

1 Allgemeines

arithmetisches Mittel: $\bar{a} = \frac{1}{T} \int_{t=0}^T a(t) dt$

Gleichrichtwert: $a_{GL} = |a(t)| = \frac{1}{T} \int_{t=0}^T |a(t)| dt$ Wechselgröße: $\bar{a} = 0$

Effektiv/RMS-wert: $A = a_{eff} = \sqrt{\frac{1}{T} \int_{t=0}^T a(t)^2 dt}$

Scheitel/Crestfaktor: $k_s = \frac{\hat{a}}{a_{eff}}$

Kreisfrequenz: $\omega = 2\pi f = \frac{2\pi}{T}$

uLiCu:

Impedanz: $\bar{Z} = \frac{\hat{u}}{\hat{i}}$; **Resistanz:** $R = \frac{\hat{u}_{//}}{\hat{i}} = Z * \cos \phi_{ui}$; **Reaktanz:** $X = \frac{\hat{u}_{\perp}}{\hat{i}} = Z * \sin \phi_{ui}$;

R-Dreieck: $Z^2 = R^2 + X^2$

iCuLi:

Admintanz: $Y = \frac{\hat{i}}{\hat{u}} = \frac{1}{\bar{Z}}$; **Konduktanz:** $G = \frac{\hat{i}_{//}}{\hat{u}} = Y * \cos \phi_{ui}$;

Suszeptanz: $B = \frac{\hat{i}_{\perp}}{\hat{u}} = -Y * \sin \phi_{ui}$;

Y-Dreieck: $Y^2 = G^2 + B^2$

2 Zweipole

Ohmscher $\phi = 0^\circ$: $Z_R = \frac{1}{Y_R} = \frac{\hat{u}}{\hat{i}} = R$; $R_R = \frac{1}{G_R} = \frac{\hat{u}_{//}}{\hat{i}} = R$; $X_R = \frac{1}{B_R} = \frac{\hat{u}_{\perp}}{\hat{i}} = 0$

Induktiver $\phi = 90^\circ (\mathbf{UvI})$: $Z_L = \frac{1}{Y_L} = \frac{\hat{u}}{\hat{i}} = \omega L$; $R_L = \frac{1}{G_L} = 0$; $X_L = -\frac{1}{B_L} = \omega L$

Kapazitiver $\phi = -90^\circ (\mathbf{IvU})$: $Z_c = \frac{1}{Y_c} = \frac{1}{\omega C}$; $R_c = \frac{1}{G_c} = 0$; $X_c = -\frac{1}{B_c} = -\frac{1}{\omega C}$

3 Schwingkreise

3.1 Reihen

Widerstand: $\underline{Z} = R + j(\omega L - \frac{1}{\omega C})$; $Z_0 = R$; $X_0 = \sqrt{\frac{L}{C}}$; **Resonanzfrequenz:** $\omega_0 = \frac{1}{\sqrt{L*C}}$;

max Strom: $I_{max} = I_0 = \frac{U}{R}$; **Güte:** $Q = \frac{\omega_0 * L}{R} = \frac{1}{\omega_0 * R * C} = \frac{X_0}{R} = \frac{f_0}{B_f} = \frac{1}{R} * \sqrt{\frac{L}{C}} = \frac{f_0}{f_{go} - f_{gu}}$

Spannungserhöhung: $U_{L0} = U_{C0} = Q * U_{R0} = Q * U_q$

Betragsgang: $Z(\omega) = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$; **Phasengang:** $\psi_Z(\omega) = \arctan \frac{\omega L * \frac{1}{\omega C}}{R}$

3.2 Parallel

Leitwert: $\underline{Y} = G + j(\omega C - \frac{1}{\omega L})$; $Y_0 = G$; $B_0 = \sqrt{\frac{C}{L}}$; **Resonanzfrequenz:** $\omega_0 = \frac{1}{\sqrt{L*C}}$;

max SPG: $U_{max} = I_0 = \frac{I}{G}$; **Güte:** $Q = \frac{\omega_0 * C}{G} = \frac{1}{\omega_0 * G * L} = \frac{B_0}{G} = \frac{f_0}{B_f} = \frac{1}{G} * \sqrt{\frac{C}{L}} = \frac{f_0}{f_{go} - f_{gu}}$

Stormerhöhung: $I_{C0} = I_{L0} = Q * I_{G0} = Q * I_q$

Betragsgang: $Y(\omega) = \sqrt{G^2 + (\omega C - \frac{1}{\omega L})^2}$; **Phasengang:** $\psi_Y(\omega) = \arctan \frac{\omega C * \frac{1}{\omega L}}{G}$

3.3 Gemeinsamem Kenngrößen

Grenzfrequenzen: $f_{go/gu} = f_0 * (\sqrt{1 + (\frac{1}{2Q})} \pm \frac{1}{2Q})$; $Q \gg 1 \Rightarrow f_{go/gu} = f_0(1 \pm \frac{1}{2Q}) = f_0 \pm \frac{B_f}{2}$

Dämpfung: $d = \frac{1}{Q}$; **Leistung:** $P_{f_{go/gu}} = \frac{1}{2} P_{f_0}$; **Güte:** $Q = \frac{|Blindleistung(\omega_0)|}{Wirkleistung(\omega_0)}$

$\frac{I(f_g)}{I_{max}} = \frac{U(f_g)}{U_{max} = \frac{1}{\sqrt{2}}}$

4 Komplexe Rechnung $i^2 = -1$

Darstellung

R-Form: $\underline{A} = a_r + i * a_i = \Re(\underline{A}) + i * \Im(\underline{A})$;

P-Form: $\underline{A} = A * e^{i*\alpha}$;

Euler: $e^{i\alpha} = \cos \alpha + i \sin \alpha$

P in R: $a_r = A * \cos \alpha$, $a_i = A * \sin \alpha$;

R in P: $A = \sqrt{a_r^2 + a_i^2}$, $\alpha = \arctan \frac{a_i}{a_r}$

Rechnen

konjugiert Komplex: $\underline{A}^* = A * e^{-i\alpha} = a_r - i * a_i$;

Addition: $\underline{C} = a_r + b_r + i * (a_i + b_i)$;

Subtraktion: $\underline{C} = a_r - b_r + i * (a_i - b_i)$;

Multiplikation: $\underline{C} = A * B * e^{i*(\alpha+\beta)}$;

Division: $\underline{C} = \frac{A}{B} * e^{i*(\alpha-\beta)}$

Inversion: $\underline{A}^{-1} = \frac{1}{\underline{A}} = \frac{1}{a_r + i*a_i} = \frac{a_r - i*a_i}{a_r^2 - a_i^2} = \frac{1}{A} * e^{-i\alpha}$

Komplexe Teiler ;

Stromteiler: $\frac{\underline{I}_a}{\underline{I}_e} = \frac{\underline{Y}_2}{\underline{Y}_1 + \underline{Y}_2} = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2}$

Spannungsteiler: $\frac{\underline{U}_a}{\underline{U}_e} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2}$

Dreick Stern Umformung

$\Delta \rightarrow \perp$: $\underline{Z}_A = \frac{\underline{Z}_{AB} * \underline{Z}_{AC}}{\underline{Z}_{AC} + \underline{Z}_{AB} + \underline{Z}_{BC}} =$;

$\perp \rightarrow \Delta$: $\underline{Z}_{AB} = \underline{Z}_A + \underline{Z}_B + \frac{\underline{Z}_A * \underline{Z}_B}{\underline{Z}_C}$

5 Leistung

Leistungsmomentanwert: $p = u * i$, bei sinus-förmig $p = P - S * \cos(2\omega t + \phi_U + \phi_I)$;

Wirkleistung: $P = U * I * \cos(\phi_U - I) = S * \cos \phi_{U-I}$

Scheinleistung: $S = U * I$;

Blindleistung: $Q = U * I * \sin \phi_{U-I}$;

Leistungsfaktor: $\lambda = \frac{P}{S}$;

P-Dreieck: $S^2 = P^2 + Q^2$;

Leistungsfaktor: $\cos \phi = \frac{P}{S}$; $\tan \phi = \frac{Q}{P}$

5.1 Leistungsanpassung

optimaler Anschlusswiderstand: $Z_{aopt} = R_a + iX_i = R_i - iX_i = \underline{Z}_i^*$, $\underline{Z}'_v = \underline{Z}_i^*$

maximale Wirkleistung: $P_{max} = \frac{U_q^2}{4 * R_i} = \frac{U_q^2}{4 * R_v} = \frac{|U_q|^2}{4 * \Re\{\underline{Z}_i\}} = \frac{(Z * I)^2}{4 * R_i}$

Wirkungsgrad: $\eta = \frac{I^2 * R_{opt}}{I^2 * (R_i + R_{opt})}$

5.2 Betragsanpassung

Widerstand: $R_a = \sqrt{R_i^2 + X_i^2} = |\underline{Z}_i| = Z_i$

Wirkleistung: $P_a = \frac{U_q^2}{2 * (Z_i + R_i)} = \frac{U_q^2}{2 * (R_a + R_i)}$

6 Spezialfälle

6.1 Belasteter Spannungsteiler

$$\frac{U_a}{U_e} = \frac{Z_{2L}}{Z_1 + Z_{2L}} = \frac{Z_L * Z_2}{Z_1 * (Z_L + Z_2) + Z_L * Z_2} = \frac{Y_1}{Y_1 + Y_2 + Y_L} \quad (1)$$

6.2 Belasteter Stromteiler

$$\frac{I_a}{I_e} = \frac{Y_{2L}}{Y_1 + Y_{2L}} = \frac{Y_L * Y_2}{Y_1 * (Y_L + Y_2) + Y_L * Y_2} = \frac{Z_1}{Z_1 + Z_2 + Z_L} \quad (2)$$

6.3 Reihe \Leftrightarrow Parallel

6.3.1 Reihe

$$2(Z_1 + Z_2) = Z_{par} \quad (3)$$

6.3.2 Parallel

$$Z_{1\parallel 2} = \frac{Z_1 * Z_2}{Z_1 + Z_2} \quad (4)$$

$$Z_{reihe1} = \Re\{Z_{1\parallel 2}\}; Z_{reihe2} = \Im\{Z_{1\parallel 2}\} \quad (5)$$

6.4 Komplexer Teiler

6.4.1 Spannungsteiler

$$Z_1 = \frac{U_e * Z_2}{U_a} - Z_2 \quad (6)$$

$$Y_1 = \frac{-U_a * Y_2}{U_a - U_e} \quad (7)$$

6.4.2 Stromteiler

$$Y_1 = \frac{I_e * Y_2}{I_a} - Y_2 \quad (8)$$

$$Z_1 = \frac{-I_a * Z_2}{I_a - I_e} \quad (9)$$