Mathematik 1 – AI

Blatt 3

A21 a) i) $e^{1+i} = e^1 \cdot e^i = e(\cos(1) + i\sin(1)) =$

$= e \cdot \cos(1) + ie\sin(1) \approx \underline{\underline{1{,}47 + 2{,}29i}}$

ii) $e^{\ln 5 + i\frac{3}{4}\pi} = e^{\ln 5} \cdot e^{i\frac{3}{4}\pi} = 5 \cdot \left(\left(\cos\left(\tfrac{3}{4}\pi\right) + i\sin\left(\tfrac{3}{4}\pi\right)\right)\right) =$

$= 5\left(-\tfrac{1}{2}\sqrt{2}\right) + i5 \cdot \tfrac{1}{2}\sqrt{2} = \underline{\underline{-\tfrac{5}{2}\sqrt{2} + \tfrac{5}{2}\sqrt{2}i}}$

iii) $e^{\sqrt{2} - i\sqrt{2}} = e^{\sqrt{2}} \cdot e^{-i\sqrt{2}} = e^{\sqrt{2}}(\cos(-\sqrt{2}) + i\sin(-\sqrt{2}))$

$= e^{\sqrt{2}}\cos(-\sqrt{2}) + ie^{\sqrt{2}}\sin(-\sqrt{2}) \approx \underline{\underline{0{,}64 - 4{,}06i}}$

iv) $e^{-2+i\ln 8} = e^{-2} e^{i\ln 8} = e^{-2}(\cos(\ln 8) + i\sin(\ln 8))$

$= e^{-2}\cos(\ln 8) + ie^{-2}\sin(\ln 8)) \approx \underline{\underline{-0{,}07 + 0{,}12i}}$

v) $e^{-i} = \cos(-1) + i\sin(-1) = \cos(1) - i\sin(1)$

$\Rightarrow e^{e^{-i}} = e^{\cos(1) - i\sin(1)} = e^{\cos(1)} e^{-i\sin(1)} =$

$= e^{\cos(1)} \cdot (\cos(-\sin(1)) + i\sin(-\sin(1)))$

$= e^{\cos(1)} \cdot \cos(\sin 1) - ie^{\cos 1} \cdot \sin(\sin 1)$

$\approx \underline{\underline{1{,}14 - 1{,}28i}}$

vi) $-e^{-i\pi} = -(-1) = 1$

$\Rightarrow e^{-e^{-i\pi}} = e^1 = e \approx \underline{\underline{2{,}718}}$

A21 b) i) $z = 1 + i$

$\Rightarrow \quad r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\varphi = \operatorname{Arg}(z) = \arccos\left(\frac{x}{r}\right) = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

$\Rightarrow \quad \operatorname{Ln}(z) = \ln(r) + i\operatorname{Arg}(z) = \underline{\underline{\ln(\sqrt{2}) + i\frac{\pi}{4}}} \approx 0{,}35 + 0{,}79i$

ii) $z = \ln(2) + i\frac{5\pi}{6}$

$\Rightarrow \quad r = |z| = \sqrt{(\ln 2)^2 + \left(\frac{5\pi}{6}\right)^2} \approx 2{,}71$

$\varphi = \arccos\left(\frac{x}{r}\right) = \arccos\left(\frac{\ln 2}{2{,}71}\right) \approx 1{,}31$

$\Rightarrow \quad \operatorname{Ln}(z) = \ln(r) + i\operatorname{Arg}(z) = \ln(2{,}71) + i\,1{,}31 \approx \underline{\underline{1{,}00 + 1{,}31i}}$

iii) $z = -\frac{\sqrt{3}}{2} + \frac{i}{2}$

$\Rightarrow \quad r = |z| = \sqrt{\left(\sqrt{3}/2\right)^2 + \left(1/2\right)^2} = \frac{1}{2}\sqrt{3+1} = 1$

$\varphi = \arccos\left(\frac{x}{r}\right) = \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5}{6}\pi$

$\Rightarrow \quad \operatorname{Ln}(z) = \ln(r) + i\operatorname{Arg}(z) = \underbrace{\ln(1)}_{0} + i\,\frac{5}{6}\pi = \underline{\underline{\frac{5}{6}\pi i}}$

iv) $z = 3e^{i\frac{1}{12}\pi} \Rightarrow r = 3, \; \varphi = \pi/12$

$\Rightarrow \quad \operatorname{Ln}(z) = \ln 3 + i\frac{\pi}{12} \approx \underline{\underline{1{,}10 + 0{,}26i}}$

v) $z = -3e^{i\frac{1}{12}\pi} = 3 \cdot e^{i\pi/12} \cdot e^{i\pi} = 3e^{i\overset{>\pi}{\frac{13}{12}\pi}} = 3 \cdot e^{i\frac{13}{12}\pi - i2\pi} =$

$= 3e^{-i\frac{11}{12}\pi} \quad \left(\operatorname{Arg}(z) = -\frac{11}{12}\pi\text{; beachte } -\pi < \operatorname{Arg}(z) \leq \pi\right)$

$\Rightarrow \quad \operatorname{Ln}(z) = \ln(r) + i\operatorname{Arg}(z) = \underline{\underline{\ln(3) - i\frac{11}{12}\pi}} \approx 1{,}10 - 2{,}88i$

vi) $z = e^{-2}e^{i\overset{>\pi}{\frac{9}{5}\pi}} = e^{-2}e^{i\,9/5\pi - i2\pi} = e^{-2}e^{-i\pi/5}$

$\Rightarrow \quad r = |z| = e^{-2}, \quad \varphi = \operatorname{Arg}(z) = -\pi/5$

$\Rightarrow \quad \operatorname{Ln}(z) = \ln|z| + i\operatorname{Arg}(z) = \ln(e^{-2}) - i\pi/5 = \underline{\underline{-2 - i\pi/5}}$

A22 $\boxed{e^z = w \iff z = \mathrm{Ln}(w) + i2\pi k \quad (k \in \mathbb{Z})}$

a) $2e^z = \sqrt{3} - i \iff e^z = \frac{1}{2}\sqrt{3} - \frac{1}{2} i$

$\iff z = \mathrm{Ln}(\frac{1}{2}\sqrt{3} - \frac{1}{2} i) + i2\pi k \quad (k \in \mathbb{Z})$

Bestimmung von $\mathrm{Ln}(\frac{1}{2}\sqrt{3} - \frac{1}{2} i)$:

$w = \frac{1}{2}\sqrt{3} - \frac{1}{2} i$

$r = |w| = \sqrt{(\frac{1}{2}\sqrt{3})^2 + (\frac{1}{2})^2} = \frac{1}{2}\sqrt{3+1} = 1;$

$\varphi = \mathrm{Arg}(w) = -\arccos(\frac{x}{r}) = -\arccos(\frac{1}{2}\sqrt{3}) = -\frac{\pi}{6}$

$\Rightarrow \mathrm{Ln}(w) = \ln(r) + i\varphi = \underbrace{\ln(1)}_{0} - i\frac{\pi}{6} = -i\frac{\pi}{6}$

$L = \{-i\frac{\pi}{6} + 2\pi i k \mid k \in \mathbb{Z}\}$

b) $3e^{2\mathrm{Ln}(z)} = -2i \quad | :3$

$e^{2\mathrm{Ln}(z)} = -\frac{2}{3} i \qquad 2 \cdot \mathrm{Ln}(z) = \mathrm{Ln}(z^2) + 2\pi i k$ für ein $k \in \mathbb{Z}$

$\iff e^{\mathrm{Ln}(z^2) + 2\pi i k} = -\frac{2}{3} i$

$\iff e^{\mathrm{Ln}(z^2)} \cdot \underbrace{e^{2\pi i k}}_{1} = -\frac{2}{3} i$

$\iff e^{\mathrm{Ln}(z^2)} = -\frac{2}{3} i$

$\iff z^2 = \underbrace{-\frac{2}{3} i}_{w}$

Die Lösungen sind die (komplexen) Quadratwurzeln von

$w = -\frac{2}{3} i = \frac{2}{3} \cdot (-i) = \frac{2}{3} \cdot e^{-i\frac{\pi}{2}}$

$r = |w| = \frac{2}{3}, \quad \varphi = \mathrm{Arg}(w) = -\frac{\pi}{2}$

$\Rightarrow w = \frac{2}{3} e^{-i\frac{\pi}{2}}$

2-te Wurzeln: $z_0 = \sqrt{\frac{2}{3}} e^{-i\frac{\pi}{4}}, \quad z_1 = \sqrt{\frac{2}{3}} e^{-i\frac{\pi}{4} + i\pi} = \sqrt{\frac{2}{3}} e^{i\frac{3}{4}\pi}$

$\Rightarrow L = \{\sqrt{\frac{2}{3}} e^{-i\frac{\pi}{4}}, \sqrt{\frac{2}{3}} e^{i\frac{3}{4}\pi}\}$

A22

c) $e^{2z} - e^z + 1 = 0$

$\Leftrightarrow (e^z)^2 - e^z + 1 = 0$

Subst.: $w = e^z$

$w^2 - w + 1 = 0$

$\Rightarrow \quad w_{1/2} = \frac{+1 \pm \sqrt{1^2 - 4}}{2} = \frac{1}{2} \pm \frac{i}{2}\sqrt{3}$

Resubst.:

1). $e^z = w_1 \Leftrightarrow z = \mathrm{Ln}(w_1) + 2\pi i k \quad (k \in \mathbb{Z})$

$w_1 = \frac{1}{2} + \frac{i}{2}\sqrt{3}$

$r = |w_1| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{2}\sqrt{1+3} = 1$

$\varphi = \mathrm{Arg}(w_1) = +\arccos\left(\frac{x}{r}\right) = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\Rightarrow \quad w_1 = e^{i\pi/3}$

$\Rightarrow \quad \mathrm{Ln}(w_1) = \ln(r) + i\varphi = \underbrace{\ln(1)}_{0} + i\frac{\pi}{3} = i\frac{\pi}{3}$

$L_1 = \{\mathrm{Ln}(w_1) + i2\pi k \mid k \in \mathbb{Z}\} = \{i\frac{\pi}{3} + i2\pi k \mid k \in \mathbb{Z}\}$

2) $e^z = w_2 \Leftrightarrow z = \mathrm{Ln}(w_2) + 2\pi i k \quad (k \in \mathbb{Z})$

$w_2 = \frac{1}{2} - \frac{i}{2}\sqrt{3}$

$r = |w_2| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{2}\sqrt{1^2+3} = 1$

$\varphi = \mathrm{Arg}(w_2) = -\arccos\left(\frac{x}{r}\right) = -\arccos\left(\frac{1}{2}\right) = -\frac{\pi}{3}$

$\Rightarrow \quad w_2 = e^{-i\pi/3}$

$\Rightarrow \quad \mathrm{Ln}(w_2) = \ln(r) + i\varphi = \underbrace{\ln(1)}_{0} - i\frac{\pi}{3} = -i\frac{\pi}{3}$

$L_2 = \{\mathrm{Ln}(w_2) + i2\pi k \mid k \in \mathbb{Z}\} = \{-i\frac{\pi}{3} + i2\pi k \mid k \in \mathbb{Z}\}$

$\Rightarrow L = L_1 \cup L_2 = \{i\frac{\pi}{3} + i2\pi k \mid k \in \mathbb{Z}\} \cup \{-i\frac{\pi}{3} + i2\pi k \mid k \in \mathbb{Z}\}$

A23 a, $A(t) = 2\sin(0{,}1\,s^{-1} \cdot t - \pi/2)$

$= A_0 \sin(\omega t + \alpha_0)$

mit $A_0 = 2$, $\omega = 0{,}1\,s^{-1}$, $\alpha_0 = -\pi/2$

komplexe Form:

(o.E.) $\underline{A}(t) = \underline{A}_0 e^{i\omega t}$ mit $\underline{A}_0 = A_0 e^{i\alpha_0}$

ohne Einheiten

$= 2e^{-i\pi/2} e^{i0{,}1t} = 2e^{i(0{,}1t - \pi/2)}$

$B(t) = 6{,}0 \cdot 10^{-3} \sin(10\,s^{-1} \cdot t - \pi/8)$

$B_0 = 6{,}0 \cdot 10^{-3}$, $\omega = 10\,s^{-1}$, $\beta_0 = -\pi/8$

komplexe Form: (o.E.)

$\underline{B}(t) = \underline{B}_0 e^{i\omega t}$ mit $\underline{B}_0 = B_0 e^{i\beta_0}$

$= 6{,}0 \cdot 10^{-3} e^{-i\frac{\pi}{8}} e^{i10t} =$

$= 6{,}0 \cdot 10^{-3} e^{i(10t - \pi/3)}$

b, $\underline{A}(t) = (-1+i) e^{\frac{i}{50} s^{-1} \cdot t} = \underline{A}_0 e^{i\omega t}$

mit $\underline{A}_0 = -1 + i$, $\omega = \frac{1}{50} s^{-1}$

$r = |\underline{A}_0| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\varphi = \mathrm{Arg}(\underline{A}_0) = \arccos\left(\frac{x}{r}\right) = \arccos\left(\frac{-1}{\sqrt{2}}\right) = \frac{3}{4}\pi$

$\Rightarrow \underline{A}_0 = \sqrt{2} e^{i\frac{3}{4}\pi}$ $\Rightarrow$ reelle Amplitude $A_0 = \sqrt{2}$

Phase $\alpha_0 = \frac{3}{4}\pi$

reelle Form: (o.E.)

$A(t) = A_0 \sin(\omega t + \alpha_0) = \sqrt{2} \sin\left(\frac{1}{50} t + \frac{3}{4}\pi\right)$

A23

zu b) $\underline{B}(t) = -32 e^{-i\frac{\pi}{8}} e^{i 20 s^{-1} \cdot t} = \underline{B}_o e^{i\omega t}$

mit $\underline{B}_o = -32 e^{-i\pi/8}$, $\omega = 20 s^{-1}$

$\underline{B}_o = -32 e^{-i\pi/8} = 32 e^{-i\pi/8} e^{i\pi} = 32 e^{i\frac{7}{8}\pi}$

$\Rightarrow r = |\underline{B}_o| = 32$, $\varphi = Arg(\underline{B}_o) = \frac{7}{8}\pi \; (= \beta_o)$

reelle Form: (o.E.)

$B_o(t) = B_o \sin(\omega t + \beta_o) = 32 \sin(20t + \frac{7}{8}\pi)$

A24 a) $T = 25s$, $A_o = 2cm$, $\alpha_o = -1{,}5\pi$

$\omega = \frac{2\pi}{T}$

reelle Form (o.E.):

$A(t) = A_o \sin(\omega t + \alpha_o) = 2 \sin(\frac{2\pi}{50} t - \frac{3}{2}\pi)$

komplexe Form (o.E.):

$\underline{A}(t) = \underline{A}_o \cdot e^{i\omega t} = 2 \cdot e^{-i\frac{3}{2}\pi} e^{i\frac{2\pi}{50}t}$

b) $f = 85Hz$, $A_o = 1{,}00\,mm$, $\alpha_o = 0{,}8\pi$

$\Rightarrow \omega = 2\pi f = 2\pi 85 s^{-1} = 170\pi s^{-1}$

reelle Form (o.E.)

$A(t) = A_o \cdot \sin(\omega t + \alpha_o) = 1{,}0 \cdot \sin(170\pi \cdot t + 0{,}8\pi)$

komplexe Form (o.E.):

$\underline{A}(t) = \underline{A}_o \cdot e^{i\omega t} = 1{,}0 \cdot e^{i 0{,}8\pi} \cdot e^{i 170\pi \cdot t}$

A25 $u_1(t) = \hat{u}_1 \cdot \sin(\omega t) = 100V \cdot \sin(314 s^{-1} t)$

$u_2(t) = \hat{u}_2 \cdot \sin(\omega t + \alpha_2) = 200V \cdot \sin(314 s^{-1} t + \frac{5}{6}\pi)$

Ges.: Amplitude und Phase der Überlagerung $u(t) = u_1(t) + u_2(t)$

komplexe Darst.: $\underline{u}_1(t) = 100 \cdot e^{i 314 \cdot t}$
(o.E.)
$\underline{u}_2(t) = 200 \cdot e^{i \frac{5}{6}\pi} \cdot e^{i 314 \cdot t}$

Addition der komplexen Darstellungen

$$\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t) = 100 \cdot e^{i 314 \cdot t} + 200 \cdot e^{i \frac{5}{6}\pi} e^{i 314 t}$$

$$= \underbrace{\left(100 + 200 e^{i \frac{5}{6}\pi}\right)}_{\hat{\underline{u}}} e^{i 314 t}$$

$$\hat{\underline{u}} = 100 + 200 \cdot e^{i \frac{5}{6}\pi} = 100 + 200 \left(\cos \tfrac{5}{6}\pi + i \sin \tfrac{5}{6}\pi\right) =$$

$$= 100 + 200 \underbrace{\cos \tfrac{5}{6}\pi}_{-\frac{1}{2}\sqrt{3}} + i\, 200 \underbrace{\sin \tfrac{5}{6}\pi}_{1/2} =$$

$$= 100 - 100\sqrt{3} + i\, 100 = 100 \cdot (1 - \sqrt{3} + i)$$

=>

$$|\hat{\underline{u}}| = 100 \sqrt{(1-\sqrt{3})^2 + 1^2} = 123{,}93$$

$$\mathrm{Arg}(\underline{u}) = + \arccos\left(\frac{x}{r}\right) = \arccos\left(\frac{100 - 100\sqrt{3}}{123{,}93}\right) = 2{,}20$$

$$\Rightarrow \quad \underline{u}(t) = 123{,}93 \cdot e^{i 2{,}20} \cdot e^{i 314 t}$$

=> reelle Darstellung der Überlagerung

$$u(t) = 123{,}93 V \cdot \sin(314 t + 2{,}20)$$

→ reelle Amplitude : 123,93 V , Phase : 2,20

(A26) a) $\neg(\neg A \vee B) \vee A$

$(1 \hat{=} w, \; 0 \hat{=} f)$

A	B	$\neg A \vee B$	$\neg(\neg A \vee B) \vee A$
1	1	1	1
1	0	0	1
0	1	1	0
0	0	1	0

b) $(A \wedge B) \rightarrow \neg(A \vee B)$

A	B	$A \wedge B$	$\neg(A \vee B)$	$(A \wedge B) \rightarrow \neg(A \vee B)$
1	1	1	0	0
1	0	0	0	1
0	1	0	0	1
0	0	0	1	1

c) $\neg(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$

A	B	$\neg(A \rightarrow B)$	$\neg B \rightarrow \neg A$	$\neg(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$
1	1	0	1	0
1	0	1	0	0
0	1	0	1	0
0	0	0	1	0

d) $\neg(A \wedge \neg B \wedge C)$

A	B	C	$A \wedge \neg B$	$\neg(A \wedge \neg B \wedge C)$
1	1	1	0	1
1	1	0	0	1
1	0	1	1	0
1	0	0	1	1
0	1	1	0	1
0	1	0	0	1
0	0	1	0	1
0	0	0	0	1

e) $(A \to \neg B) \leftrightarrow ((A \vee C) \wedge B)$ (*)

A	B	C	$A \to \neg B$	$A \vee C$	$(A \vee C) \wedge B$	(*)
1	1	1	0	1	1	0
1	1	0	0	1	1	0
1	0	1	1	1	0	0
1	0	0	1	1	0	0
0	1	1	1	1	1	1
0	1	0	1	0	0	0
0	0	1	1	1	0	0
0	0	0	1	0	0	0

f) $(A \to B) \wedge (B \to C) \to (A \to C)$ (**)

A	B	C	$A \to B$	$B \to C$	$(A \to B) \wedge (B \to C)$	$A \to C$	(**)
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	0	1	0	1	0	1	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

A27 a) $(A \leftrightarrow B) \leftrightarrow (A \wedge B) \vee (\neg A \wedge \neg B)$ (#)

A	B	$A \leftrightarrow B$	$A \wedge B$	$\neg A \wedge \neg B$	$(A \wedge B) \vee (\neg A \wedge \neg B)$	(#)
1	1	1	1	0	1	1
1	0	0	0	0	0	1
0	1	0	0	0	0	1
0	0	1	0	1	1	1

$\to$ Tautologie

b) $B \land \neg(A \lor B)$

A	B	$A \lor B$	$B \land \neg(A \lor B)$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	0

=> Kontradiktion

c) $(A \to (B \land \neg B)) \to \neg A$ (##)

A	B	$B \land \neg B$	$A \to (B \land \neg B)$	(##)
1	1	0	0	1
1	0	0	0	1
0	1	0	1	1
0	0	0	1	1

=> Tautologie

d) $A \lor (B \land \neg C) \to A \land B \land \neg C$ (###)

A	B	C	$B \land \neg C$	$A \lor (B \land \neg C)$	$A \land B \land \neg C$	(###)
1	1	1	0	1	0	0
1	1	0	1	1	1	1
1	0	1	0	1	0	0
1	0	0	0	1	0	0
0	1	1	0	0	0	1
0	1	0	1	1	0	0
0	0	1	0	0	0	1
0	0	0	0	0	0	1

=> weder Tautologie noch Kontradiktion

Distributiv-G. $\quad 1 \hat{=} w \quad 0 \hat{=} f$

A28 a) $(A \vee B) \wedge (A \vee C \vee \neg B) \equiv$

$$\equiv \underbrace{(A \wedge A)}_{A} \vee (A \wedge C) \vee (A \wedge \neg B) \vee (B \wedge A) \vee (B \wedge C) \vee \underbrace{(B \wedge \neg B)}_{0 \text{ (Kontradiktion)}} \equiv$$

$$\equiv \underbrace{A \vee (A \wedge C)}_{A} \vee \underbrace{(A \wedge \neg B) \vee (A \wedge B)}_{A \wedge \underbrace{(\neg B \vee B)}_{1 \text{ (Tautologie)}}} \vee (B \wedge C) \vee 0 \equiv$$

$(X \vee 0 \equiv X,$
$X \wedge 1 \equiv X)$

$$\equiv A \vee (A \wedge 1) \vee (B \wedge C) \equiv$$

$$\equiv \underbrace{A \vee A}_{A} \vee (B \wedge C) \equiv \underline{\underline{A \vee (B \wedge C)}};$$

b) $\neg (A \vee C) \vee (A \rightarrow B) \equiv (\neg A \wedge \neg C) \vee (\neg A \vee B) \overset{\text{D-Gesetz}}{\equiv}$

$$\equiv [\neg A \vee (\neg A \vee B)] \wedge [\neg C \vee (\neg A \vee B)]$$

$$\equiv (\neg A \vee B) \wedge (\neg A \vee B \vee \neg C) \equiv$$

$$\equiv (\neg A \vee B \vee 0) \wedge (\neg A \vee B \vee \neg C) \overset{\text{D-Gesetz}}{\equiv}$$

$$\equiv (\neg A \vee B) \vee (\underbrace{0 \wedge \neg C}_{0}) = \underline{\underline{\neg A \vee B}};$$

c) $(A \wedge \neg B) \leftrightarrow (B \vee A) \equiv$

$X \equiv Y \Leftrightarrow$
$[(X \wedge Y) \vee (\neg X \wedge \neg Y)]$

$$\equiv [(A \wedge \neg B) \wedge (B \vee A)] \vee [\neg (A \wedge \neg B) \wedge \neg (B \vee A)]$$

$$\equiv [\underbrace{(A \wedge \underbrace{\neg B \wedge B}_{0})}_{0} \vee (A \wedge \neg B \wedge A)] \vee [(\neg A \vee B) \wedge (\neg B \wedge \neg A)] =$$

$$\equiv [0 \vee (A \wedge \neg B)] \vee [(\neg A \wedge \neg B \wedge \neg A) \vee (\underbrace{B \wedge \neg B \wedge \neg A}_{0})]$$

$$\equiv (A \wedge \neg B) \vee [(\neg A \wedge \neg B) \vee 0] \equiv (\underbrace{A \vee \neg A}_{1}) \wedge \neg B = \underline{\underline{\neg B}}$$

d) $(((A \to B) \to A) \to A) \vee B \equiv$

$\equiv (\neg((A \to B) \to A) \vee A) \vee B \equiv$

$\equiv (\neg(\neg(A \to B) \vee A) \vee A) \vee B \equiv$

$\equiv (\neg(\neg(\neg A \vee B) \vee A) \vee A) \vee B \equiv$

$\equiv (\neg((A \wedge \neg B) \wedge \neg A) \vee A) \vee B \equiv$

$\equiv (\neg(\underbrace{\underbrace{A \wedge \neg A}_{0} \wedge \neg B}_{0}) \vee A) \vee B \equiv$

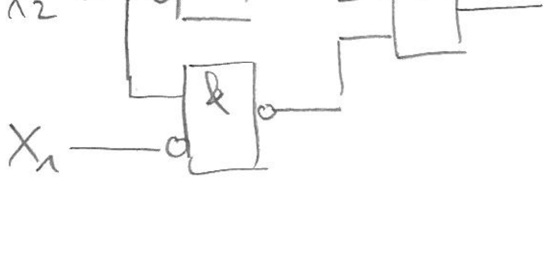
$\equiv \neg(\underbrace{0 \vee A}_{A}) \vee B = \underline{\underline{\neg A \vee B}}$

A29 a) $\neg(\neg X_1 \wedge X_2) \vee \neg(X_1 \vee \neg X_2)$

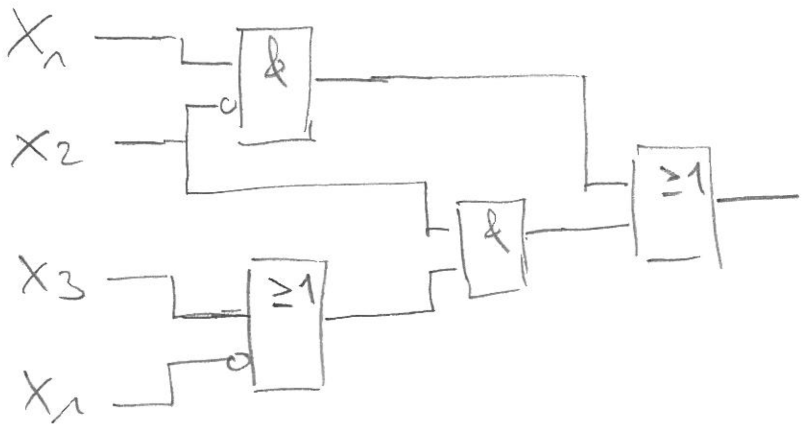
b) $\neg\left[(X_1 \oplus \neg X_2) \wedge (\neg X_1 \vee \neg X_2)\right] \vee \neg(X_1 \wedge X_2)$

c) $\neg\left\{\left[X_1 \vee \neg(X_2 \vee \neg X_3)\right] \vee \left[(\neg X_2 \wedge X_3) \wedge \neg X_1\right]\right\}$

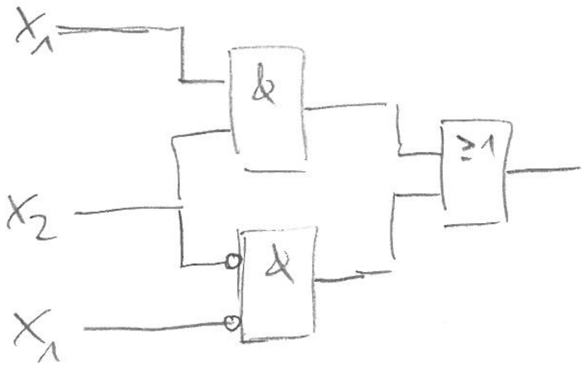
A30 a) $(X_1 \wedge \neg X_2) \vee \neg(\neg X_1 \wedge X_2)$



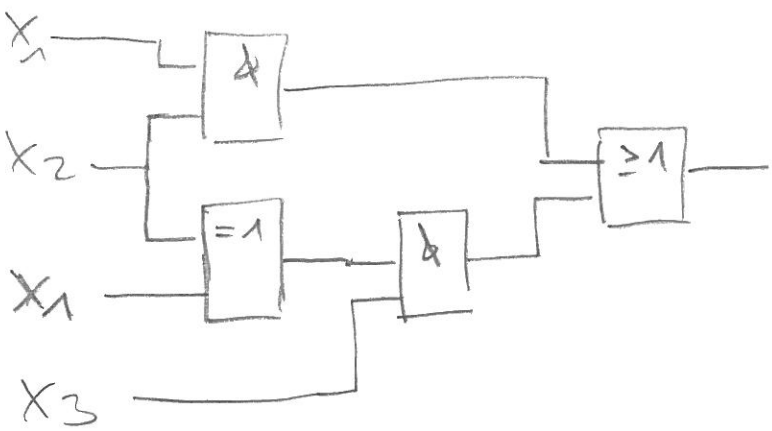
b) $(x_1 \wedge \neg x_2) \vee (x_2 \wedge (x_3 \vee \neg x_1))$



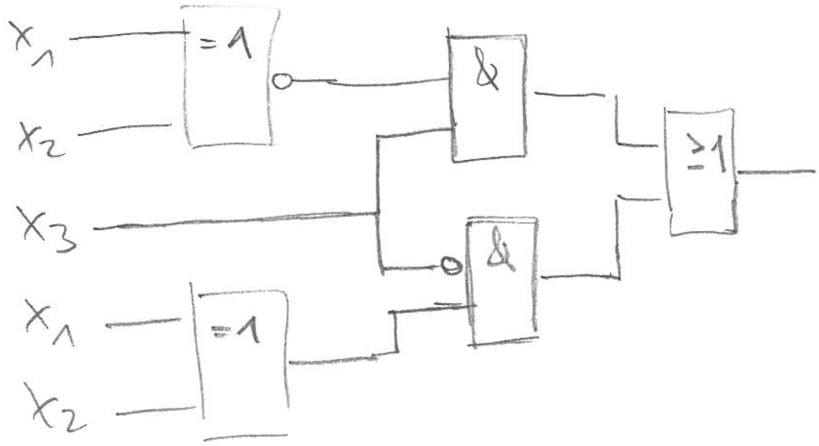
c) $x_1 \leftrightarrow x_2 \; (\equiv (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2))$



d) $(x_1 \wedge x_2) \vee (x_3 \wedge (x_1 \oplus x_2))$



e) $(x_3 \land \neg(x_1 \oplus x_2)) \lor ((x_1 \oplus x_2) \land \neg x_3)$



f) $\neg(\neg x_1 \rightarrow x_2) \equiv \neg(\neg\neg x_1 \lor x_2) \equiv$

$\equiv \neg(x_1 \lor x_2)$