Part 1: The Integral Key

Unlock the six-character key $K = C_1C_2C_3C_4C_5C_6$. Each character C_n is determined by the corresponding integral I_n.

Character C_1 (Uppercase English Letter)

Let $I_1 = \int_0^1 \int_0^1 (x+y)^2 dxdy$. Then $C_1 = \text{Letter}\left(\left\lfloor\left(I_1 + \frac{5}{6}\right) \cdot 3 + 0.5\right\rfloor\right)$.

Character C_2 (Arabic Digit 0-9)

Let $I_2 = \int_0^\infty \frac{dx}{(x+1)\sqrt{x}}$. Then $C_2 = \lfloor I_2^2 \rfloor \pmod{10}$.

Character C_3 (Uppercase English Letter)

Let $I_3 = \int_0^1 \arcsin(x)dx$. Then $C_3 = \text{Letter}\left(\left\lfloor\left(I_3 + 2 - \frac{\pi}{2}\right) \cdot 10 + 0.5\right\rfloor\right)$.

Character C_4 (Arabic Digit 0-9)

Let $I_4 = \frac{1}{\pi}\int_{-\infty}^\infty \frac{\sin^2(x)}{x^2}dx$. Then $C_4 = (I_4 + 4)^2 \pmod{10}$.

Character C_5 (Uppercase English Letter)

Let $I_5 = \int_0^\infty x^2 e^{-x} dx$. Then $C_5 = \text{Letter}\left((I_5+1)^2 + 2\right)$.

Character C_6 (Arabic Digit 0-9)

Let $I_6 = \int_0^{\ln(2)} \frac{e^x - 1}{e^x + 1}dx$. Then $C_6 = \lfloor e^{I_6} \cdot 8 \rfloor \pmod{10}$.

Notes for Part 1:

- $\lfloor \cdot \rfloor$ denotes the floor function.
- Letter(n) refers to the n-th uppercase letter of the English alphabet (e.g., A=1, B=2, ..., Z=26).
- The digits C_2, C_4, C_6 should be Arabic numerals from 0 to 9.
- $\pmod{10}$ denotes the modulo 10 operation.

Part 2: Extension of the Integral Key

After successfully finding the key $K = C_1C_2C_3C_4C_5C_6$, you discover a hidden compartment. It requires an advanced three-character key $K' = C_7C_8C_9$ to unlock. The instructions are below, referencing your previously calculated results (i.e., $I_1 = 7/6, I_2 = \pi, I_3 = \pi/2 - 1, I_4 = 1, I_5 = 2, I_6 = \ln(9/8)$ and $C_1 = F$ (numerical value 6), $C_2 = 9, C_3 = J$ (numerical value 10), $C_4 = 5, C_5 = K$ (numerical value 11), $C_6 = 9$):

Character C_7 (Uppercase English Letter)

Let $I_7 = \int_0^1 \frac{\ln(1+x)}{1+x^2} dx$. Let $N_1 = \text{Value}(C_1)$ be the numerical value of C_1 from the previous key (A=1, B=2, ...). Then $C_7 = \text{Letter}\left(\left\lfloor \frac{I_7}{\pi \ln \sqrt[N_1]{2}} \cdot 24 + \frac{C_2}{30} \right\rfloor\right)$. (Here C_2 is the digit from the previous key).

Character C_8 (Arabic Digit 0-9)

Let $f(t) = I_5 t^2 + I_4 t + C_6$, where I_4, I_5 are the integral values from the previous puzzle, and C_6 is the digit from the previous key. Let $I_8 = \int_0^\infty \frac{1}{f(t)} dt$. Then $C_8 = \left\lfloor \frac{I_8 \cdot \sqrt{4I_5C_6 - I_4^2}}{2\pi} \cdot (C_4 + C_2 - 1) \right\rfloor \pmod{10}$. (Here C_2, C_4 are digits from the previous key). (Note: The formula for C_8 was slightly modified to ensure its reasonableness. The originally derived factor was $\sqrt{I_5C_6 - I_4^2/4}$, which equals $\frac{1}{2}\sqrt{4I_5C_6 - I_4^2}$. The term $(C_4 + C_2)$ was changed to $(C_4 + C_2 - 1)$ to adjust the final value.)

Character C_9 (Uppercase English Letter)

Let $I_9(a) = \int_0^\infty e^{-x} \sin(ax) dx$. Let $I_{9A} = I_9(\text{Value}(C_3))$ and $I_{9B} = I_9(\text{Value}(C_5))$, where C_3, C_5 are letters from the previous key. Then $C_9 = \text{Letter}\left(\left\lfloor (I_{9A} \cdot \text{Value}(C_3) + I_{9B} \cdot \text{Value}(C_5)) \cdot I_1 +\right.\right.$ If the result of the floor function modulo 26 is 0, then take Z (which corresponds to 26). (Here I_1, I_6 are integral values from the previous puzzle. 0.5 was added to fine-tune the result).

Notes for Part 2:

- $\lfloor \cdot \rfloor$ denotes the floor function.
- $\text{Letter}(n)$ refers to the n-th uppercase letter of the English alphabet (e.g., A=1, B=2, ..., Z=26).
- $\text{Value}(L)$ refers to the numerical value of the letter L (A=1, ..., Z=26).
- $\pmod{m}$ denotes the modulo m operation.

- All previous integral values and character values are as initially given in the problem or as you have already calculated.