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Mathematik 1 - AI

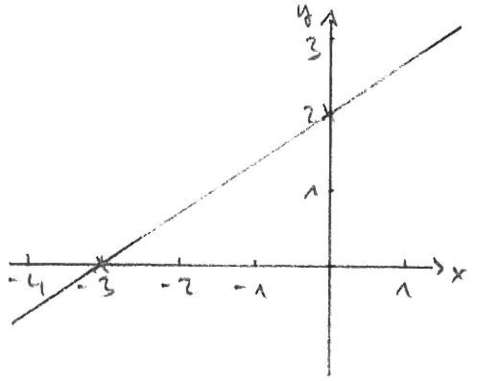
Blatt 5

(A41) a) Gl. / Ungl. in den Unbestimmten x, y.

i) $2x - 3y + 6 = 0$ (Gerade)

Schnittpunkte mit den Koordinatenachsen

- mit x-Achse: $y = 0$: $2x + 6 = 0 \Leftrightarrow x = -3$
- mit y-Achse: $x = 0$: $-3y + 6 = 0 \Leftrightarrow y = 2$

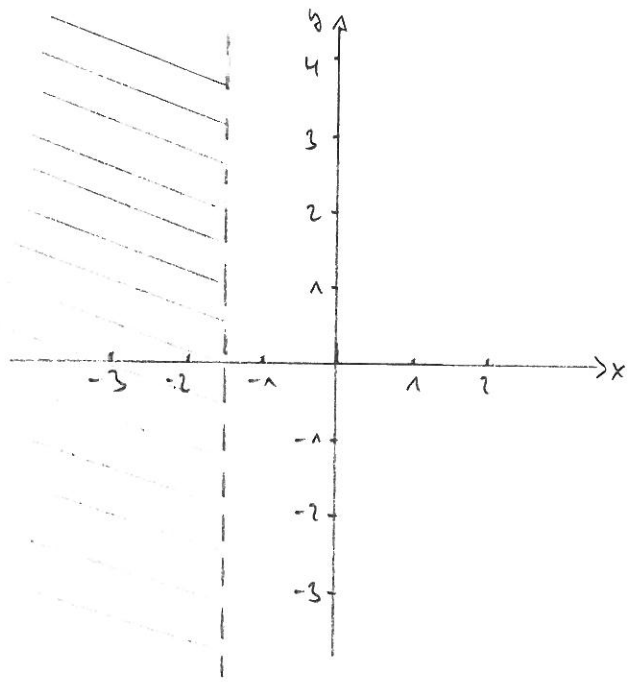


ii) $x + \frac{3}{2} < 0$ (Halbebene)

Halbebene wird von der Geraden $x + \frac{3}{2} = 0 \Leftrightarrow x = -\frac{3}{2}$ (senkrechte Gerade) begrenzt.

Halbebene: $x + \frac{3}{2} < 0 \Leftrightarrow x < -\frac{3}{2}$

$\Rightarrow$ Halbebene = Gebiet links der Geraden $x = -\frac{3}{2}$



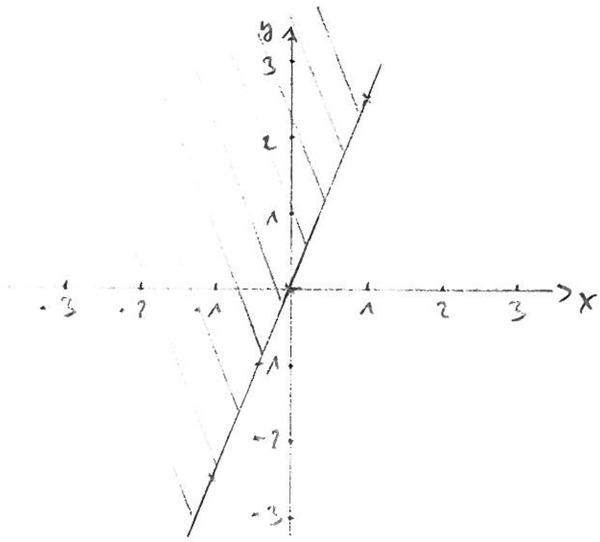
A41) iii) $-\frac{x}{2} + \frac{y}{5} \geq 0$ (Halbebene)

Halbebene wird von der Geraden $-\frac{x}{2} + \frac{y}{5} = 0 \Leftrightarrow y = \frac{5}{2}x$

(Ursprungsgerade mit Steigung $m = \frac{5}{2}$) begrenzt.

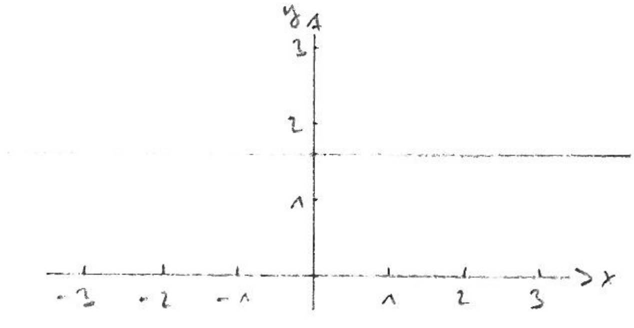
Halbebene: $-\frac{x}{2} + \frac{y}{5} \geq 0 \Leftrightarrow \frac{y}{5} \geq \frac{x}{2} \Leftrightarrow y \geq \frac{5}{2}x$

$\Rightarrow$ Halbebene = Gebiet oberhalb der Geraden $y = \frac{5}{2}x$



iv) $-\frac{5}{8}y + 1 = 0$ (Gerade)

$-\frac{5}{8}y + 1 = 0 \Leftrightarrow \frac{5}{8}y = 1 \Leftrightarrow y = \frac{8}{5}$ (waagrechte Gerade)

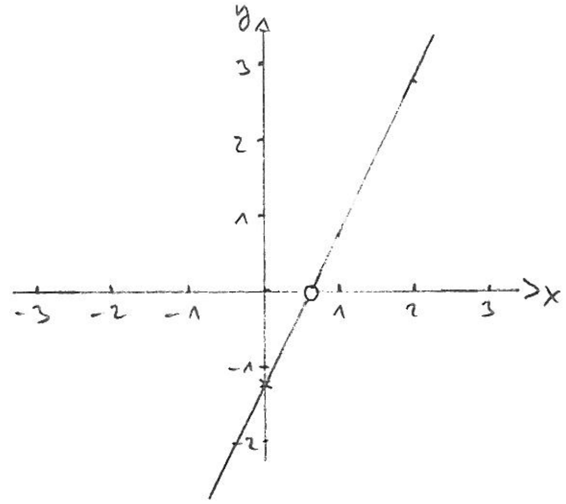


v) $\frac{5-8x}{2y} = -2$ (Gerade mit „Loch")

$\frac{5-8x}{2y} = -2 \Leftrightarrow 5 - 8x = -4y \Leftrightarrow y = 2x - \frac{5}{4}$ $(y \neq 0)$

Steigung $m = 2$, y-Abschnitt $t = -\frac{5}{4}$

A41 zu v)

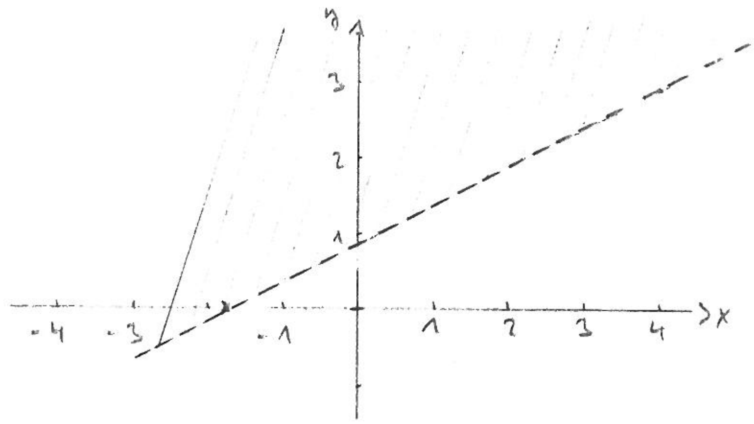


vi) $-\frac{1}{2}(12x+3) > \frac{3}{4}(12-16y)$ (Halbebene)

$-\frac{1}{2}(12x+3) > \frac{3}{4}(12-16y) \Leftrightarrow -6x - \frac{3}{2} > 9 - 12y$

$\Leftrightarrow 12y > 6x + \frac{21}{2} \Leftrightarrow y > \frac{1}{2}x + \frac{7}{8}$

Halbebene = Gebiet oberhalb der Geraden $y = \frac{1}{2}x + \frac{7}{8}$ (Steigung $m = \frac{1}{2}$, y-Abschnitt $b = \frac{7}{8}$)



b) Die Lösungsmenge von $\frac{x_1}{a_1} + \frac{x_2}{a_2} + \frac{x_3}{a_3} = 1$ ist eine Ebene im $\mathbb{R}^3$, da die Gl. linear (mit 3 Unbestimmten) ist.

Schnittpunkt

- mit x_1-Achse: $x_2 = x_3 = 0$: $\frac{x_1}{a_1} + 0 + 0 = 1 \Leftrightarrow x_1 = a_1$
- mit x_2-Achse: $x_1 = x_3 = 0$: $0 + \frac{x_2}{a_2} + 0 = 1 \Leftrightarrow x_2 = a_2$
- mit x_3-Achse: $x_1 = x_2 = 0$: $0 + 0 + \frac{x_3}{a_3} = 1 \Leftrightarrow x_3 = a_3$

A41

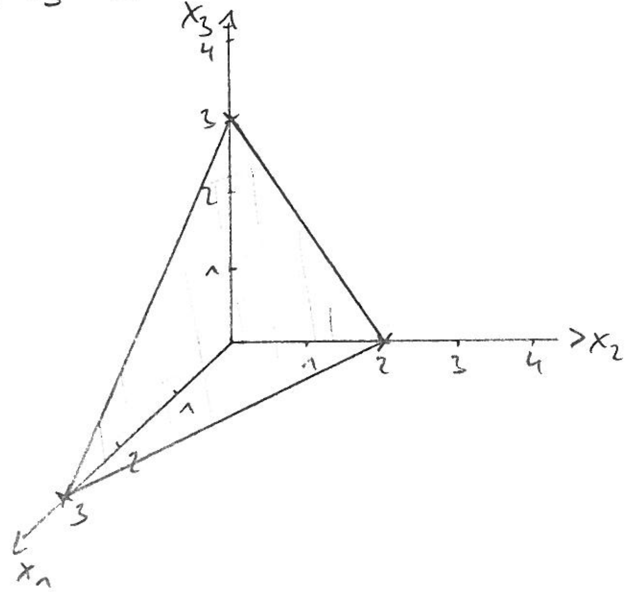
c) i) $2x_1 + 3x_2 + 2x_3 = 6 \quad | :6$

$\Leftrightarrow \frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = 1$

$\Leftrightarrow \frac{x_1}{3} + \frac{x_2}{2} + \frac{x_3}{3} = 1$

$\Rightarrow$ Schnittpunkte mit den Koordinatenachsen (nach A20b)

$x_1 = 3, \; x_2 = 2, \; x_3 = 3$

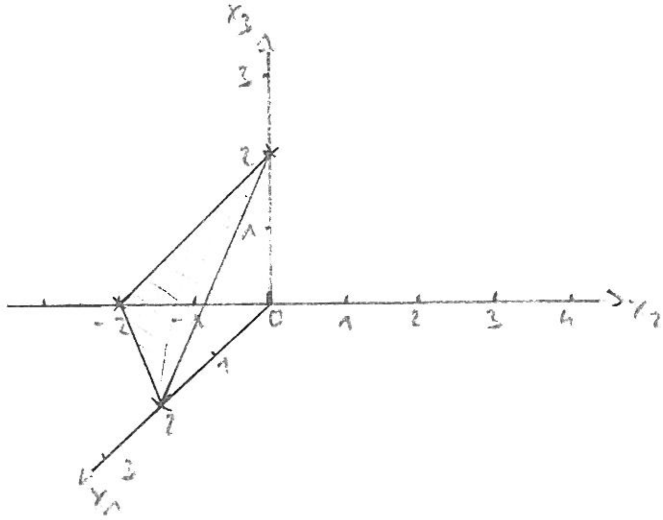


ii) $x_1 - x_2 + x_3 = 2 \quad | :2$

$\Leftrightarrow \frac{x_1}{2} - \frac{x_2}{2} + \frac{x_3}{2} = 1$

$\Rightarrow$ Schnittpunkte mit den Koordinatenachsen (nach A20b)

$x_1 = 2, \; x_2 = -2, \; x_3 = 2$



A41

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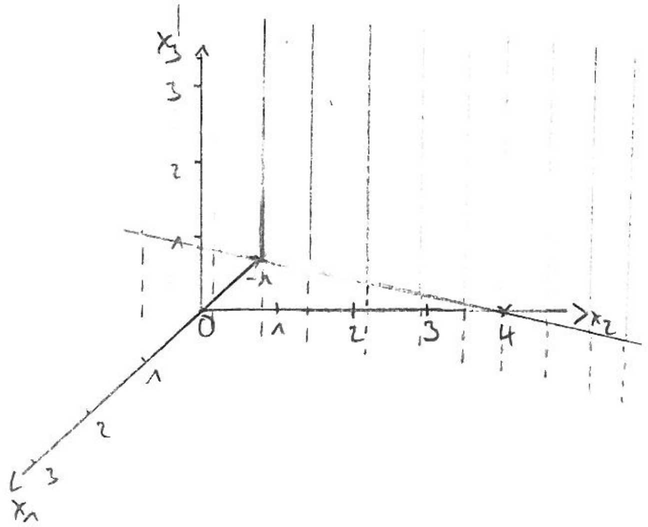
c) iii) $x_2 = 4 + 4x_1$ (Ebene parallel zur x_3-Achse)

$$x_2 = 4 + 4x_1 \quad | -4x_1$$

$$\Leftrightarrow -4x_1 + x_2 = 4 \quad | :4$$

$$\Leftrightarrow -x_1 + \frac{x_2}{4} = 1 \quad \Leftrightarrow \quad \frac{x_1}{-1} + \frac{x_2}{4} = 1$$

$\Rightarrow$ Schnittpunkte mit der x_1- und x_2-Achse: $x_1 = -1$, $x_2 = 4$



A42 a) i) $8x^2+6x-27=0$

Diskr. $D = 36-4\cdot 8\cdot(-27) = 36+864 = 900$

$\Rightarrow x_{1,2} = \frac{-6\pm\sqrt{900}}{16} = \frac{-6\pm 30}{16}$

$\Rightarrow x_1 = \frac{3}{2}, \ x_2 = -\frac{9}{4} \quad \Rightarrow \mathbb{L} = \left\{\frac{3}{2}; -\frac{9}{4}\right\}$

ii) $-2x^2+11x<29$

$\Leftrightarrow 2x^2-11x+29>0$

Nst von $2x^2-11x+29$ bestimmen: Diskr. $D = 121-4\cdot 2\cdot 29 = -111 < 0$

$\Rightarrow 2x^2-11x+29$ hat keine Nst

Setzt man $x=0$ ein, so erhält man $29 (>0) \Rightarrow 2x^2-11x+29>0$

für jedes $x\in\mathbb{R}$

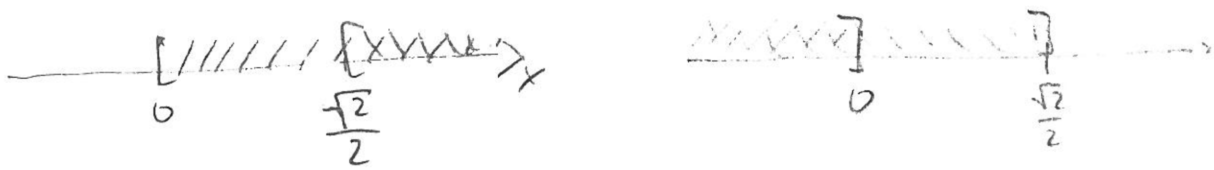
$\Rightarrow \mathbb{L} = \mathbb{R}$

iii) $x(2x-\sqrt{2})\geq 0$

$x(2x-\sqrt{2})\geq 0 \Leftrightarrow$

$(x\geq 0$ und $\underbrace{2x-\sqrt{2}\geq 0}_{x\geq\frac{\sqrt{2}}{2}})$ oder $(x\leq 0$ und $\underbrace{2x-\sqrt{2}\leq 0}_{x\leq\frac{\sqrt{2}}{2}})$

$x\geq\frac{\sqrt{2}}{2}$ $\qquad\qquad$ $x\leq 0$



$\Rightarrow \mathbb{L} = \left\{x\leq 0 \text{ oder } x\geq\frac{\sqrt{2}}{2}\right\} = \{x\leq 0\}\cup\left\{x\geq\frac{\sqrt{2}}{2}\right\}$

$= \left]-\infty; 0\right]\cup\left[\frac{\sqrt{2}}{2}; \infty\right[$

A42 zu iii) Alternative Lösung mittels VZT

$f(x) = x(2x - \sqrt{2}) = x \cdot 2(x - \frac{\sqrt{2}}{2}) = 2x(x - \frac{\sqrt{2}}{2})$

Nst.: $x_1 = 0$, $x_2 = \frac{\sqrt{2}}{2}$

VZT:

Faktor	$-\infty < x < 0$	$x = 0$	$0 < x < \frac{\sqrt{2}}{2}$	$x = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} < x < \infty$
2	+	+	+	+	+
x	−	0	+	+	+
$x - \frac{\sqrt{2}}{2}$	−	−	−	0	+
$f(x)$	+	0	−	0	+

$\underbrace{-\infty < x \le 0}_{\ge 0}$, $\underbrace{\frac{\sqrt{2}}{2} \le x < \infty}_{\ge 0}$

$\Rightarrow \mathbb{L} =]-\infty; 0] \cup [\frac{\sqrt{2}}{2}; \infty[$

iv) $2x^4 + 3x^2 - 14 \le 0$

Nst. von $P(x) = 2x^4 + 3x^2 - 14$:

$2x^4 + 3x^2 - 14 = 0$ (biquadratische Gl.)

Subst. $x^2 = u$

$2u^2 + 3u - 14 = 0 \Rightarrow u_{1/2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-14)}}{4}$

$= \frac{-3 \pm 11}{4}$

$\Rightarrow u_1 = 2$, $u_2 = -\frac{7}{2}$

$2u^2 + 3u - 14 = 2(u-2)(u+\frac{7}{2})$

Rücksubst.: $u = x^2$

$2x^4 + 3x^2 - 14 = 2(x^2 - 2)\underbrace{(x^2 + \frac{7}{2})}_{>0} = 2(x - \sqrt{2})(x + \sqrt{2})(x^2 + \frac{7}{2})$

$\Rightarrow$ unzerlegbar

VZT:

Faktor	$-\infty < x < -\sqrt{2}$	$x = -\sqrt{2}$	$-\sqrt{2} < x < \sqrt{2}$	$x = \sqrt{2}$	$\sqrt{2} < x < \infty$
2	+	+	+	+	+
$x + \sqrt{2}$	−	0	+	+	+
$x - \sqrt{2}$	−	−	−	0	+
$x^2 + \frac{7}{2}$	+	+	+	+	+
$P(x)$	+	0	−	0	+

$\underbrace{-\sqrt{2} \le x \le \sqrt{2}}_{\le 0}$

$\Rightarrow \mathbb{L} = [-\sqrt{2}; \sqrt{2}]$

A42 v) $64x^6 - 204x^3 - 27 = 0$

Subst.: $x^3 = u$

$64u^2 - 204u - 27 = 0 \qquad u_{1|2} = \frac{204 \pm \sqrt{204^2 + 4 \cdot 64 \cdot 27}}{2 \cdot 64} =$

$= \frac{204 \pm 12 \cdot \sqrt{337}}{128} = \frac{51 \pm 3\sqrt{337}}{32}$

$\underbrace{u_1 = \frac{51}{32} + \frac{3}{32}\sqrt{337}}_{>0}, \quad u_2 = \frac{51}{32} - \frac{3}{32}\sqrt{337} < 0$

Rücksubst.: $u = x^3$

$u_1 = x_1^3 \iff x_1 = \sqrt[3]{u_1} = \sqrt[3]{\frac{51}{32} + \frac{3}{32}\sqrt{337}}$

$u_2 = x_2^3 \underset{(u_2<0)}{\iff} x_2 = -\sqrt[3]{-u_2} = -\sqrt[3]{\frac{3}{32}\sqrt{337} - \frac{51}{32}}$

vi) $-(-2x+12)(-\frac{1}{2}x+3) \geq 0$

$-(-2x+12) \cdot (-\frac{1}{2}x+3) = -(-2)(x-6) \cdot (-\frac{1}{2}) \cdot (x-6) =$

$= -(x-6)^2 \geq 0 \iff \underbrace{(x-6)^2}_{\geq 0} \leq 0 \iff (x-6)^2 = 0$

$\iff x = 6$

$\Rightarrow \mathbb{L} = \{6\}$

b)

i) $x^3 - 3x^2 + 4 > 0$

Nst. von $f(x) = x^3 - 3x^2 + 4$.

Probiere alle Teiler von 4: $\pm 1, \pm 2, \pm 4$

$f(-1) = 0$

$\Rightarrow$ Polynomdivision $f(x) : (x+1)$

$$\begin{array}{l}
(x^3 - 3x^2 + 0x + 4) : (x+1) = x^2 - 4x + 4 \\
\underline{-(x^3 + x^2)} \\
\quad -4x^2 + 0x \\
\quad \underline{-(-4x^2 - 4x)} \\
\qquad 4x + 4 \\
\qquad \underline{-(4x+4)} \\
\qquad\quad 0
\end{array}$$

(A42)

$$\Rightarrow \quad x^3-3x^2+4 = (x+1)\underbrace{(x^2-4x+4)}_{(x-2)^2 \text{ (bin. Formel)}}$$

$$= (x+1)(x-2)^2$$

$\Rightarrow$ Nst $x_1 = -1$, $x_2 = 2$

VZT

Faktor	$-\infty < x < -1$	$x=-1$	$-1<x<2$	$x=2$	$2<x<\infty$
$x+1$	$-$	0	$+$	$+$	$+$
$(x-2)^2$	$+$	$+$	$+$	0	$+$
$f(x)$	$-$	0	$+$	0	$+$
			>0		>0

$\Rightarrow \quad \mathbb{L} = \,]-1;2[\, \cup \,]2;\infty[$

ii) $x^3+3x^2-x-3=0$

Nst. von $f(x) = x^3+3x^2-x-3$

Probiere die Teiler von 3: ± 1, ± 3 $\quad \Rightarrow f(1) = 0$

Polynomdivision: $f(x) : (x-1)$

$$\begin{array}{l}
(x^3+3x^2-x-3):(x-1) = x^2+4x+3 \\
-(x^3-x^2) \\
\hline
\quad 4x^2-x \\
\quad -(4x^2-4x) \\
\hline
\qquad 3x-3
\end{array}$$

$$\Rightarrow (x^3+3x^2-x-3) = (x-1)\underbrace{(x^2+4x+3)}_{D = 16-12 = 4>0 \;\Rightarrow\; \text{zerlegbar}}$$

Nst. von x^2+4x+3:

$$x_{1/2} = \frac{-4 \pm \sqrt{16-12}}{2} = \frac{-4\pm 2}{2} = -2 \pm 1 \Rightarrow x_1 = -1,\; x_2 = -3$$

$\Rightarrow \quad \mathbb{L} = \{-3; -1; 1\}$

(A2.1)

iii) $-x^3+2x^2-3x+2<0$

Nst. von $f(x)=-x^3+2x^2-3x+2$

Probiere alle Teiler von 2: $\pm1, \pm2$ $\Rightarrow f(1)=0$

$\Rightarrow$ Polynomdivision $f(x):(x-1)$

$$
\begin{array}{l}
(-x^3+2x^2-3x+2):(x-1)=-x^2+x-1 \\
\underline{-(-x^3+x^2)} \\
\quad\quad x^2-3x \\
\quad\quad \underline{-(x^2-x)} \\
\quad\quad\quad -2x+2 \\
\quad\quad\quad \underline{-(-2x+2)} \\
\quad\quad\quad\quad 0
\end{array}
$$

$\Rightarrow -x^3+2x^2-3x+2=(x-1)(\underbrace{-x^2+x-1}_{})$

$D=1-4\cdot(-1)\cdot(-1)=1-4<0$

$\Rightarrow$ (über $\mathbb{R}$) unzerlegbar

$-x^2+x-1<0$ für jedes $x\in\mathbb{R}$

VZT:

Faktor	$-\infty<x<1$	$x=1$	$1<x<\infty$
$x-1$	$-$	0	$+$
$-x^2+x-1$	$-$	$-$	$-$
$f(x)$	$+$	0	$-$

≤ 0

$\Rightarrow \mathbb{L}=[1;\infty[$

iv) $x^6-x^4<0$

$f(x)=x^6-x^4=x^4(x^2-1)=x^4(x-1)(x+1)$ $\Rightarrow$ Nst. $-1, 0, 1$

VZT:

Faktor	$-\infty<x<-1$	$x=-1$	$-1<x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<\infty$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
x^4	$+$	$+$	$+$	0	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$-$	0	$+$
$f(x)$	$+$	0	$-$	0	$-$	0	$+$
			<0		<0		

$\Rightarrow \mathbb{L}=]-1;0[\,\cup\,]0;1[$

A21

v) $-2x^5 - 2x^4 + 2x + 8 > 0$

Nst. von $f(x) = -2x^5 - 2x^4 + 2x + 8$

Probiere alle Teiler von 8: $\pm 1, \pm 2, \pm 4, \pm 8$ => $f(1) = 0$

=> Polynomdivision $f(x) : (x-1)$

$$
\begin{array}{l}
(-2x^5 - 8x^4 + 0x^3 + 0x^2 + 2x + 8) : (x-1) = -2x^4 - 10x^3 - 10x^2 - 10x - 8 \\
-(-2x^5 + 2x^4) \\
\quad -10x^4 + 0x^3 \\
\quad -(-10x^4 + 10x^3) \\
\qquad -10x^3 + 0x^2 \\
\qquad -(-10x^3 + 10x^2) \\
\qquad\quad -10x^2 + 2x \\
\qquad\quad -(-10x^2 + 10x) \\
\qquad\qquad -8x + 8
\end{array}
$$

$$
\Rightarrow f(x) = (x-1)(-2x^4 - 10x^3 - 10x^2 - 10x - 8) =
$$

$$
= (-2)(x-1)(x^4 + 5x^3 + 5x^2 + 5x + 4)
$$

Nst. von $f_1(x) = x^4 + 5x^3 + 5x^2 + 5x + 4$

Probiere alle Teiler von 4: $\pm 1, \pm 2, \pm 4$ => $f_1(-1) = 0$

=> Polynomdivision $f_1(x) : (x+1)$

$$
\begin{array}{l}
(x^4 + 5x^3 + 5x^2 + 5x + 4) : (x+1) = x^3 + 4x^2 + x + 4 \\
-(x^4 + x^3) \\
\quad 4x^3 + 5x^2 \\
\quad -(4x^3 + 4x^2) \\
\qquad x^2 + 5x \\
\qquad -(x^2 + x) \\
\qquad\quad 4x + 4 \\
\qquad\quad -(4x + 4) \\
\qquad\qquad 0
\end{array}
$$

A.42 zu v)

$$\Rightarrow \quad f_1(x) = (x+1)\,\underbrace{(x^3+4x^2+x+4)}_{f_2(x)}$$

Nst von $f_2(x) = x^3+4x^2+x+4$

Probiere alle Teiler von 4 : $\pm 1, \pm 2, \pm 4 \Rightarrow f_2(-4) = 0$

$\Rightarrow$ Polynomdivision $f_2(x) : (x+4)$

$$\begin{array}{l} (x^3+4x^2+x+4) : (x+4) = x^2+0x+1 \\ -(x^3+4x^2) \\ \hline \quad 0x^2+x \\ \quad -(0x^2+0x) \\ \hline \qquad x+4 \end{array}$$

$$\Rightarrow f_2(x) = (x+4)\,\underbrace{(x^2+1)}_{>0} \Rightarrow \text{(über } \mathbb{R}\text{) unzerlegbar}$$

insgesamt folgt: Nst: $-4, -1, +1$

$$f(x) = (-2)(x-1)(x+1)(x+4)(x^2+1)$$

VZT:

Faktor	$-\infty < x < -4$	$x=-4$	$-4<x<-1$	$x=-1$	$-1<x<1$	$x=1$	$1<x<\infty$
-2	−	−	−	−	−	−	−
$x+4$	−	0	+	+	+	+	+
$x+1$	−	−	−	0	+	+	+
$x-1$	−	−	−	−	−	0	+
x^2+1	+	+	+	+	+	+	+
$f(x)$	+ (>0)	0	−	0	+ (>0)	0	−

$\Rightarrow \mathbb{L} =]-\infty; -4[\; \cup \;]-1; 1[$

(A42) vi) $x^5 - 3x^4 + 4x^3 - 4x^2 \geq 0$

Nst. von $f(x) = x^5 - 3x^4 + 4x^3 - 4x^2$

($\hat{=}$ niedrigste Potenz ausklammern)

$$f(x) = x^5 - 3x^4 + 4x^3 - 4x^2 = x^2 \underbrace{(x^3 - 3x^2 + 4x - 4)}_{f_1(x)}$$

Nst. von $f_1(x) = x^3 - 3x^2 + 4x - 4$

Probiere alle Teiler von 4 : $\pm 1, \pm 2, \pm 4 \Rightarrow f_1(2) = 0$

$\Rightarrow$ Polynomdivision $f_1(x) : (x-2)$

$$(x^3 - 3x^2 + 4x - 4) : (x-2) = x^2 - x + 2$$

$$\begin{array}{l} -(x^3 - 2x^2) \\ \hline \quad -x^2 + 4x \\ \quad -(-x^2 + 2x) \\ \hline \qquad 2x - 4 \\ \qquad -(2x - 4) \\ \hline \qquad\quad 0 \end{array}$$

$\Rightarrow f_1(x) = (x-2)\underbrace{(x^2 - x + 2)}$

$D = 1 - 4 \cdot 2 < 0 \Rightarrow$ (über $\mathbb{R}$) unzerlegbar

$x^2 - x + 2 > 0$ für jedes x

insgesamt folgt:

$f(x) = x^2 (x-2)(x^2 - x + 2)$ Nst: 0, 2

VZT:

Faktor	$-\infty < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < \infty$
x^2	+	0	+	+	+
$x - 2$	−	−	−	0	+
$x^2 - x + 2$	+	+	+	+	+
$f(x)$	−	0	−	0	+
		$\underbrace{}_{\geq 0}$		$\underbrace{}_{\geq 0}$	

$\Rightarrow \mathbb{L} = \{0\} \cup [2; \infty[$

A43 Kegelschnitte $Ax^2+By^2+Cx+Dy+E=0$ ($A\neq 0$ oder $B\neq 0$)

Kreis:	$A=B$
Ellipse:	$A\cdot B>0$, $A\neq B$
Hyperbel	$A\cdot B<0$
Parabel	$A=0$ oder $B=0$

a) $2y^2-9x=-12y$

$\Leftrightarrow \quad -9x+2y^2+12y=0$

$A=0$
$\Rightarrow$ Parabel;

$-9x+\underbrace{2y^2+12y}_{\text{quadrat. Ergänzung}}=0$ Hauptform: $(y-y_S)^2=2p(x-x_S)$

$$2y^2+12y=2(y^2+6y)=2\left(y^2+6y+3^2-3^2\right)=2\left[(y+3)^2-9\right]=$$
$$=2(y+3)^2-18$$

$-9x+2y^2+12y=0$

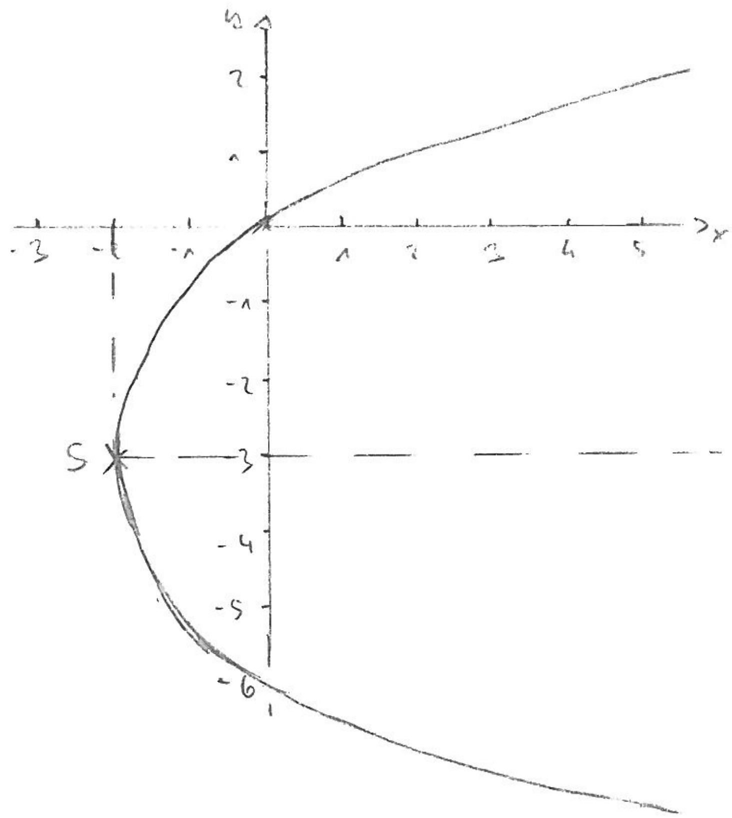
$\Leftrightarrow \quad -9x+2(y+3)^2-18=0$

$\Leftrightarrow \quad 2(y+3)^2=9x+18$

$\Leftrightarrow \quad 2(y+3)^2=9(x+2) \quad |:2$

$\Leftrightarrow \quad (y+3)^2=\frac{9}{2}(x+2)$

Scheitel $S(-2;-3)$; Parabelachse $\parallel$ x-Achse; $p=\frac{9}{4}>0$ (nach rechts geöffnet)



A.43 b) $\overset{A}{9}x^2 + \overset{B}{16}y^2 - 18x = 135$

$A \cdot B > 0$ und $A \neq B \Rightarrow$ Ellipse

Hauptform: $\frac{(x-x_M)^2}{a^2} + \frac{(y-y_M)^2}{b^2} = 1$

$9x^2 + 16y^2 - 18x = 135$

$\Leftrightarrow \underbrace{9x^2 - 18x}_{\text{quadrat. Ergänzung}} + 16y^2 = 135$

$9x^2 - 18x = 9(x^2 - 2x) = 9(x^2 - 2x + 1^2 - 1^2) = 9[(x-1)^2 - 1]$

$= 9(x-1)^2 - 9$

$9x^2 - 18x + 16y^2 = 135$

$\Leftrightarrow 9(x-1)^2 - 9 + 16y^2 = 135 \quad | +9$

$\Leftrightarrow 9(x-1)^2 + 16y^2 = 144 \quad | :144$

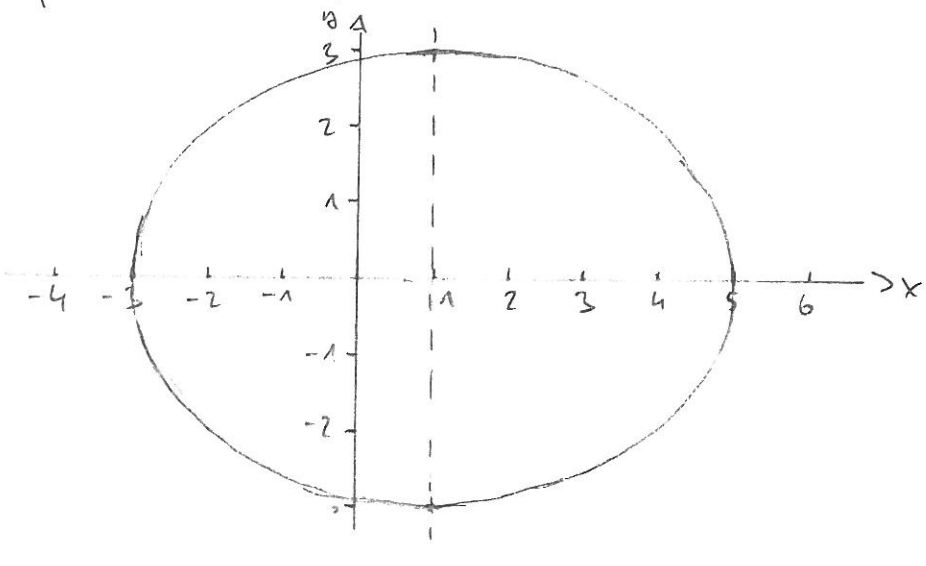
$\Leftrightarrow \frac{9(x-1)^2}{144} + \frac{16y^2}{144} = 1$

$\Leftrightarrow \frac{(x-1)^2}{\frac{144}{9}} + \frac{16y^2}{\frac{144}{16}} = 1$ $\qquad \frac{144}{9} = \frac{12^2}{3^2} = \left(\frac{12}{3}\right)^2 = 4^2$

$\qquad \frac{144}{16} = \frac{12^2}{4^2} = \left(\frac{12}{4}\right)^2 = 3^2$

$\Leftrightarrow \frac{(x-1)^2}{4^2} + \frac{y^2}{3^2} = 1$

Mittelpunkt $M(1,0)$; Halbachsen $a=4$, $b=3$



A43 c) $\overset{A}{x^2} + \overset{B}{y^2} - 2x + 4y - 20 = 0$

$A = B = 1 \Rightarrow$ Kreis

Hauptform: $(x - x_M)^2 + (y - y_M)^2 = r^2$

$$x^2 + y^2 - 2x + 4y - 20 = 0$$

$$\Leftrightarrow \underbrace{x^2 - 2x}_{\text{quadrat. Ergänzung}} + \underbrace{y^2 + 4y}_{\text{quadrat. Ergänzung}} - 20 = 0$$

$$x^2 - 2x = x^2 - 2x + 1 - 1 = (x-1)^2 - 1$$

$$y^2 + 4y = y^2 + 4y + 2^2 - 2^2 = (y+2)^2 - 4$$

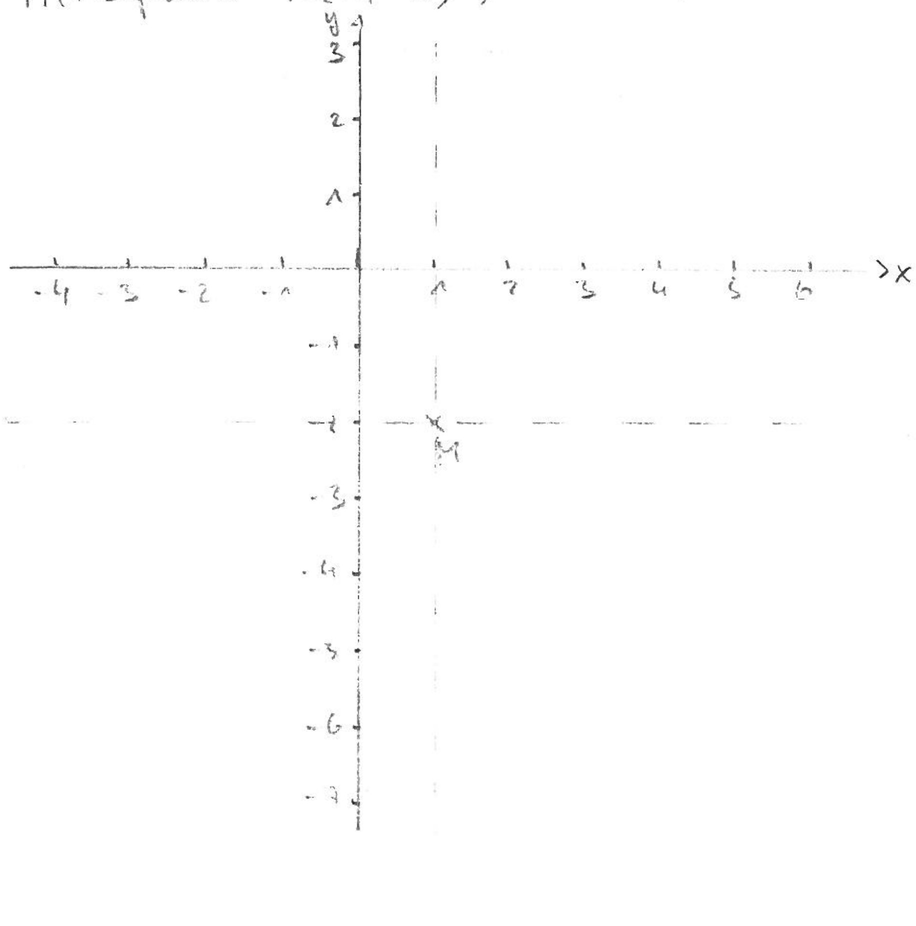
$\Rightarrow$

$$x^2 - 2x + y^2 + 4y - 20 = 0$$

$$\Leftrightarrow (x-1)^2 - 1 + (y+2)^2 - 4 - 20 = 0$$

$$\Leftrightarrow (x-1)^2 + (y+2)^2 = 25$$

Mittelpunkt $M(1; -2)$, Radius $r = 25$



A43) d) $x^2 + 2y = 4x$

$\underbrace{x^2 - 4x}_{\text{quadrat. Ergänzung}} + 2y = 0 \qquad B = 0 \Rightarrow$ Parabel

$x^2 - 4x = x^2 - 4x + 2^2 - 2^2 = (x-2)^2 - 4$

$\Rightarrow$

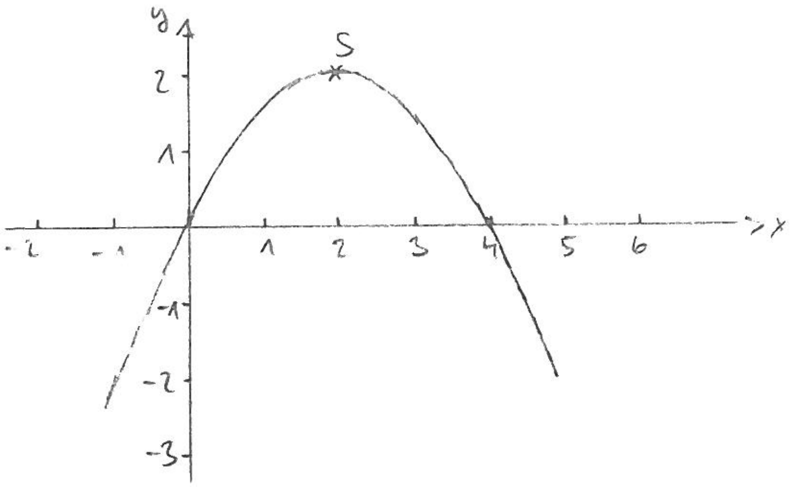
$(x-2)^2 - 4 + 2y = 0$

$\Leftrightarrow (x-2)^2 = -2y + 4$

$\Leftrightarrow (x-2)^2 = \underset{<0}{-2}(y-2)$

$\Rightarrow$ Parabelachse $\parallel$ y-Achse ; nach unten geöffnet

Scheitel $S(2;2)$



e) $-\overset{A}{9}x^2 + 36x + \overset{B}{4}y^2 + 16y = 56$

$A \cdot B < 0 \Rightarrow$ Hyperbel

$\underbrace{-9x^2 + 36x}_{\text{quadrat. Ergänzung}} + \underbrace{4y^2 + 16y}_{\text{quadrat. Ergänzung}} = 56$

$-9x^2 + 36x = -9(x^2 - 4x) = -9(x^2 - 4x + 2^2 - 2^2) = -9[(x-2)^2 - 4] = -9(x-2)^2 + 36$

$4y^2 + 16y = 4(y^2 + 4y) = 4(y^2 + 4y + 2^2 - 2^2) = 4[(y+2)^2 - 4] = 4(y+2)^2 - 16$

A43 zu e)

$$-9(x-2)^2+36+4(y+2)^2-16=56 \quad \Big| -36+16$$

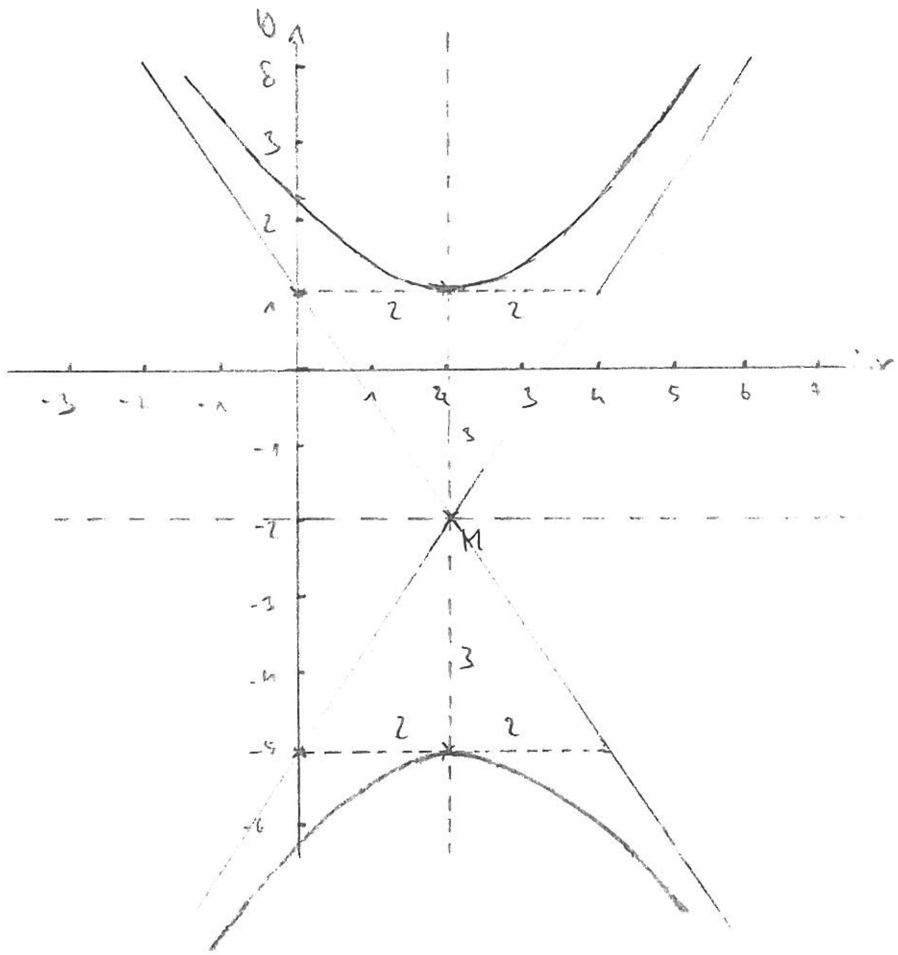
$$\Leftrightarrow -9(x-2)^2+4(y+2)^2=36 \quad \Big| :36$$

$$\Leftrightarrow -\frac{(x-2)^2}{36/9}+\frac{(y+2)^2}{36/4}=1$$

$$\Leftrightarrow -\frac{(x-2)^2}{2^2}+\frac{(y+2)^2}{3^2}=1$$

Mittelpunkt $M(2;-2)$
Halbachsen $a=2$, $b=3$

Vorzeichen $\Rightarrow$ Hyperbel der Form 



A43 zu f)

$100x^2 + 36y^2 - 400x + 108y = -361$; $A \cdot B > 0 \Rightarrow$ Ellipse

$\Leftrightarrow \underbrace{100x^2 - 400x}_{\text{quadrat. Ergänzung}} \underbrace{+36y^2 + 108y}_{\text{quadrat. Ergänzung}} = -361$

$100x^2 - 400x = 100(x^2 - 4x + 2^2 - 2^2) = 100(x-2)^2 - 400$

$36y^2 + 108 = 36(y^2 + 3y + \square - \square) = 36\left(y^2 + 3y + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) =$

$= 36\left[\left(y + \frac{3}{2}\right)^2 - \frac{9}{4}\right] = 36\left(y + \frac{3}{2}\right)^2 - 81$

$\Rightarrow$

$100(x-2)^2 - 400 + 36\left(y + \frac{3}{2}\right)^2 - 81 = -361 \quad | +400 + 81$

$\Leftrightarrow 100(x-2)^2 + 36\left(y + \frac{3}{2}\right)^2 = 400 + 81 - 361$

$\Leftrightarrow 100(x-2)^2 + 36\left(y + \frac{3}{2}\right)^2 = 120 \quad | :120$

$\Leftrightarrow \frac{(x-2)^2}{\frac{120}{100}} + \frac{\left(y + \frac{3}{2}\right)^2}{\frac{120}{36}} = 1$ $\quad \left(\sqrt{120} = 2\sqrt{30}\right)$

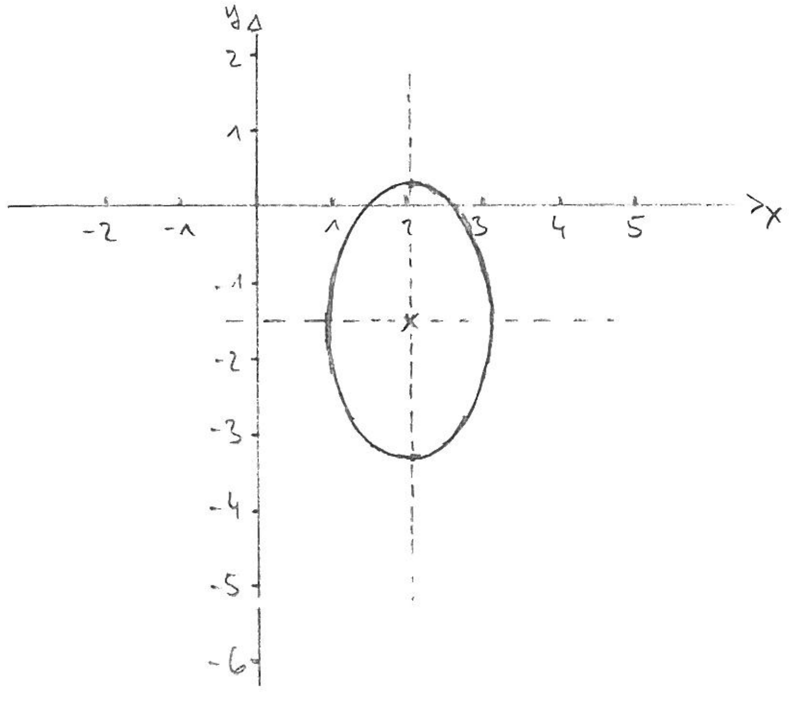
$\Leftrightarrow \frac{(x-2)^2}{\frac{2\sqrt{30}}{10}} + \frac{\left(y + \frac{3}{2}\right)^2}{\frac{2 \cdot \sqrt{30}}{6}} = 1$

$\Leftrightarrow \frac{(x-2)^2}{\frac{\sqrt{30}}{5}} + \frac{\left(y + \frac{3}{2}\right)^2}{\frac{\sqrt{30}}{3}}$

Mittelpunkt $M\left(2; -\frac{3}{2}\right)$

Halbachsen: $a = \frac{\sqrt{30}}{5} \approx 1{,}1$

$b = \frac{\sqrt{30}}{3} \approx 1{,}8$



A44 a) $\frac{4}{x} - \frac{1}{2x} + 2 = \frac{1}{4}$ $\quad \mathbb{G} = \mathbb{R} \setminus \{0\}$

Die Nenner sind $x, 2x, 4$ $\Rightarrow$ HN $= 4x$

$$\frac{16 - 2 + 8x}{HN} = \frac{x}{HN} \quad \Big| \cdot (HN \neq 0)$$

$\Leftrightarrow \quad 14 + 8x = x$

$\Leftrightarrow \quad 7x = -14$

$\Leftrightarrow \quad x = -2 \in \mathbb{G}$

Probe: $\frac{4}{-2} - \frac{1}{2\cdot(-2)} + 2 = -2 + \frac{1}{4} + 2 = \frac{1}{4}$ ✓

$\Rightarrow$

$\mathbb{L} = \{-2\}$

b) $\frac{3}{5x} - \frac{1}{3} = \frac{2}{3x} - \frac{4}{9}$ $\quad \mathbb{G} = \mathbb{R} \setminus \{0\}$

Die Nenner sind $5x, 3, 3x, 9$ $\Rightarrow$ HN $= 45x$

$$\frac{27 - 15x}{HN} = \frac{30 - 20x}{HN} \quad \Big| \cdot HN (\neq 0)$$

$27 - 15x = 30 - 20x$

$\Leftrightarrow \quad 5x = 3$

$\Leftrightarrow \quad x = \frac{3}{5} \in \mathbb{G}$

Probe:

l.S. $\frac{3}{5 \cdot 3/5} - \frac{1}{3} = 1 - \frac{1}{3} = \frac{2}{3}$ ✓

r.S. $\frac{2}{3 \cdot 3/5} - \frac{4}{9} = \frac{10}{9} - \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$ ✓

$\Rightarrow \mathbb{L} = \{\frac{3}{5}\}$

c) $\frac{18}{2x-3} + \frac{3}{3-2x} = 5$ $\quad \mathbb{G} = \mathbb{R} \setminus \{\frac{3}{2}\}$

Wg. $3 - 2x = -(2x-3)$ gilt: $\frac{3}{3-2x} = \frac{3}{-(2x-3)} = -\frac{3}{2x-3}$

$$\frac{18}{2x-3} - \frac{3}{2x-3} = 5 \quad \Big| \cdot 2x-3 (\neq 0)$$

$\Leftrightarrow \quad 18 - 3 = 5(2x-3)$

$\Leftrightarrow \quad 18 - 3 = 10x - 15$

$\Leftrightarrow \quad 10x = 30$

$\Leftrightarrow \quad x = 3 \in \mathbb{G}$

Probe:

l.S. $\frac{18}{2\cdot3-3} + \frac{3}{3-2\cdot3} = \frac{18}{3} + \frac{3}{-3} =$

$= \frac{18}{3} - \frac{3}{3} = \frac{15}{3} = 5$ ✓

r.S. 5 ✓

$\Rightarrow \mathbb{L} = \{3\}$

A 44.

d) $\frac{2}{1-x} - \frac{3}{5-5x} + \frac{5}{4x-4} = \frac{3}{10}$

Nenner	EF
$1-x$	-20
$5-5x = 5(1-x)$	-4
$4x-4 = -4(1-x)$	5
$10 = 2\cdot 5$	$-2(1-x)$

$\mathbb{G} = \mathbb{R}\setminus\{1\}$

$\Rightarrow HN = -20(1-x)$

$$\frac{2\cdot(-20) - 3\cdot(-4) + 5\cdot 5}{HN} = \frac{3(-2)(1-x)}{HN} \quad | \cdot HN (\neq 0)$$

$\Leftrightarrow -40 + 12 + 25 = -6(1-x)$

$\Leftrightarrow -3 = -6 + 6x$

$\Leftrightarrow 6x = 3 \quad | : 6$

$\Leftrightarrow x = \frac{1}{2} \in \mathbb{G}$

Probe:

l.S. $\frac{2}{1-1/2} - \frac{3}{5-5/2} + \frac{5}{4/2-4} = \frac{2}{1/2} - \frac{3}{5/2} + \frac{5}{-2} = 4 - \frac{6}{5} - \frac{5}{2} = \frac{40}{10} - \frac{12}{10} - \frac{25}{10} = \frac{3}{10}$ ✓

r.S. $3/10$ ✓

$\Rightarrow \mathbb{L} = \{$

e) $\frac{11}{x^2-25} + \frac{3x-9}{5-x} + \frac{2x+28}{3x+15} = 0$

Nenner	EF
$x^2-25 = (x+5)(x-5)$	-3
$5-x = -(x-5)$	$3(x+5)$
$3x+15 = 3(x+5)$	$-(x-5)$
$HN = -3(x+5)(x-5)$	

$\mathbb{G} = \mathbb{R}\setminus\{\pm 5\}$

$$\frac{11\cdot(-3) + (3x-9)\cdot 3(x+5) + (2x+28)\cdot(-(x-5))}{HN} = 0 \quad | \cdot HN(\neq 0)$$

$\Leftrightarrow -33 + 9x^2 + 45x - 27x - 135 - 2x^2 + 10x - 28x + 140 = 0$

$\Leftrightarrow 7x^2 - 28 = 0$

$\Leftrightarrow x^2 = 4$

$\Leftrightarrow x = \pm 2 \in \mathbb{G}$

Probe: $\boxed{x=2}$:

l.S. $\frac{11}{4-25} + \frac{6-9}{5-2} + \frac{4+28}{6+15} = -\frac{11}{21} - \frac{3}{3} + \frac{32}{21} = 0$ ✓ r.S ✓

$\boxed{x=-2}$

l.S. $\frac{11}{4-25} + \frac{-6-9}{5+2} + \frac{-4+28}{-6+15} = -\frac{11}{21} - \frac{15}{7} + \frac{24}{9} = 0$ ✓ r.S. ✓

$\Rightarrow \mathbb{L} = \{-2; 2\}$

A44

f) $\dfrac{6x^2-23x-3}{x^2-2x-15} - \dfrac{10x-15}{x+3} = \dfrac{30-4x}{x-5}$

Nenner $x^2-2x-15$ faktorisieren:

Nst.: $x_{1/2} = \dfrac{2 \pm \sqrt{4+4\cdot 15}}{2} = \dfrac{2 \pm \sqrt{64}}{2} = \dfrac{2 \pm 8}{2} = 1 \pm 4$

$\Rightarrow x_1 = 5,\ x_2 = -3$

Nenner	EF
$x^2-2x-15 = (x+3)(x-5)$	1
$x+3$	$x-5$
$x-5$	$x+3$

$HN = (x+3)(x-5)$

$\mathbb{G} = \mathbb{R} \setminus \{-3;5\}$

$\Rightarrow$

$$\frac{6x^2-23x-3-(10x-15)(x-5)}{HN} = \frac{(30-4x)(x+3)}{HN} \quad \Big|\cdot HN\ (\neq 0)$$

$\Leftrightarrow 6x^2-23x-3-10x^2+65x-75 = -4x^2+18x+90$

$\Leftrightarrow 24x-168=0$

$\Leftrightarrow x = 7 \in \mathbb{G}$

Probe: l.S. $\dfrac{6\cdot 7^2-23\cdot 7-3}{7^2-2\cdot 7-15} - \dfrac{10\cdot 7-15}{7+3} = \dfrac{130}{20} - \dfrac{55}{10} = \dfrac{20}{20} = 1$ ✓

r.S. $\dfrac{30-4\cdot 7}{7-5} = \dfrac{30-28}{2} = \dfrac{2}{2} = 1$ ✓

$\Rightarrow \mathbb{L} = \{7\}$.

A44

g) $\left(\frac{1}{x}-2\right):\left(\frac{x}{3}+1\right)+\frac{x-1}{x+3}=1-\frac{2x+5}{x^2+3x}$

$$\Leftrightarrow \quad \frac{\frac{1}{x}-2}{\frac{1}{3}x+1}+\frac{x-1}{x+3}=1-\frac{2x+5}{x^2+3x}$$

$$\Leftrightarrow \quad \frac{3\cdot\left(\frac{1}{x}-2\right)}{3\cdot\left(\frac{1}{3}x+1\right)}+\frac{x-1}{x+3}=1-\frac{2x+5}{x^2+3x}$$

$$\Leftrightarrow \quad \frac{\frac{3}{x}-6}{x+3}+\frac{x-1}{x+3}=1-\frac{2x+5}{x^2+3x}$$

Nenner	EF
$x+3$	x
$x^2+3x = x(x+3)$	1

HN $= x(x+3)$

$\mathbb{G} = \mathbb{R}\setminus\{-3;0\}$

$$\frac{\left(\frac{3}{x}-6+x-1\right)\cdot x}{HN}=\frac{x^2+3x-(2x+5)}{HN} \quad \Big|\cdot HN\,(\neq 0)$$

$$\Leftrightarrow \quad 3-6x+x^2-x = x^2+3x-2x-5$$

$$\Leftrightarrow \quad -7x+3 = x-5$$

$$\Leftrightarrow \quad 8x=8$$

$$\Leftrightarrow \quad x=1\in\mathbb{G}$$

Probe:

l.S.: $(1-2):\left(\frac{1}{3}+1\right)+\frac{0}{4}=(-1)\cdot\frac{3}{4}=-\frac{3}{4}$ ✓

r.S.: $1-\frac{2+5}{1+3}=1-\frac{7}{4}=-\frac{3}{4}$ ✓

$\Rightarrow \mathbb{L}=\{1\}.$

A44

h) $\dfrac{x}{x+\dfrac{1}{x+\frac{1}{x}}} : \dfrac{1}{x^2+2} = x^2+1 \qquad (x \neq 0)$

$$\Leftrightarrow \quad \dfrac{x}{x+\dfrac{1}{\frac{x^2+1}{x}}} \cdot \dfrac{x^2+2}{1} = x^2+1$$

$$\Leftrightarrow \quad \dfrac{x\,(x^2+2)}{x+\dfrac{x}{x^2+1}} = x^2+1$$

$$\Leftrightarrow \quad \dfrac{x\,(x^2+2)}{\dfrac{x(x^2+1)+x}{x^2+1}} = x^2+1$$

$$\Leftrightarrow \quad \dfrac{x\,(x^2+1)\,(x^2+2)}{x(x^2+1)+x} = x^2+1$$

$$\Leftrightarrow \quad \dfrac{\cancel{x}\,(x^2+1)\,(x^2+2)}{\cancel{x}\,\cancel{(x^2+2)}} = x^2+1$$

$$\Leftrightarrow \quad x^2+1 = x^2+1$$

$$\Leftrightarrow \quad 0 = 0$$

$\mathbb{L} = \mathbb{R}\setminus\{0\}$

A45 a) $\sqrt{4x+5} = x+2$

$$\sqrt{4x+5} = x+2 \quad |\uparrow^2$$

$$\Rightarrow \quad 4x+5 = (x+2)^2$$

$$\Leftrightarrow \quad 4x+5 = x^2+4x+4 \quad |-4x-4$$

$$\Leftrightarrow \quad x^2 = 1 \quad |\pm\sqrt{}$$

$$\Leftrightarrow \quad x = \pm 1$$

Probe: $\boxed{x=1}$: l.S. $\sqrt{4+5} = \sqrt{9} = 3$

r.S. $1+2 = 3$ ✓

$\boxed{x=-1}$: l.S. $\sqrt{-4+5} = \sqrt{1} = 1$

r.S. $-1+2 = 1$ ✓

$$\Rightarrow \quad \mathbb{L} = \{-1; +1\}$$

b) $\sqrt{x^2+2} = 3x$

$$\sqrt{x^2+2} = 3x \quad |\uparrow^2$$

$$\Rightarrow \quad x^2+2 = 9x^2 \quad |-x^2$$

$$\Leftrightarrow \quad 8x^2 = 2 \quad |:8$$

$$\Leftrightarrow \quad x^2 = \frac{1}{4} \quad |\pm\sqrt{}$$

$$\Leftrightarrow \quad x = \pm\frac{1}{2}$$

Probe: $\boxed{x=\frac{1}{2}}$: l.S. $\sqrt{\frac{1}{4}+2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

r.S. $3\cdot\frac{1}{2} = \frac{3}{2}$ ✓

$\boxed{x=-\frac{1}{2}}$: l.S. $\sqrt{\frac{1}{4}+2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

r.S. $3\cdot(-\frac{1}{2}) = -\frac{3}{2}$ ↯

$$\Rightarrow \quad \mathbb{L} = \left\{\frac{3}{2}\right\}$$

A45 c) $\sqrt{2x+8} - \sqrt{5+x} = 1$

$$\sqrt{2x+8} - \sqrt{5+x} = 1 \quad | +\sqrt{5+x}$$

$$\Leftrightarrow \sqrt{2x+8} = 1 + \sqrt{5+x} \quad | \uparrow^2$$

$$\Rightarrow 2x+8 = (1+\sqrt{5+x})^2$$

$$\Leftrightarrow 2x+8 = 1 + 2\sqrt{5+x} + 5 + x \quad | -x-6$$

$$\Leftrightarrow 2\sqrt{5+x} = x+2 \quad | \uparrow^2$$

$$\Rightarrow 4(5+x) = (x+2)^2$$

$$\Leftrightarrow 20 + 4x = x^2 + 4x + 4 \quad | -4x-4$$

$$\Leftrightarrow x^2 = 16 \quad | \pm\sqrt{}$$

$$\Leftrightarrow x = \pm 4$$

Probe: $\boxed{x=4}$: l.S. $\sqrt{8+8} - \sqrt{5+4} = \sqrt{16} - \sqrt{9} = 4-3 = 1$

r.S. 1 ✓

$\boxed{x=-4}$: l.S. $\sqrt{-8+8} - \sqrt{5-4} = \sqrt{0} - \sqrt{1} = -1$

r.S. 1 ↯

$$\Rightarrow \mathbb{L} = \{4\}$$

d) $\sqrt{x^2+x} = 1 + \sqrt{x^2-x}$

$$\sqrt{x^2+x} = 1 + \sqrt{x^2-x} \quad | \uparrow^2$$

$$\Rightarrow x^2+x = (1+\sqrt{x^2-x})^2$$

$$\Leftrightarrow x^2+x = 1 + 2\sqrt{x^2-x} + x^2 - x \quad | -x^2+x-1$$

$$\Leftrightarrow 2\sqrt{x^2-x} = 2x-1 \quad | \uparrow^2$$

$$\Rightarrow 4(x^2-x) = (2x-1)^2$$

$$\Leftrightarrow 4x^2-4x = 4x^2-4x+1 \quad | -4x^2+4x$$

$$\Leftrightarrow 0 = 1 \quad ↯$$

$$\Rightarrow \mathbb{L} = \emptyset \; (= \{\})$$

A45

e) $\sqrt{\dfrac{x-1}{x+1}} + \sqrt{\dfrac{x+1}{x-1}} = \dfrac{5}{2} \qquad (x \neq \pm 1)$

$$\sqrt{\frac{x-1}{x+1}} + \sqrt{\frac{x+1}{x-1}} = \frac{5}{2} \quad \Big|\, (\;)^2$$

$$\Rightarrow \left(\sqrt{\frac{x-1}{x+1}} + \sqrt{\frac{x+1}{x-1}}\right)^2 = \frac{25}{4}$$

$$\Leftrightarrow \frac{x-1}{x+1} + 2\cdot\sqrt{\frac{x-1}{x+1}}\cdot\sqrt{\frac{x+1}{x-1}} + \frac{x+1}{x-1} = \frac{25}{4}$$

$$\Leftrightarrow \frac{x-1}{x+1} + 2\underbrace{\sqrt{\frac{(x-1)(x+1)}{(x+1)(x-1)}}}_{1} + \frac{x+1}{x-1} = \frac{25}{4}$$

$$\Leftrightarrow \frac{x-1}{x+1} + 2 + \frac{x+1}{x-1} = \frac{25}{4} \quad \Big|\, -2$$

$$\Leftrightarrow \frac{x-1}{x+1} + \frac{x+1}{x-1} = \frac{17}{4}$$

$$\Leftrightarrow \frac{(x-1)^2 + (x+1)^2}{(x+1)\cdot(x-1)} = \frac{17}{4} \quad \Big|\cdot (x+1)(x-1)\ (\neq 0)$$

$$\Leftrightarrow x^2 - 2x + 1 + x^2 + 2x + 1 = \frac{17}{4}(x+1)(x-1)$$

$$\Leftrightarrow 2x^2 + 2 = \frac{17}{4}x^2 - \frac{17}{4} \quad \Big|\, -2x^2 + \frac{17}{4}$$

$$\Leftrightarrow \frac{9}{4}x^2 = \frac{25}{4} \quad \Big|\cdot \frac{4}{9}$$

$$\Leftrightarrow x^2 = \frac{25}{9} \quad \Big|\, \pm\sqrt{\;}$$

$$\Leftrightarrow x = \pm\frac{5}{3}$$

<u>Probe:</u> $\boxed{x = \frac{5}{3}}$ l.S. $\sqrt{\dfrac{\frac{5}{3}-1}{\frac{5}{3}+1}} + \sqrt{\dfrac{\frac{5}{3}+1}{\frac{5}{3}-1}} = \sqrt{\dfrac{\frac{2}{3}}{\frac{8}{3}}} + \sqrt{\dfrac{\frac{8}{3}}{\frac{2}{3}}} = \sqrt{\dfrac{2}{8}} + \sqrt{\dfrac{8}{2}}$

$= \sqrt{\frac{1}{4}} + \sqrt{4} = \frac{1}{2} + 2 = \frac{5}{2}$

r.S. $\frac{5}{2}$ ✓

$\boxed{x = -\frac{5}{3}}$ l.S. $\sqrt{\dfrac{-\frac{5}{3}-1}{-\frac{5}{3}+1}} + \sqrt{\dfrac{-\frac{5}{3}+1}{-\frac{5}{3}-1}} = \sqrt{\dfrac{-\frac{8}{3}}{-\frac{2}{3}}} + \sqrt{\dfrac{-\frac{2}{3}}{-\frac{8}{3}}} = \sqrt{\dfrac{8}{2}} + \sqrt{\dfrac{2}{8}}$

$= \sqrt{4} + \sqrt{\frac{1}{4}} = 2 + \frac{1}{2} = \frac{5}{2}$

r.S. $\frac{5}{2}$ ✓

$\Rightarrow \mathbb{L} = \left\{-\frac{5}{3}; \frac{5}{3}\right\}$

A45

f) $5\sqrt{x-1} - 2\sqrt{2x+5} = \sqrt{3x-5}$

$5\sqrt{x-1} - 2\sqrt{2x+5} = \sqrt{3x-5} \quad |\ ^2$

$\Rightarrow \left(5\sqrt{x-1} - 2\sqrt{2x+5}\right)^2 = 3x-5$

$\Leftrightarrow 25(x-1) - 20\cdot\sqrt{x-1}\sqrt{2x+5} + 4(2x+5) = 3x-5$

$\Leftrightarrow 25x - 25 - 20\cdot\sqrt{2x^2+3x-5} + 8x + 20 = 3x-5$

$\Leftrightarrow 33x - 5 - 20\sqrt{2x^2+3x-5} = 3x-5 \quad |\ -3x+5+20\sqrt{2x^2+3x-5}$

$\Leftrightarrow 30x = 20\sqrt{2x^2+3x-5} \quad |\ ^2$

$\Rightarrow 900x^2 = 400(2x^2+3x-5)$

$\Leftrightarrow 900x^2 = 800x^2 + 1200x - 2000 \quad |\ -800x^2 - 1200x + 2000$

$\Leftrightarrow 100x^2 - 1200x + 2000 = 0 \quad |\ :100$

$\Leftrightarrow x^2 - 12x + 20 = 0$

$x_{1/2} = \frac{12 \pm \sqrt{144-80}}{2} = \frac{12 \pm \sqrt{64}}{2} = 6 \pm 4$

$x_1 = 10;\ x_2 = 2$

<u>Probe</u>: $\boxed{x=10}$ l.S. $5\sqrt{9} - 2\cdot\sqrt{25} = 5\cdot3 - 2\cdot5 = 15 - 10 = 5$

r.S. $\sqrt{25} = 5$ ✓

$\boxed{x=2}$: l.S. $5\sqrt{1} - 2\sqrt{9} = 5 - 2\cdot3 = 5 - 6 = -1$

r.S. $\sqrt{6-5} = \sqrt{1} = 1$ ↯

$\Rightarrow \mathbb{L} = \{10\}$

A45

g) $\sqrt{x^2+2+\sqrt{7-6x}}+x=1$

$$\sqrt{x^2+2+\sqrt{7-6x}}+x=1 \quad |-x$$

$$\Leftrightarrow \sqrt{x^2+2+\sqrt{7-6x}}=1-x \quad |\uparrow^2$$

$$\Rightarrow x^2+2+\sqrt{7-6x}=(1-x)^2$$

$$\Leftrightarrow x^2+2+\sqrt{7-6x}=1-2x+x^2 \quad |-x^2-2$$

$$\Leftrightarrow \sqrt{7-6x}=-1-2x \quad |\uparrow^2$$

$$\Rightarrow 7-6x=(-1-2x)^2$$

$$\Leftrightarrow 7-6x=1+4x+4x^2 \quad |-7+6x$$

$$\Leftrightarrow 4x^2+10x-6=0 \quad |:2$$

$$\Leftrightarrow 2x^2+5x-3=0$$

$$x_{1/2}=\frac{-5\pm\sqrt{25+24}}{4}=\frac{-5\pm 7}{4}$$

$$x_1=\frac{1}{2}; \quad x_2=-3$$

Probe:

$\boxed{x=\frac{1}{2}}$: l.S. $\sqrt{\frac{1}{4}+2+\sqrt{7-3}}+\frac{1}{2}=\sqrt{\frac{9}{4}+2}=\sqrt{\frac{17}{4}}\approx 2{,}06$

r.S. 1 ↯

$\boxed{x=-3}$ l.S. $\sqrt{9+2+\sqrt{25}}-3=\sqrt{11+5}-3=\sqrt{16}-3=1$

r.S. 1 ✓

$$\Rightarrow \mathbb{L}=\{-3\}$$

A45 h)
$$\frac{7\cdot\sqrt{\frac{x+1}{x}}+5}{7\sqrt{\frac{x}{x+1}}-5}=6$$

Wg. $\sqrt{\frac{x}{x+1}}=\frac{1}{\sqrt{\frac{x+1}{x}}}$ ist es naheliegend, die Substitution

$u=\sqrt{\frac{x+1}{x}}$ durchzuführen. $(x\neq 0,\ x\neq -1)$

Subst. $u=\sqrt{\frac{x+1}{x}}\ (\geq 0)$

$$\frac{7u+5}{7\cdot\frac{1}{u}-5}=6 \quad |\cdot(7\cdot\tfrac{1}{u}-5)$$

$$\Leftrightarrow\ 7u+5=6\cdot(7\cdot\tfrac{1}{u}-5)$$

$$\Leftrightarrow\ 7u+5=42\cdot\tfrac{1}{u}-30 \quad |\cdot u\ (\neq 0)$$

$$\Leftrightarrow\ 7u^2+5u=42-30u \quad |+30u-42$$

$$\Leftrightarrow\ 7u^2+35u-42=0$$

$$u_{1/2}=\frac{-35\pm\sqrt{35^2+4\cdot7\cdot42}}{14}=\frac{-35\pm49}{14}$$

$u_1=1;\ u_2=-6$

Rücksubst. $\sqrt{\frac{x+1}{x}}=u$

$\boxed{u_1=1}$

$$\sqrt{\frac{x+1}{x}}=1 \quad |(\,)^2$$

$$\Rightarrow\ \frac{x+1}{x}=1$$

$$\Leftrightarrow\ x+1=x \quad |-x$$

$$\Leftrightarrow\ 1=0\ ↯$$

$\boxed{u_2=-6}$

$$\underbrace{\sqrt{\frac{x+1}{x}}}_{\geq 0}=-6\ ↯$$

$\Rightarrow\ \mathbb{L}=\emptyset$

A45 k) $x^2 = \sqrt{6x}\sqrt{3x} + 10x + 20 \qquad (x \geq 0)$

$$x^2 = \sqrt{6x}\sqrt{5x} + 10x + 20$$

$$\Leftrightarrow x^2 = \sqrt{30x^2} + 10x + 20$$

$(x \geq 0)$ $$\Leftrightarrow x^2 = \sqrt{30}\,x + 10x + 20$$

$$\Leftrightarrow x^2 - (\sqrt{30} + 10)x - 20 = 0$$

$$x_{1|2} = \frac{\sqrt{30} + 10 \pm \sqrt{D}}{2} \quad \text{mit}$$

$$D = (-\sqrt{30} + 10)^2 + 4 \cdot 20 = 30 + 20\sqrt{30} + 100 + 80 = 20\sqrt{30} + 210$$

$$x_1 = \frac{\sqrt{30} + 10 + \sqrt{20\sqrt{30} + 210}}{2} \approx 16,6265$$

Da $x_2 < 0$ und aber $x \geq 0$ gelten muss, gibt es nur die Lsg. x_1

$$\mathbb{L} = \left\{ \frac{\sqrt{30} + 10 + \sqrt{20\sqrt{30} + 210}}{2} \right\}$$

A46 a) $10\cdot\left(\frac{5}{9}\right)^{4-x} = 2^{2x+1}$

$$10\cdot\left(\frac{5}{9}\right)^{4-x} = 2^{2x+1} \quad \Big| \lg(\ldots)$$

$$\Leftrightarrow \lg\left(10\cdot\left(\frac{5}{9}\right)^{4-x}\right) = \lg\left(2^{2x+1}\right)$$

$$\Leftrightarrow \underbrace{\lg 10}_{1} + (4-x)\lg\frac{5}{9} = (2x+1)\lg 2$$

$$\Leftrightarrow 1 + 4\cdot\lg\frac{5}{9} - x\lg\frac{5}{9} = 2x\lg 2 + \lg 2 \quad \Big| + x\lg\frac{5}{9} - \lg 2$$

$$\Leftrightarrow 1 + 4\cdot\lg\frac{5}{9} - \lg 2 = 2x\lg 2 + x\lg\frac{5}{9}$$

$$\Leftrightarrow 1 + 4\lg\frac{5}{9} - \lg 2 = x\left(2\lg 2 + \lg\frac{5}{9}\right) \quad \Big| :\left(2\lg 2 + \lg\frac{5}{9}\right)$$

$$\Leftrightarrow x = \frac{1 + 4\lg\frac{5}{9} - \lg 2}{2\lg 2 + \lg\frac{5}{9}} \approx -0{,}9289 \qquad \mathbb{L} = \left\{\frac{1+4\lg\frac{5}{9}-\lg 2}{2\lg 2+\lg\frac{5}{9}}\right\}$$

b) $4^{2x-3}\cdot 32^{1-x} = \frac{1}{8}$

$$4^{2x-3}\cdot 32^{1-x} = \frac{1}{8} \quad \Big| \ln(\ldots)$$

$$\Leftrightarrow \ln\left(4^{2x-3}\cdot 32^{1-x}\right) = \ln\frac{1}{8}$$

$$\Leftrightarrow \ln\left(4^{2x-3}\right) + \ln\left(32^{1-x}\right) = \ln\frac{1}{8}$$

$$\Leftrightarrow (2x-3)\ln 4 + (1-x)\ln 32 = \ln\frac{1}{8}$$

$$\Leftrightarrow 2x\cdot\ln 4 - 3\ln 4 + \ln 32 - x\ln 32 = \ln\frac{1}{8} \quad \Big| +3\ln 4 - \ln 32$$

$$\Leftrightarrow 2x\cdot\ln 4 - x\cdot\ln 32 = \ln\tfrac{1}{8} + 3\ln 4 - \ln 32$$

$$\Leftrightarrow x\cdot(2\ln 4 - \ln 32) = \ln\frac{4^3}{8\cdot 32}$$

$$\Leftrightarrow x\cdot\ln\left(\frac{4^2}{32}\right) = \ln\frac{1}{4}$$

$$\Leftrightarrow x\cdot\ln\frac{1}{2} = \ln\frac{1}{4} \quad \Big| :\ln\frac{1}{2}$$

$$\Leftrightarrow x = \frac{\ln\frac{1}{4}}{\ln\frac{1}{2}} = \frac{-\ln 4}{-\ln 2} = \frac{\ln 4}{\ln 2} = \frac{\ln 2^2}{\ln 2} = \frac{2\ln 2}{\ln 2} = \underline{\underline{2}} \qquad \mathbb{L} = \{2\}$$

A 46 c) $4 \cdot 2^{\sqrt{x}} = 0{,}5^{-x} \qquad (x \geq 0)$

$$4 \cdot 2^{\sqrt{x}} = 0{,}5^{-x} \quad | \ln(\ldots)$$

$$\Leftrightarrow \ln(4 \cdot 2^{\sqrt{x}}) = \ln(0{,}5^{-x})$$

$$\Leftrightarrow \ln 4 + \sqrt{x} \cdot \ln 2 = -x \ln 0{,}5 \quad | + x \ln 0{,}5$$

$$\Leftrightarrow x \cdot \ln 0{,}5 + \sqrt{x} \cdot \ln 2 + \ln 4 = 0$$

Subst.: $u = \sqrt{x} \ (\geq 0)$

$$(\ln 0{,}5) u^2 + (\ln 2) u + \ln 4 = 0 \qquad (\text{quadrat. Gl.})$$

$$u_{1/2} = \frac{-\ln 2 \pm \sqrt{D}}{2 \cdot \ln 0{,}5} \quad \text{mit } D = (\ln 2)^2 - 4 \underbrace{\ln 0{,}5}_{<0} \cdot \ln 4 > 0$$

Wg. $\ln 0{,}5 = \ln \frac{1}{2} = -\ln 2$ und $\ln 4 = \ln 2^2 = 2 \cdot \ln 2$ gilt

$$D = (\ln 2)^2 + 4 \cdot (-\ln 2) \cdot 2 \cdot \ln 2 = (\ln 2)^2 + 8 \cdot (\ln 2)^2 = 9 \cdot \ln 2^2$$

$$\Rightarrow u_{1/2} = \frac{-\ln 2 \pm \sqrt{9 \cdot (\ln 2)^2}}{-2 \ln 2} = \frac{-\ln 2 \pm 3 \cdot \ln 2}{-2 \cdot \ln 2} =$$

$$= \frac{\ln 2 (-1 \pm 3)}{-2 \ln 2} = \frac{-1 \pm 3}{-2}$$

$$\Rightarrow u_1 = -1 \ ; \ u_2 = 2$$

Da $u = \sqrt{x} \geq 0$ gelten muss, scheidet $u_1 = -1$ aus.

Rücksubst.: $\sqrt{x} = u$

$\underline{u_2 = 2}$: $\quad \sqrt{x} = 2 \quad | (\ldots)^2$

$$\underset{(x \geq 0)}{\Leftrightarrow} x = 4$$

Probe: $\underline{x = 4}$: l.S. $4 \cdot 2^{\sqrt{4}} = 4 \cdot 2^2 = 16$ $\quad \Rightarrow \mathbb{L} = \{4\}$

r.S. $0{,}5^{-4} = 2^4 = 16$ ✓

A46 d) $\frac{2^{5x}}{7^{x+2}} = 10$

$$\frac{2^{5x}}{7^{x+2}} = 10 \quad \Big| \cdot 7^{x+2} \; (\neq 0)$$

$$\Leftrightarrow \quad 2^{5x} = 10 \cdot 7^{x+2} \quad \big| \ln(\ldots)$$

$$\Leftrightarrow \quad \ln(2^{5x}) = \ln(10 \cdot 7^{x+2})$$

$$\Leftrightarrow \quad 5x \cdot \ln 2 = \ln 10 + (x+2)\ln 7$$

$$\Leftrightarrow \quad 5x \ln 2 = \ln 10 + x \cdot \ln 7 + 2\ln 7 \quad \big| -x\ln 7$$

$$\Leftrightarrow \quad 5x \ln 2 - x\ln 7 = \ln 10 + 2\ln 7$$

$$\Leftrightarrow \quad x(5\ln 2 - \ln 7) = \ln(10 \cdot 7^2)$$

$$\Leftrightarrow \quad x \cdot \ln\frac{2^5}{7} = \ln 490 \quad \Big| : \ln\frac{2^5}{7} \; \left(= \ln\frac{32}{7}\right)$$

$$\Leftrightarrow \quad x = \frac{\ln 490}{\ln \frac{32}{7}} \approx 4{,}076 \qquad \mathbb{L} = \left\{\frac{\ln 490}{\ln \frac{32}{7}}\right\}$$

e) $(\sqrt{2})^{x+3} = \frac{3 \cdot 13^{4-x}}{(\sqrt{5})^x}$

$$(\sqrt{2})^{x+3} = \frac{3 \cdot 13^{4-x}}{(\sqrt{5})^x} \quad \Big| \cdot (\sqrt{5})^x \; (\neq 0)$$

$$\Leftrightarrow \quad (\sqrt{2})^{x+3} (\sqrt{5})^x = 3 \cdot 13^{4-x} \qquad (\sqrt{2} = 2^{1/2};\ \sqrt{5} = 5^{1/2})$$

$$\Leftrightarrow \quad 2^{\frac{1}{2}x + \frac{3}{2}} \cdot 5^{1/2 x} = 3 \cdot 13^{4-x} \quad \big| \ln(\ldots)$$

$$\Leftrightarrow \quad \ln\left(2^{1/2 x + 3/2}\right) + \ln 5^{1/2 x} = \ln 3 + \ln\left(13^{4-x}\right)$$

$$\Leftrightarrow \quad \left(\tfrac{1}{2}x + \tfrac{3}{2}\right)\ln 2 + \tfrac{1}{2}x \ln 5 = \ln 3 + (4-x)\ln 13$$

$$\Leftrightarrow \quad \tfrac{1}{2}x \cdot \ln 2 + \tfrac{3}{2}\ln 2 + \tfrac{1}{2}x \ln 5 = \ln 3 + 4 \cdot \ln 13 - x\ln 13 \quad \Big| -\tfrac{3}{2}\ln 2 + x\ln 13$$

$$\Leftrightarrow \quad \tfrac{1}{2}x \ln 2 + \tfrac{1}{2}x \ln 5 + x\ln 13 = \ln 3 + 4\ln 13 - \tfrac{3}{2}\ln 2$$

$$\Leftrightarrow \quad x\left(\tfrac{1}{2}\ln 2 + \tfrac{1}{2}\ln 5 + \ln 13\right) = \ln 3 + 4 \cdot \ln 13 - \tfrac{3}{2}\ln 2 \quad \Big| : \left(\tfrac{1}{2}\ln 2 + \tfrac{1}{2}\ln 5 + \ln 13\right)$$

$$\Leftrightarrow \quad x = \frac{\ln 3 + 4\ln 13 - \frac{3}{2}\ln 2}{\frac{1}{2}\ln 2 + \frac{1}{2}\ln 5 + \ln 13} \approx 2{,}777 \qquad \mathbb{L} = \left\{\frac{\ln 3 + 4\ln 13 - \frac{3}{2}\ln 2}{\frac{1}{2}\ln 2 + \frac{1}{2}\ln 5 + \ln 13}\right\}$$

A46 f) $9\cdot\left(\frac{2}{3}\right)^{2x+1} + 54\left(\frac{2}{3}\right)^{x-1} = 42$

Aufgrund der Addition auf der l.S versucht man eine geeignete Substitution zu finden.

Subst.: $u = \left(\frac{2}{3}\right)^x$

$$\left(\frac{2}{3}\right)^{2x+1} = \left(\frac{2}{3}\right)^{2x}\cdot\frac{2}{3} = \left[\left(\frac{2}{3}\right)^x\right]^2\cdot\frac{2}{3} = u^2\cdot\frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{x-1} = \left(\frac{2}{3}\right)^x\cdot\left(\frac{2}{3}\right)^{-1} = u\cdot\left(\frac{2}{3}\right)^{-1} = u\cdot\frac{3}{2}$$

$\Rightarrow$ neue Gl. in der Unbestimmten u:

$$9\cdot u^2\cdot\frac{2}{3} + 54\cdot u\cdot\frac{3}{2} = 42 \quad |-42$$

$$\Leftrightarrow \quad 6u^2 + 81u - 42 = 0 \quad |:3$$

$$\Leftrightarrow \quad 2u^2 + 27u - 14 = 0$$

$$u_{1/2} = \frac{-27 \pm \sqrt{D}}{4} \quad \text{mit } D = 27^2 + 4\cdot 2\cdot 14 = 841$$

$$u_{1/2} = \frac{-27 \pm \sqrt{841}}{4} = \frac{-27 \pm 29}{4}$$

$$u_1 = \frac{1}{2}; \quad u_2 = -14$$

Da $u = \left(\frac{2}{3}\right)^x > 0$ gilt, scheidet die Lsg. $u_2 = -14$ aus.

Rücksubst.: $\left(\frac{2}{3}\right)^x = u$

$u_1 = \frac{1}{2}$

$$\left(\frac{2}{3}\right)^x = \frac{1}{2} \quad |\ln(\ldots)$$

$$\Leftrightarrow \quad \ln\left(\frac{2}{3}\right)^x = \ln\frac{1}{2}$$

$$\Leftrightarrow \quad x\cdot\ln\frac{2}{3} = \ln\frac{1}{2} \quad |:\ln\frac{2}{3}$$

$$\Leftrightarrow \quad x = \frac{\ln 1/2}{\ln 2/3} \approx 1{,}710 \qquad \mathbb{L} = \left\{\frac{\ln 1/2}{\ln 2/3}\right\}$$

A46 a) $3^x + 9^x = \sqrt{3}(\sqrt{3}+1)$

Substitution wg. Addition auf l.S.

Subst.: $u = 3^x$

$9^x = (3^2)^x = 3^{2x} = (3^x)^2 = u^2$

$\Rightarrow \quad u + u^2 = \sqrt{3}(\sqrt{3}+1)$

$\Leftrightarrow \quad u^2 + u = 3 + \sqrt{3} \quad | -(3+\sqrt{3})$

$\Leftrightarrow \quad u^2 + u - (3+\sqrt{3}) = 0$

$u_{1,2} = \dfrac{-1 \pm \sqrt{D}}{2}$ mit $D = 1 + 4(3+\sqrt{3}) = 13 + 4\sqrt{3} \; (>0)$

$= 4\cdot\sqrt{3}^2 + 4\sqrt{3} + 1 = (2\sqrt{3}+1)^2$

$u_1 = \dfrac{-1 + 2\sqrt{3} + 1}{2} = \sqrt{3}$

$u_2 = \dfrac{-1 - 2\sqrt{3} - 1}{2} = \dfrac{-2-2\sqrt{3}}{2} = -1-\sqrt{3} < 0$ (scheidet aus wg. $u = 3^x > 0$)

Rücksubst: $3^x = u$

$u_1 = \sqrt{3}$: $3^x = \sqrt{3}$

$\Leftrightarrow 3^x = 3^{1/2}$

$\Leftrightarrow x = 1/2$

$\mathbb{L} = \{\frac{1}{2}\}$

A46 h) $4^{x+2} - 5\cdot 2^{x+3} = 24$

Substitution wg. Differenz auf l.S.

Subst.: $u = 2^x$

$$4^{x+2} = 4^x \cdot 4^2 = (2^2)^x \cdot 16 = 2^{2x}\cdot 16 = (2^x)^2 \cdot 16 = u^2 \cdot 16$$

$$2^{x+3} = 2^x \cdot 2^3 = u \cdot 8$$

$\Rightarrow$ neue Gl. in der Unbestimmten u:

$$16u^2 - 40u = 24 \quad | -24$$

$$\Leftrightarrow 16u^2 - 40u - 24 = 0 \quad | :2$$

$$\Leftrightarrow 2u^2 - 5u - 3 = 0$$

$$u_{1/2} = \frac{5 \pm \sqrt{D}}{4} \quad \text{mit } D = 25 + 4\cdot 2\cdot 3 = 49$$

$$u_{1/2} = \frac{5 \pm 7}{4}$$

$$u_1 = 3$$

$$u_2 = -\frac{1}{2} \quad (\text{scheidet wg. } u = 2^x > 0 \text{ aus})$$

Rücksubst.: $2^x = u$

$\boxed{u_1 = 3}$ $\quad 2^x = 3 \quad | \ln(\ldots)$

$$\Leftrightarrow x \cdot \ln 2 = \ln 3 \quad | :\ln 2$$

$$\Leftrightarrow x = \frac{\ln 3}{\ln 2} \approx 1{,}585$$

$$\mathbb{L} = \left\{ \frac{\ln 3}{\ln 2} \right\}$$

A 46 k) $\left(\frac{1}{2}\right)^{x-1} \cdot \left(8^{x+1} - 4^5\right) = 16\left(4^x - 8\right)$

Substitution wg. Differenzen in den Klammern.

<u>Subst.:</u> $u = 2^x \ (>0)$

$\left(\frac{1}{2}\right)^{x-1} = 2^{1-x} = 2 \cdot 2^{-x} = 2 \cdot u^{-1}$

$8^{x+1} = 8^x \cdot 8 = (2^3)^x \cdot 8 = (2^x)^3 \cdot 8 = 8u^3$

$4^x = (2^2)^x = (2^x)^2 = u^2$

$\Rightarrow$ neue Gl. in d. Unbekannten u

$2u^{-1} \cdot (8u^3 - 4^5) = 16(u^2 - 8) \quad | \cdot u \ (>0)$

$\Leftrightarrow \quad 2 \cdot (8u^3 - 4^5) = 16u(u^2 - 8)$

$\Leftrightarrow \quad 16u^3 - 2 \cdot 4^5 = 16u^3 - 128u \quad | - 16u^3$

$\Leftrightarrow \quad -2^{11} = -2^7 u \quad | : (-2^7)$

$\Leftrightarrow \quad u = 2^4$

<u>Rücksubst.:</u> $2^x = u$

$\underline{u = 2^4}$ $\quad 2^x = 2^4$

$\Leftrightarrow \quad x = 4$

$\mathbb{L} = \{4\}$

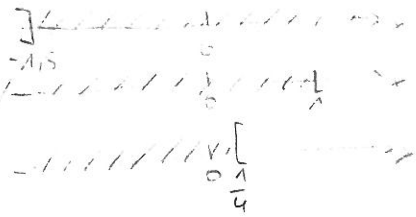
A47 Da sich beim Zusammenfassen von Logarithmen die max. Definitionsmenge (bzw. Grundmenge) der Ausgangsgleichung ändern kann (sie wird möglicherweise größer), bestimmt man entweder die max. Definitionsmenge der Ausgangsgleichung oder führt am Ende eine Probe durch.

a) $\lg(2x+3) + \lg(1-x) = \lg(1-4x)$

max. Grundmenge $\mathbb{G}$: $2x+3>0 \Leftrightarrow x>-\frac{3}{2}$

$1-x>0 \Leftrightarrow x<1$

$1-4x>0 \Leftrightarrow x<\frac{1}{4}$



$\Rightarrow \mathbb{G} = \{x>-\frac{3}{2}\} \cap \{x<1\} \cap \{x<\frac{1}{4}\} =]-\frac{3}{2}; \frac{1}{4}[$

$\lg(2x+3) + \lg(1-x) = \lg(1-4x)$ über $\mathbb{G}$

$\Leftrightarrow \lg[(2x+3)(1-x)] = \lg(1-4x) \quad | \, 10^{\wedge}$

$\Leftrightarrow (2x+3)(1-x) = 1-4x$

$\Leftrightarrow -2x^2 - x + 3 = 1-4x \quad | +2x^2+x-3$

$\Leftrightarrow 2x^2 - 3x - 2 = 0$

$x_{1/2} = \frac{3 \pm \sqrt{D}}{4}$ mit $D = 9 + 4 \cdot 2 \cdot 2 = 25$

$x_{1/2} = \frac{3 \pm 5}{4}$

$\left.\begin{array}{l} x_1 = 2 \notin \mathbb{G} \\ x_2 = -\frac{1}{2} \in \mathbb{G} \end{array}\right\} \Rightarrow \mathbb{L} = \{-\frac{1}{2}\}$

bzw. Probe:

$\boxed{x_1 = 2}$ l.S. $\lg(7) + \lg(-1)$ ↯

$\boxed{x_2 = -\frac{1}{2}}$ l.S. $\lg(2) + \lg(1{,}5) = \lg(2 \cdot 1{,}5) = \lg(3)$

r.S. $\lg(1+2) = \lg(3)$ ✓

A47 b) $\log_9(x^2+1) = \log_3(2x-1)$

Logarithmus umrechnen: $\log_9(x^2+1) = \frac{\log_3(x^2+1)}{\log_3(9)} = \frac{\log_3(x^2+1)}{2}$

$\log_9(x^2+1) = \log_3(2x-1)$ max. Grundmenge $G = \{x > \frac{1}{2}\}$

$\Leftrightarrow \frac{1}{2} \cdot \log_3(x^2+1) = \log_3(2x+1) \quad | \cdot 2$

$\Leftrightarrow \log_3(x^2+1) = 2 \cdot \log_3(2x-1)$

$\Leftrightarrow \log_3(x^2+1) = \log_3((2x-1)^2) \quad | 3^{\square}$

$\Leftrightarrow x^2+1 = (2x-1)^2$

$\Leftrightarrow x^2+1 = 4x^2-4x+1 \quad | -x^2-1$

$\Leftrightarrow 3x^2-4x = 0$

$\Leftrightarrow x(3x-4) = 0$

$x = 0 \notin G$ oder $x = \frac{4}{3} \in G \Rightarrow \mathbb{L} = \{\frac{4}{3}\}$

bzw. Probe:

$x=0$: l.S. $\log_9(1)$

r.S. $\log_3(-1)$ ↯

$x = \frac{4}{3}$: l.S. $\log_9(\frac{16}{9}+1) = \log_9(\frac{25}{9}) = \frac{\log_3(\frac{25}{9})}{2} = \frac{1}{2}\log_3(\frac{25}{9})$

$= \log_3(\sqrt{\frac{25}{9}}) = \log_3(\frac{5}{3})$

r.S. $\log_3(2 \cdot \frac{4}{3} - 1) = \log_3(\frac{8}{3} - \frac{3}{3}) = \log_3(\frac{5}{3})$ ✓

A47 c) $\operatorname{lb}(x^2-1) - \operatorname{lb}(4x-1) = \operatorname{lb}(\frac{1}{3})$

max. Grundmenge $\mathbb{G}$: $x^2-1>0 \Leftrightarrow |x|>1$

$4x-1>0 \Leftrightarrow x>\frac{1}{4}$

$\Rightarrow \mathbb{G} =]1, \infty[= \{x>1\}$

$\operatorname{lb}(x^2-1) - \operatorname{lb}(4x-1) = \operatorname{lb}(\frac{1}{3}) \quad | + \operatorname{lb}(4x-1)$

$\Leftrightarrow \operatorname{lb}(x^2-1) = \operatorname{lb}(\frac{1}{3}) + \operatorname{lb}(4x-1)$

$\Leftrightarrow \operatorname{lb}(x^2-1) = \operatorname{lb}[\frac{1}{3}(4x-1)] \quad | 2^{\wedge}$

$\Leftrightarrow x^2-1 = \frac{1}{3}(4x-1) \quad | \cdot 3$

$\Leftrightarrow 3x^2-3 = 4x-1 \quad | -4x+1$

$\Leftrightarrow 3x^2-4x-2 = 0$

$x_{1/2} = \frac{4 \pm \sqrt{D}}{6}$ mit $D = 4^2 + 4 \cdot 3 \cdot 2 = 16 + 24 = 40$

$x_{1/2} = \frac{4 \pm 2\sqrt{10}}{6} = \frac{2}{3} \pm \frac{1}{3}\sqrt{10}$

$\left.\begin{array}{l} x_1 = \frac{2+\sqrt{10}}{3} \approx 1{,}72 \in \mathbb{G} \\ x_2 = \frac{2-\sqrt{10}}{3} \approx -0{,}387 \notin \mathbb{G} \end{array}\right\} \Rightarrow \mathbb{L} = \left\{\frac{2+\sqrt{10}}{3}\right\}$

bzw. Probe:

$x_1 = \frac{2+\sqrt{10}}{3}$: l.S. $\operatorname{lb}\left[\left(\frac{2+\sqrt{10}}{3}\right)^2 - 1\right] - \operatorname{lb}\left[4 \cdot \frac{2+\sqrt{10}}{3} - 1\right] =$

$= \operatorname{lb}\left[\frac{4+4\sqrt{10}+10}{9} - 1\right] - \operatorname{lb}\left[\frac{8+4\sqrt{10}}{3} - 1\right] =$

$= \operatorname{lb}\left[\frac{5+4\sqrt{10}}{9}\right] - \operatorname{lb}\left[\frac{5+4\sqrt{10}}{3}\right] = \operatorname{lb}\left[\frac{5+4\sqrt{10}}{9} \cdot \frac{3}{5+4\sqrt{10}}\right] = \operatorname{lb}(\frac{1}{3})$

r.S. $\operatorname{lb}(1/3)$ ✓

$x_2 = \frac{2-\sqrt{10}}{3}$: l.S. $\operatorname{lb}\left[\left(\frac{2-\sqrt{10}}{3}\right)^2 - 1\right] - \operatorname{lb}\left[4 \cdot \frac{2-\sqrt{10}}{3} - 1\right] =$

$= \operatorname{lb}\left[\underbrace{\frac{5-4\sqrt{10}}{9}}_{<0\ ↯}\right] - \operatorname{lb}\left[\underbrace{\frac{5-4\sqrt{10}}{3}}_{<0\ ↯}\right]$

A47 d) $\lg 2 + \lg[(x+2)(3x+5)] = \lg(5x^2-1)$

max. Grundmenge $\mathbb{G}$:

$$(x+2)(3x+5) > 0$$

$$\Leftrightarrow [(x+2>0 \text{ und } (3x+5)>0]$$
$$\text{oder } [(x+2<0 \text{ und } (3x+5)<0]$$

$$5x^2-1>0 \Leftrightarrow |x| > \tfrac{1}{5}$$

$$\Rightarrow \mathbb{G} = \left]-\infty;-2\right[\cup \left]-\tfrac{5}{3};-\tfrac{1}{5}\right[\cup \left]\tfrac{1}{5};\infty\right[$$
$$= \{x<-2\} \cup \{-\tfrac{5}{3}<x<-\tfrac{1}{5}\} \cup \{x>\tfrac{1}{5}\}$$

$$\lg 2 + \lg[(x+2)(3x+5)] = \lg(5x^2-1)$$

$$\Leftrightarrow \lg[2(x+2)(3x+5)] = \lg(5x^2-1) \quad | \, 10^{\wedge}$$

$$\Leftrightarrow 2(x+2)(3x+5) = 5x^2-1$$

$$\Leftrightarrow 6x^2+22x+20 = 5x^2-1 \quad | -5x^2+1$$

$$\Leftrightarrow x^2+22x+21 = 0$$

$$x_{1|2} = \frac{-22 \pm \sqrt{D}}{2} \quad \text{mit } D = 22^2 - 4\cdot 21 = 400$$

$$x_{1|2} = \frac{-22 \pm 20}{2} = -11 \pm 10$$

$$\left.\begin{array}{l} x_1 = -1 \in \mathbb{G} \\ x_2 = -21 \in \mathbb{G} \end{array}\right\} \Rightarrow \mathbb{L} = \{-21;-1\}$$

bzw. Probe: $x_1 = -1$: l.S. $\lg 2 + \lg(1\cdot 2) = \lg 2 + \lg 2 = \lg 4$
r.S. $\lg(5-1) = \lg 4$ ✓

$x_2 = -21$: l.S. $\lg 2 + \lg[(-19)\cdot(-58)] = \lg 2 + \lg(1102) = \lg(2204)$
r.S. $\lg(5\cdot 21^2 - 1) = \lg(2204)$ ✓

A47 c) $\lg(x^2+4) - \log_{\sqrt{10}}(3x+2) = 0$

max. Grundmenge $\mathbb{G}$: $x^2+4 > 0$ gilt stets

$3x+2 > 0 \Leftrightarrow x > -\frac{2}{3}$

$\Rightarrow \mathbb{G} = \{x > -\frac{2}{3}\}$

Logarithmus umrechnen: $\log_{\sqrt{10}}(3x+2) = \dfrac{\log_{10}(3x+2)}{\log_{10}(\sqrt{10})} = \dfrac{\log_{10}(3x+2)}{\frac{1}{2}}$

$= 2 \cdot \lg(3x+2)$

$$\lg(x^2+4) - \log_{\sqrt{10}}(3x+2) = 0 \quad (\text{üb. } \mathbb{G})$$

$$\Leftrightarrow \lg(x^2+4) - 2\lg(3x+2) = 0 \quad | + 2\lg(3x+2)$$

$$\Leftrightarrow \lg(x^2+4) = 2\lg(3x+2)$$

$$\Leftrightarrow \lg(x^2+4) = \lg[(3x+2)^2] \quad | 10^{\square}$$

$$\Leftrightarrow x^2+4 = (3x+2)^2$$

$$\Leftrightarrow x^2+4 = 9x^2+12x+4 \quad | -x^2-4$$

$$\Leftrightarrow 8x^2+12x = 0 \quad | :4$$

$$\Leftrightarrow 2x^2+3x = 0$$

$$\Leftrightarrow x(2x+3) = 0$$

$$\left.\begin{array}{l} x_1 = 0 \in \mathbb{G} \\ x_2 = -\frac{3}{2} \notin \mathbb{G} \end{array}\right\} \Rightarrow \mathbb{L} = \{0\}$$

bzw. Probe:

$x_1 = 0$ | l.S. $\lg(4) - \log_{\sqrt{10}}(2) = \lg(4) - 2\lg(2) = \lg(4) - \lg(4) = 0$

r.S. $= 0$ ✓

$x_2 = -\frac{3}{2}$ | l.S. $\lg(\frac{9}{4}+4) - \log_{\sqrt{10}}(\underbrace{-\frac{9}{4}+2}_{<0})$ ↯

A47 d) $\lg(2x^2+x-5) + \log_{0,1}(x^2+1) = \lg 2$

$\underbrace{x^2+1}_{>0}$

max. Grundmenge G: $2x^2+x-5 > 0 \Leftrightarrow x < \frac{-1-\sqrt{41}}{4} \approx -1{,}85$ oder $x > \frac{-1+\sqrt{41}}{4} \approx 1{,}35$

$G = \left]-\infty; \underbrace{\frac{-1-\sqrt{41}}{4}}_{-1{,}85}\right[\cup \left]\underbrace{\frac{-1+\sqrt{41}}{4}}_{1{,}35}; \infty\right[$

$\lg(2x^2+x-5) + \log_{0,1}(x^2+1) = \lg 2$

Logarithmus umrechnen: $\log_{0,1}(x^2+1) = \frac{\log_{10}(x^2+1)}{\log_{10}(0,1)} = \frac{\lg(x^2+1)}{-1}$

$= -\lg(x^2+1)$

$\lg(2x^2+x-5) - \lg(x^2+1) = \lg 2 \quad | + \lg(x^2+1)$

$\Leftrightarrow \lg(2x^2+x-5) = \lg 2 + \lg(x^2+1)$

$\Leftrightarrow \lg(2x^2+x-5) = \lg(2x^2+2) \quad | 10^{\wedge}$

$\Leftrightarrow 2x^2+x-5 = 2x^2+2 \quad | -2x^2-2$

$\Leftrightarrow x-7 = 0$

$\Leftrightarrow x = 7 \in G \qquad \mathbb{L} = \{7\}$

bzw. Probe: $\boxed{x=7}$

l.S. $\lg(98+7-5) + \log_{0,1}(50) = \lg(100) - \lg(50) = \lg\left(\frac{100}{50}\right) = \lg 2$

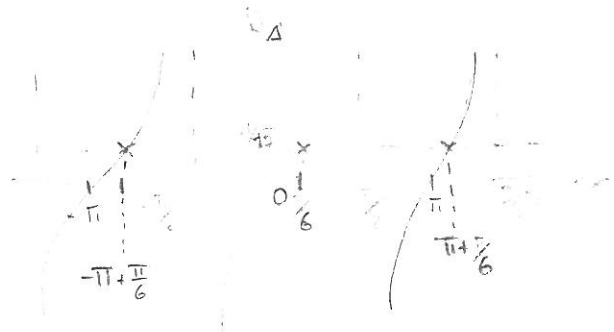
r.S. $\lg 2$ ✓

A48

a) $\sqrt{3}\tan(x) - 1 = 0 \qquad \mathbb{G} = [-\pi; \pi]$

$\sqrt{3}\tan(x) - 1 = 0 \qquad | +1 \quad | : \sqrt{3}$

$\Leftrightarrow \qquad \tan x = \frac{1}{\sqrt{3}}$



$\Rightarrow$ (mit TR): $\quad x_1 = \frac{\pi}{6} \quad (\hat{=} 30°)$

aus Grafik: $\quad x_2 = -\pi + \frac{\pi}{6} = -\frac{5}{6}\pi$

$\mathbb{L} = \left\{ -\frac{5}{6}\pi ; \frac{\pi}{6} \right\}$

b) $4\sin^2 x = 3, \qquad \mathbb{G} = [0; 2\pi]$

$4\sin^2 x = 3 \quad | :4$

$\Leftrightarrow \quad \sin^2 x = \frac{3}{4}$

$\Leftrightarrow \quad |\sin x| = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}$

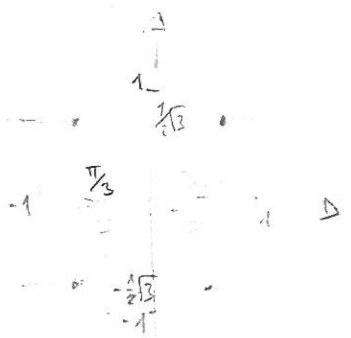
$\Leftrightarrow \quad \sin x = \pm\frac{1}{2}\sqrt{3}$

in $\mathbb{G} = [0; 2\pi]$ gilt:

$\sin x = \frac{1}{2}\sqrt{3} \Leftrightarrow x = \frac{\pi}{3} \quad (\hat{=} 60°)$ oder $x = \pi - \frac{\pi}{3} = \frac{2}{3}\pi \quad (\hat{=} 120°)$

$\sin x = -\frac{1}{2}\sqrt{3} \Leftrightarrow x = 2\pi - \frac{\pi}{3} \quad (\hat{=} 300°)$ oder $x = \pi + \frac{\pi}{3} = \frac{4}{3}\pi \quad (\hat{=} 240°)$

$= \frac{5}{3}\pi$



$\Rightarrow \quad \mathbb{L} = \left\{ \frac{\pi}{3} ; \frac{2}{3}\pi ; \frac{4}{3}\pi ; \frac{5}{3}\pi \right\}$

A48

c) $\sin(\underbrace{2x+1}_{\alpha}) = \frac{1}{2}$; $\mathbb{G} = [-2\pi; 2\pi]$

$\sin\alpha = \frac{1}{2} \Leftrightarrow \alpha = \frac{\pi}{6} + k\cdot 2\pi$ oder $\alpha = \frac{5}{6}\pi + k\cdot 2\pi$ $(k \in \mathbb{Z})$

Wg. $\mathbb{G} = [-2\pi; 2\pi]$ gilt für alle Lösungen x: $-2\pi \le x \le 2\pi$

$$-2\pi \le x \le 2\pi \quad | \cdot 2$$
$$\Leftrightarrow -4\pi \le 2x \le 4\pi \quad | +1$$
$$\Leftrightarrow -4\pi + 1 \le \underbrace{2x+1}_{\alpha} \le 4\pi + 1$$
$$\Leftrightarrow -4\pi + 1 \le \alpha \le 4\pi + 1 \quad (*)$$

Wir prüfen, für welche $k \in \mathbb{Z}$ $\alpha = \frac{\pi}{6} + k\cdot 2\pi$ bzw. $\alpha = \frac{5}{6}\pi + k\cdot 2\pi$ die Doppelungleichung $(*)$ erfüllt und beginnen mit $k = -2$:

$\boxed{k=-2}$: Nur $\alpha_1 = -4\pi + \frac{5}{6}\pi$ erfüllt $(*)$ (da $\frac{5}{6}\pi > 1$)

$\boxed{k=-1}$: $\alpha_2 = -2\pi + \frac{\pi}{6}$ und

$\alpha_3 = -2\pi + \frac{5}{6}\pi$ erfüllen $(*)$

$\boxed{k=0}$: $\alpha_4 = \frac{\pi}{6}$ und

$\alpha_5 = \frac{5}{6}\pi$ erfüllen $(*)$

$\boxed{k=1}$: $\alpha_6 = 2\pi + \frac{\pi}{6}$ und

$\alpha_7 = 2\pi + \frac{5}{6}\pi$ erfüllen $(*)$

$\boxed{k=2}$: Nur $\alpha_8 = 4\pi + \frac{\pi}{6}$ erfüllt $(*)$ (da $\frac{\pi}{6} < 1$)

Daraus erhält man die Lösungen $x_i = \frac{1}{2}\alpha_i - \frac{1}{2}$ $(i = 1, 2, \dots, 8)$

$x_1 = -2\pi + \frac{5}{12}\pi - \frac{1}{2} \approx -5{,}4742$

$x_2 = -\pi + \frac{\pi}{12} - \frac{1}{2} \approx -3{,}3798$

$x_3 = -\pi + \frac{5}{12}\pi - \frac{1}{2} \approx -2{,}3326$ $\quad \mathbb{L} = \{x_1, x_2, \dots, x_8\}$

$x_4 = \frac{\pi}{12} - \frac{1}{2} \approx -0{,}2382$

$x_5 = \frac{5}{12}\pi - \frac{1}{2} \approx 0{,}8090$

$x_6 = \pi + \frac{\pi}{12} - \frac{1}{2} \approx 2{,}9034$

$x_7 = \pi + \frac{5}{12}\pi - \frac{1}{2} \approx 3{,}9506$

$x_8 = 2\pi + \frac{\pi}{12} - \frac{1}{2} \approx 6{,}0450$

A48 d) $\sin x - \sin(\pi - x) = 0, \quad \mathbb{G} = [0, 2\pi]$

Da $\sin(\pi - x) = \sin[-(x-\pi)] = -\sin(x-\pi)$ gilt, ergibt man $\sin(\pi - x)$ aus $\sin x$ durch Verschiebung um π (nach rechts) entlang der x-Achse und anschließende Spiegelung an der x-Achse, und stimmt somit mit $\sin x$ überein,

d.h. $\sin(x) = \sin(\pi - x)$ für alle $x \in \mathbb{R}$.

$\Rightarrow$ $\sin x - \sin(\pi - x)$ ist die Nullfunktion

$\Rightarrow$ $\mathbb{L} = \mathbb{G} = [0, 2\pi]$

c) $2\cos^2 x - 3\cos x = 0 \qquad \mathbb{G} = [0, 2\pi]$

Subst.: $u = \cos x$

$$2u^2 - 3u = 0$$

$\Leftrightarrow$ $u(2u - 3) = 0$

$\Leftrightarrow$ $u = 0$ oder $u = \frac{3}{2}$

Wg. $-1 \leq \cos x \leq 1$ und somit $-1 \leq u \leq 1$ scheidet die Lsg. $u = \frac{3}{2}$ aus.

Rücksubst.: $\cos x = u$

$\boxed{u = 0}$ $\cos x = 0 \qquad \mathbb{G} = [0, 2\pi]$

$\Leftrightarrow$ $x = \frac{\pi}{2}$ oder $x = \frac{3}{2}\pi$

$\Rightarrow$ $\mathbb{L} = \left\{ \frac{\pi}{2} ; \frac{3}{2}\pi \right\}$

A 48

f) $\sin(x+\frac{\pi}{2}) + \cos(x+\frac{\pi}{2}) = 0$ $\mathbb{G} = \mathbb{R}$

$\underbrace{\sin(x+\frac{\pi}{2})}_{\cos(x)} + \cos(x+\frac{\pi}{2}) = 0$

$\Leftrightarrow \cos(x) + \cos(x+\frac{\pi}{2}) = 0$

Anwendung der allg. Formel $\cos(x_1)+\cos(x_2) = 2\cos\left(\frac{x_1+x_2}{2}\right)\cos\left(\frac{x_1-x_2}{2}\right)$

gibt

$\cos(x)+\cos(x+\frac{\pi}{2}) = 2\cos\left(\frac{x+x+\pi/2}{2}\right)\cos\left(\frac{x-x-\pi/2}{x}\right)$

$= 2\cos(x+\frac{\pi}{4})\underbrace{\cos(-\frac{\pi}{4})}_{\frac{1}{2}\sqrt{2}} = \sqrt{2}\cos(x+\frac{\pi}{4})$

$\Rightarrow$

$\cos(x)+\cos(x+\frac{\pi}{2})$

$\Leftrightarrow \sqrt{2}\cos(x+\frac{\pi}{4}) = 0 \quad | : \sqrt{2}$

$\Leftrightarrow \cos(\underbrace{x+\frac{\pi}{4}}_{\alpha}) = 0$

$\cos\alpha = 0 \Leftrightarrow \alpha = \frac{\pi}{2} + k\cdot\pi \quad (k\in\mathbb{Z})$

Man erhält die Lösungen

$x = \alpha - \frac{\pi}{4} = \frac{\pi}{4} + k\cdot\pi \quad (k\in\mathbb{Z})$

$\Rightarrow \mathbb{L} = \{\frac{\pi}{4} + k\cdot\pi \mid k\in\mathbb{Z}\}$

A49

a) $a^2x = 3a^2 - 2a$

1. Fall: $a = 0$

$$a^2x = 3a^2 - 2a$$

$$\Leftrightarrow \quad 0 = 0$$

$\mathbb{L} = \mathbb{R}$

2. Fall: $a \neq 0$

$$a^2x = 3a^2 - 2a \quad | : a^2$$

$$\Leftrightarrow \quad x = \frac{3a^2 - 2a}{a^2}$$

$$\Leftrightarrow \quad x = 3 - \frac{2}{a}$$

$\mathbb{L} = \{ 3 - \frac{2}{a} \}$

b) $\frac{1}{a-2x} - \frac{a-1}{a^2+2ax} = \frac{4a-1}{a^2-4x^2}$

Nenner faktorisieren	EF
$a - 2x$	$a(a+2x)$
$a^2+2ax = a(a+2x)$	$(a-2x)$
$a^2-4x^2 = (a-2x)(a+2x)$	a
$HN = a(a+2x)(a-2x)$	

$(a \neq 0)$ $\mathbb{G} = \{ x \neq \pm \frac{a}{2} \}$

muss gelten, sonst ist 2. Nenner $= 0$.

$$\frac{1}{a-2x} - \frac{a-1}{a^2+2ax} = \frac{4a-1}{a^2-4x^2}$$

$$\Leftrightarrow \quad \frac{a(a+2x) - (a-1)\cdot(a-2x)}{HN} = \frac{(4a-1)\cdot a}{HN} \quad | \cdot HN (\neq 0)$$

$$\Leftrightarrow \quad a(a+2x) - (a-1)(a-2x) = (4a-1)\cdot a$$

$$\Leftrightarrow \quad a^2 + 2ax - a^2 + 2ax + a - 2x = 4a^2 - a \quad | -a$$

$$\Leftrightarrow \quad 4ax - 2x = 4a^2 - 2a$$

A49 zu b)

$4ax - 2x = 4a^2 - 2a$

$\Leftrightarrow \quad 2x(2a-1) = 4a^2 - 2a$

<u>1. Fall</u>: $a = \frac{1}{2}$

$2x(2a-1) = 4a^2 - 2a$

$\Leftrightarrow \quad 0 = 4 \cdot \frac{1}{4} - 2 \cdot \frac{1}{2}$

$\Leftrightarrow \quad 0 = 0$

$\mathbb{L} = \mathbb{G} = \{x \neq \pm \frac{a}{2}\} = \{x \neq \pm \frac{1}{4}\} = \mathbb{R} \setminus \{\pm \frac{1}{4}\}$

<u>2. Fall</u>: $a \neq \frac{1}{2}$

$2x(2a-1) = 4a^2 - 2a \quad | : 2(2a-1)$

$\Leftrightarrow \quad x = \frac{4a^2 - 2a}{2(2a-1)}$

$\Leftrightarrow \quad x = \frac{2a(2a-1)}{2(2a-1)}$

$\Leftrightarrow \quad x = a$

Ist $a = 0$, so ist $x = 0 \notin \mathbb{G} = \{x \neq 0\}$ (da $\pm \frac{a}{2} = 0$)

und daher $\mathbb{L} = \emptyset$

Ist $a \neq 0$, so ist $x = a \neq \pm \frac{a}{2}$, d.h. $x = a \in \mathbb{G}$

und daher $\mathbb{L} = \{a\}$

Zusammengefasst:

$a = \frac{1}{2}$ | $a \neq \frac{1}{2}$

$a = \frac{1}{2}$: $\mathbb{L} = \mathbb{R} \setminus \{\pm \frac{1}{4}\}$

$a \neq \frac{1}{2}$: $a = 0$ | $a \neq 0$

$a = 0$: $\mathbb{L} = \emptyset$

$a \neq 0$: $\mathbb{L} = \{a\}$

A49 c) $\dfrac{x+a}{x-a} + \dfrac{x-2a}{x+a} = \dfrac{9}{4}$

max. Grundmenge $\mathbb{G} = \{x \neq \pm a\} = \mathbb{R} \setminus \{\pm a\}$

$HN = 4(x+a)(x-a)$

$$\frac{x+a}{x-a} + \frac{x-2a}{x+a} = \frac{9}{4}$$

$$\Leftrightarrow \frac{(x+a)\cdot 4(x+a) + (x-2a)\cdot 4(x-a)}{HN} = \frac{9(x+a)(x-a)}{HN} \quad | \cdot HN (\neq 0)$$

$$\Leftrightarrow 4(x+a)^2 + 4(x-2a)(x-a) = 9(x^2-a^2)$$

$$\Leftrightarrow 4(x^2+2ax+a^2) + 4(x^2-3ax+2a^2) = 9x^2-9a^2$$

$$\Leftrightarrow 4x^2+8ax+4a^2+4x^2-12ax+8a^2 = 9x^2-9a^2$$

$$\Leftrightarrow 8x^2-4ax+12a^2 = 9x^2-9a^2 \quad | -8x^2+4ax-12a^2$$

$$\Leftrightarrow x^2+4ax-21a^2 = 0$$

$x_{1/2} = \dfrac{-4a \pm \sqrt{D}}{2}$ mit $D = (4a)^2 + 4\cdot 21a^2 = 100a^2$

und $\pm\sqrt{D} = \pm 10|a| = \pm 10a$ (Betrag kann wg. „$\pm$" wegfallen)

$$\Rightarrow x_{1/2} = \frac{-4a \pm 10a}{2}$$

$x_1 = 3a$, $x_2 = -7a$

1. Fall: $a = 0$

$x_1 = x_2 = 0 = a \notin \mathbb{G}$. Somit gilt $\mathbb{L} = \emptyset$

2. Fall: $a \neq 0$

Dann ist $x_1 \neq x_2$ und $x_1 \neq \pm a$, $x_2 \neq \pm a$,

d.h. $x_1, x_2 \in \mathbb{G}$. Es gilt dann $\mathbb{L} = \{-7a; 3a\}$

A49 d) $\sqrt{2x^2+2ax+a^2} = x+2a$

max. Grundmenge $\mathbb{G} = \{x \in \mathbb{R} \mid 2x^2+2ax+a^2 \geq 0\}$

Diskriminante von $2x^2+2ax+a^2$: $D = (2a)^2 - 4\cdot 2\cdot a^2 = 4a^2 - 8a^2 = -4a^2$

<u>1. Fall</u>: $a = 0$

$$\sqrt{2x^2+2ax+a^2} = x+2a$$

$$\Leftrightarrow \sqrt{2x^2} = x \qquad (\mathbb{G} = \mathbb{R})$$

$$\Leftrightarrow \sqrt{2}\,|x| = x$$

$(\sqrt{x^2} = |x|)$

Da l.S. ≥ 0 gilt, kann $x<0$ nicht gelten.

Also muss $x \geq 0$ sein und in diesem Fall ist $|x| = x$

$$\sqrt{2}\,|x| = x$$

$$\Leftrightarrow \sqrt{2}\,x = x$$

$$\Leftrightarrow \sqrt{2}\,x - x = 0$$

$$\Leftrightarrow (\sqrt{2}-1)x = 0 \qquad | : (\sqrt{2}-1)$$

$$\Leftrightarrow x = 0$$

$\mathbb{L} = \{0\}$.

<u>2. Fall</u>: $a \neq 0$

Für die Diskriminante D von $2x^2+2ax+a^2$ gilt dann $D = -4a^2 < 0$

Somit besitzt $2x^2+2ax+a^2$ keine (reellen) Nst. und hat daher

ein konstantes Vorzeichen, setzt man $x=0$, so erhält man $a^2 > 0$,

d.h. $2x^2+2ax+a^2 > 0$ für jedes $x \in \mathbb{R}$.

Somit ist $\mathbb{G} = \mathbb{R}$.

A 49 zu d)

$$\sqrt{2x^2+2ax+a^2} = x+2a \quad |\,(\;)^2$$

$$\Rightarrow \quad 2x^2+2ax+a^2 = (x+2a)^2$$

$$\Leftrightarrow \quad 2x^2+2ax+a^2 = x^2+4ax+4a^2 \quad |-x^2-4ax-4a^2$$

$$\Leftrightarrow \quad x^2-2ax-3a^2 = 0$$

$$x_{1/2} = \frac{2a \pm \sqrt{D}}{2} \quad \text{mit } D = (2a)^2 + 4\cdot 3a^2 = 16a^2;$$

und somit ist $\pm\sqrt{D} = \pm 4a$

$$x_{1/2} = \frac{2a \pm 4a}{2}$$

$x_1 = 3a$, $x_2 = -a$, $\mathbb{G} = \mathbb{R}$

Da Quadrieren keine Äquivalenzumformung ist, muss noch eine Probe durchgeführt werden.

Probe:

$\boxed{x_1 = 3a}$ l.S. $\sqrt{2\cdot(3a)^2+2a\cdot 3a+a^2} = \sqrt{18a^2+6a^2+a^2} = \sqrt{25a^2} = 5|a|$

r.S. $3a+2a = 5a$

$x_1 = 3a$ ist eine Lsg., falls $a>0$ gilt und keine Lsg. für $a<0$.

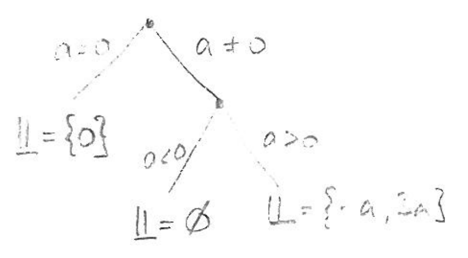
$\boxed{x_2 = -a}$ l.S. $\sqrt{2\cdot(-a)^2+2a(-a)+a^2} = \sqrt{a^2} = |a|$

r.S. $-a+2a = a$

$x_2 = -a$ ist eine Lsg. falls $a>0$ gilt und keine Lsg. für $a<0$

$\Rightarrow \quad \mathbb{L} = \{-a; 3a\}$ falls $a>0$ und $\mathbb{L} = \emptyset$, falls $a<0$

Zusammengefasst:



A50