M2
Continuous Optimization
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Projects

In a region Ω of $\mathbb{R}^2$ consider K antennas at coordinates $a_k \in \mathbb{R}^2, 1 \leq k \leq K$. At a point $x \in \mathbb{R}^2$ assume that the strength of the signal received from antenna k is

$$1/(1 + \|x - a_k\|^2).$$

Fix T "target locations" $b_1, ..., b_T \in \mathbb{R}^2$. We want all targets to be "well covered by the antennas" : the goal is to find the best place to install a *new* antenna, so that the minimal signal strength among all target locations is the highest possible.

1. Write a code for a function `signal_strength` that, given a and x, computes the strength of signal coming from a at position x.
2. Write code for a function `total_signal_strength` that, given all the locations of the antennas, and a location x, computes the total signal strength at x (sum of the strength of all signals coming from all antennas)
3. Plot (the contour lines of) `total_signal_strength`.
4. Write code for `min_strength_at_targets` that, given all the locations of the antennas and the targets, computes the minimal signal received among all targets.
5. Write down the optimization problem to solve. Is it smooth ?
6. Show that it may be rewritten as a smooth optimization problem by adding an auxiliary scalar variable z (take inspiration from the paragraph on smoothness from Nocedal & Wright, chapter 12).
7. Using `scipy.optimize`, solve the problem numerically.

For points 3 and 7, start by simple examples with few targets and few antennas in a square region, before scaling up.