int right = 0; right<posList.size(); right++){
15         while(posList.get(right)>posList.get(left)+k){
16             left++;
17         }
18         ans = Math.max(ans, right-left+1);
19     }
20 }
21 return ans;
22 }
```