Simulation for model checking

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Basic ideas

- Way back in the first week we talked about simulating data from distributions in R using the *rfoo* functions.
- In general simulations are way more flexible/useful
  - For bootstrapping as we saw in week 7
  - For evaluating models
  - For testing different hypotheses
  - For sensitivity analysis
- At minimum it is useful to simulate
  - A best case scenario
  - A few examples where you know your approach won't work
  - The importance of simulating the extremes

Simulating data from a model

Suppose that you have a regression model

$$Y_i = b_0 + b_1 X_i + e_i$$

Here is an example of generating data from this model where X_i and e_i are normal:

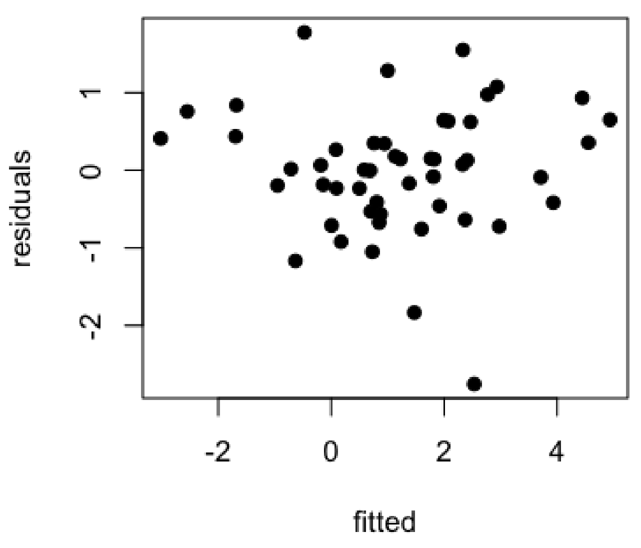
```
set.seed(44333)
x <- rnorm(50)
e <- rnorm(50)
b0 <- 1; b1 <- 2
y <- b0 + b1*x + e
```

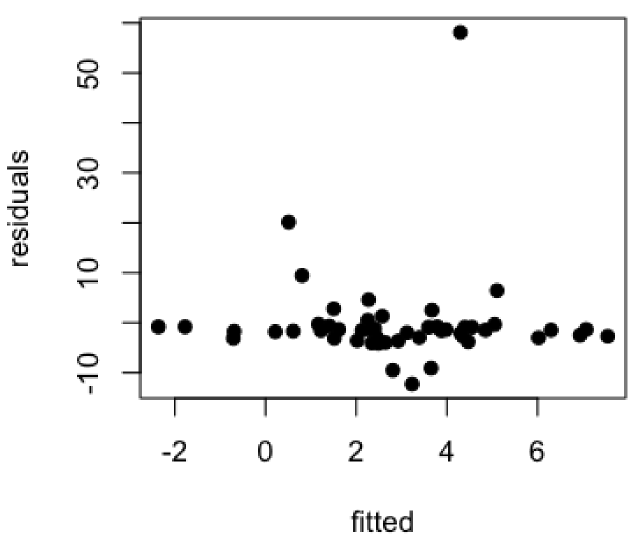
Violating assumptions

```
set.seed(44333)
x <- rnorm(50)
e <- rnorm(50); e2 <- rcauchy(50)
b0 <- 1; b1 <- 2
y <- b0 + b1*x + e; y2 <-  b0 + b1*x + e2
```

Violating assumptions

```
par(mfrow=c(1,2))
plot(lm(y ~ x)$fitted,lm(y~x)$residuals,pch=19,xlab="fitted",ylab="residuals")
plot(lm(y2 ~ x)$fitted,lm(y2~x)$residuals,pch=19,xlab="fitted",ylab="residuals")
```





5/15

Repeated simulations

```
set.seed(44333)
betaNorm <- betaCauch <- rep(NA,1000)
for(i in 1:1000){
  x <- rnorm(50); e <- rnorm(50); e2 <- rcauchy(50); b0 <- 1; b1 <- 2
  y <-  b0 + b1*x + e; y2 <- b0 + b1*x + e2
  betaNorm[i] <- lm(y ~ x)$coeff[2]; betaCauch[i] <- lm(y2 ~ x)$coeff[2]
}
quantile(betaNorm)
```

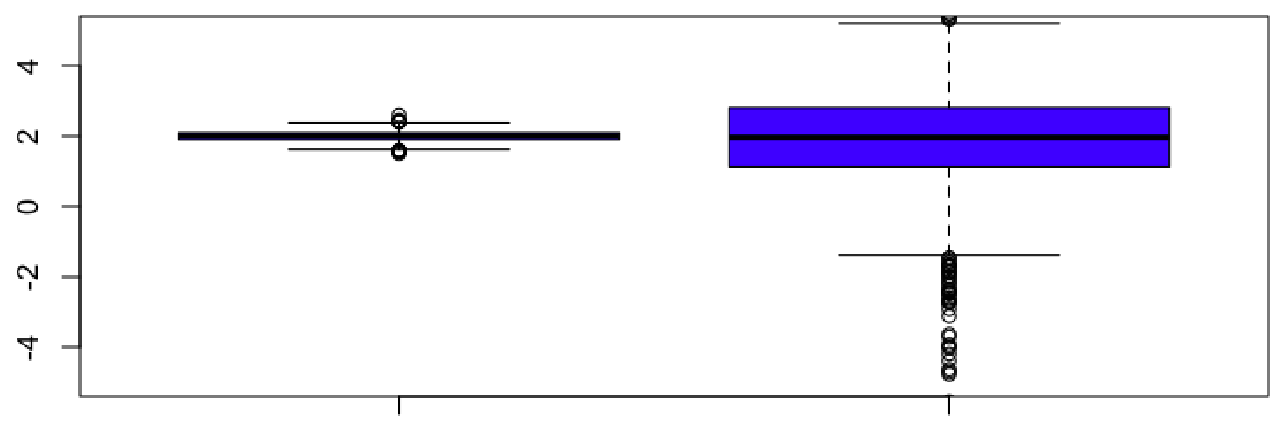
```
   0%   25%   50%   75%  100%
1.500 1.906 2.013 2.100 2.596
```

```
quantile(betaCauch)
```

```
      0%      25%      50%      75%     100%
-278.352    1.130    1.965    2.804  272.391
```

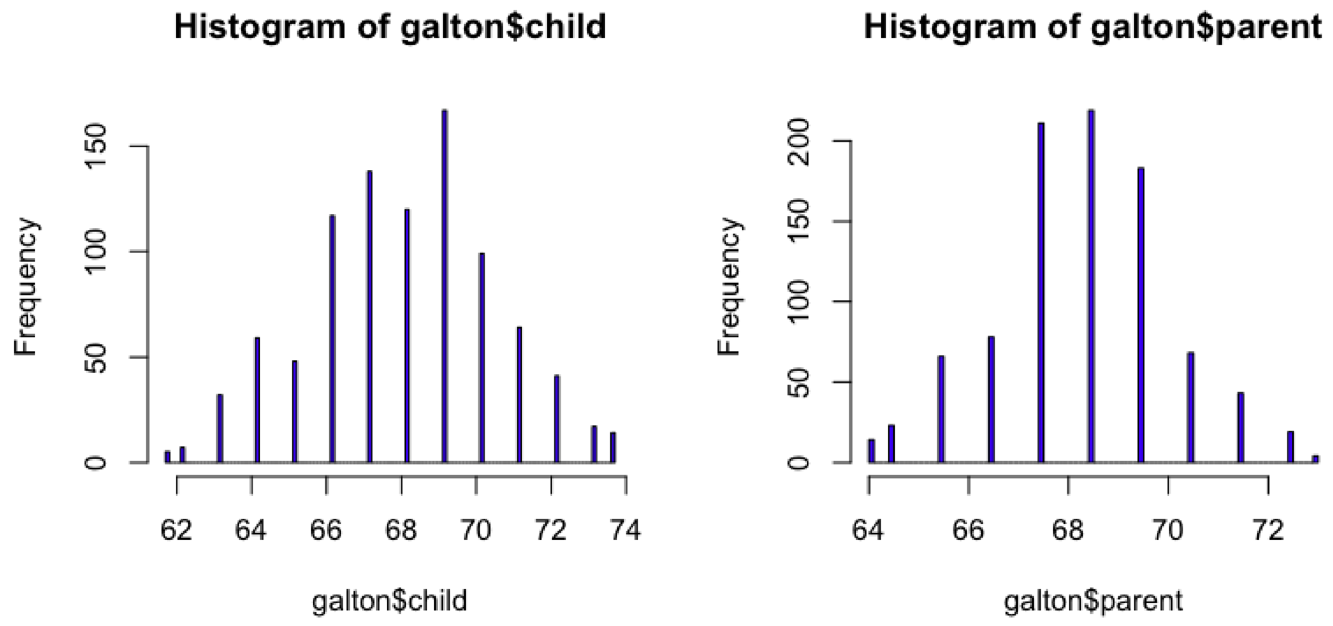
Monte Carlo Error

```
boxplot(betaNorm,betaCauch,col="blue",ylim=c(-5,5))
```



Simulation based on a data set

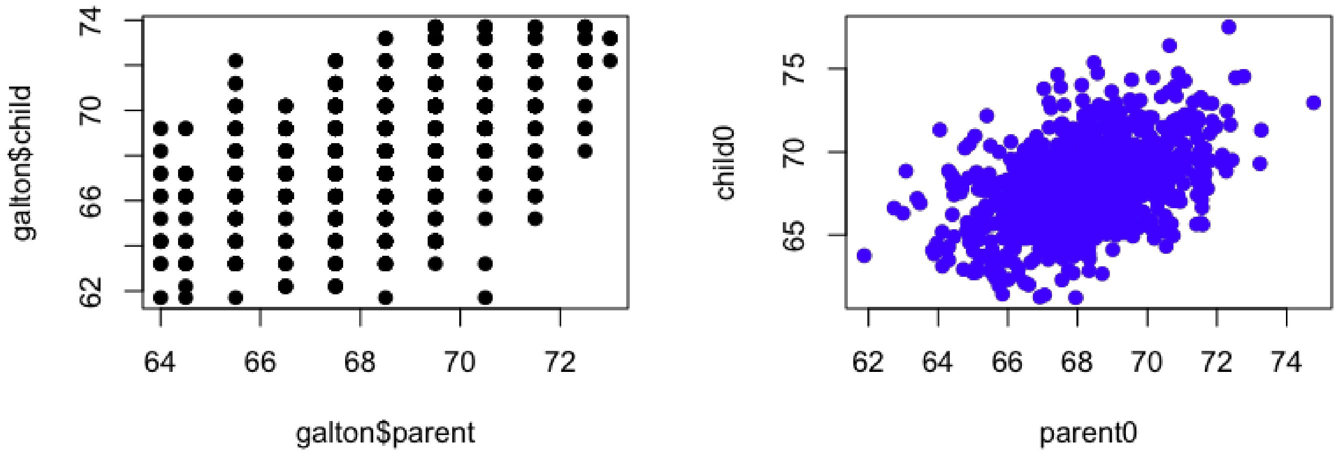
```r
library(UsingR); data(galton); nobs <- dim(galton)[1]
par(mfrow=c(1,2))
hist(galton$child,col="blue",breaks=100)
hist(galton$parent,col="blue",breaks=100)
```



8/15

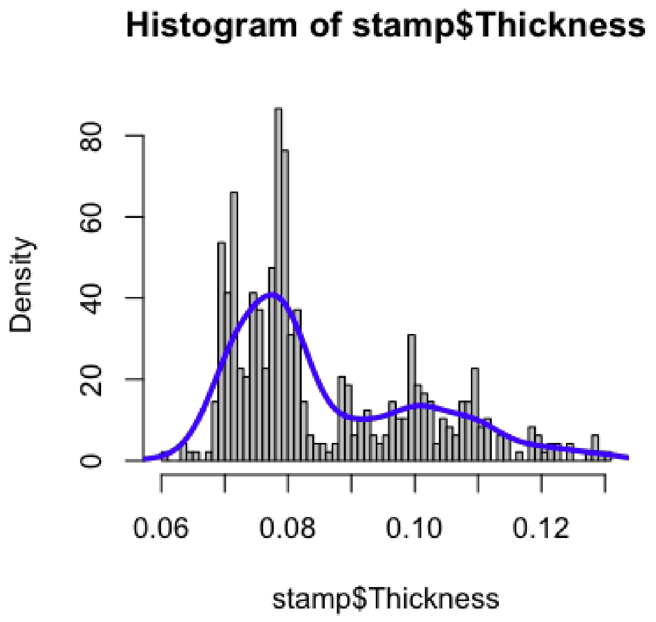
Calculating means,variances

```r
lm1 <- lm(galton$child ~ galton$parent)
parent0 <- rnorm(nobs,sd=sd(galton$parent),mean=mean(galton$parent))
child0 <- lm1$coeff[1] + lm1$coeff[2]*parent0 + rnorm(nobs,sd=summary(lm1)$sigma)
par(mfrow=c(1,2))
plot(galton$parent,galton$child,pch=19)
plot(parent0,child0,pch=19,col="blue")
```



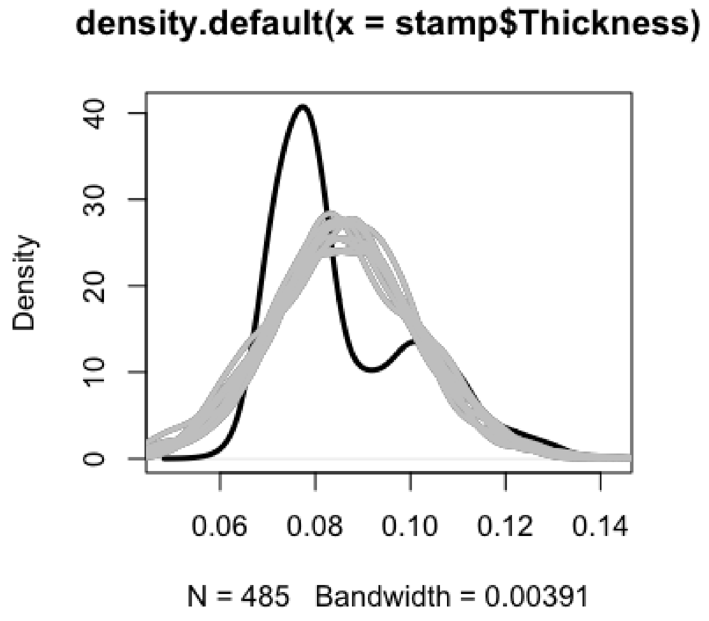
Simulating more complicated scenarios

```
library(bootstrap); data(stamp); nobs <- dim(stamp)[1]
hist(stamp$Thickness,col="grey",breaks=100,freq=F)
dens <- density(stamp$Thickness)
lines(dens,col="blue",lwd=3)
```

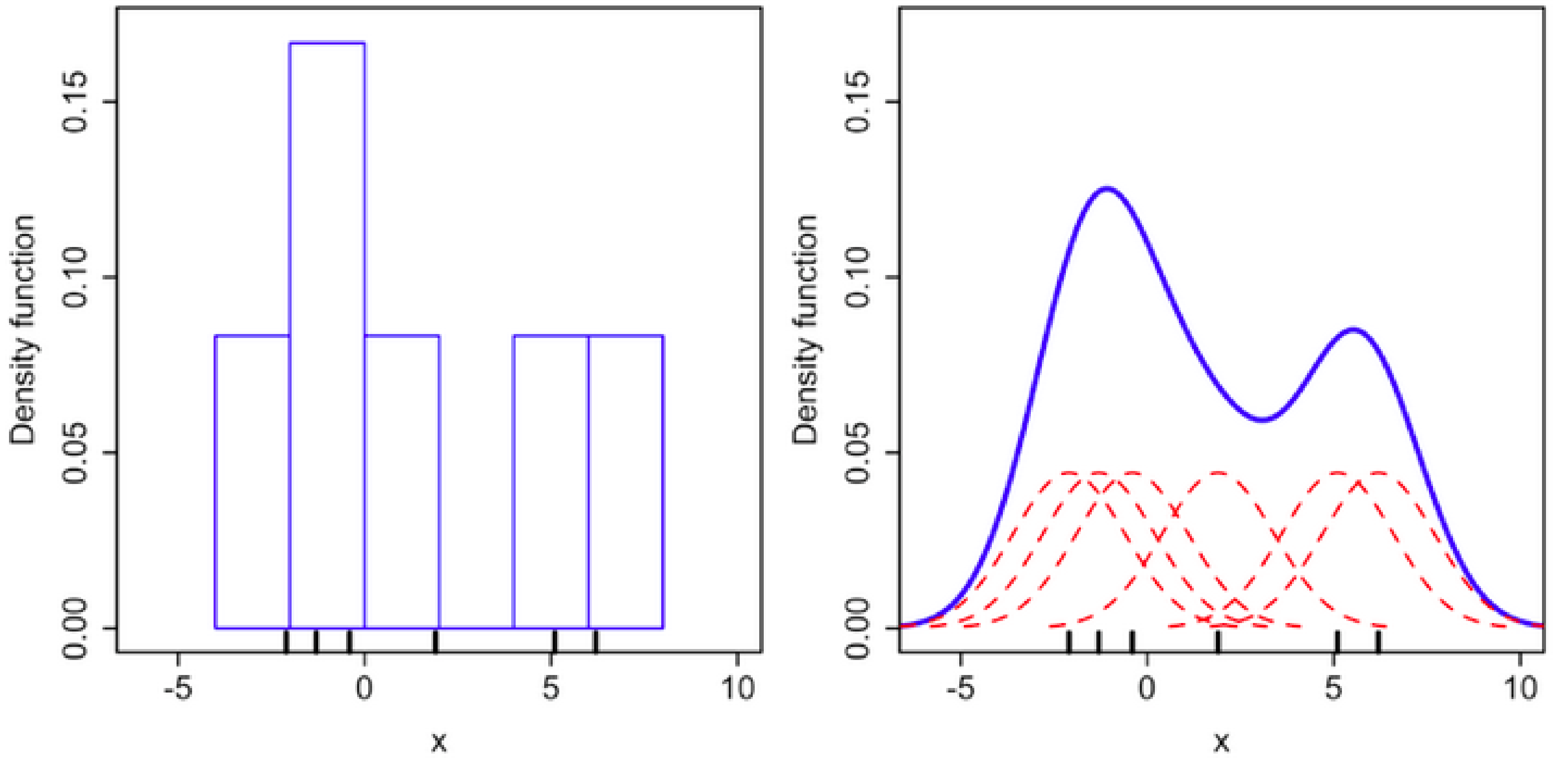


A simulation that is too simple

```
plot(density(stamp$Thickness),col="black",lwd=3)
for(i in 1:10){
  newThick <- rnorm(nobs,mean=mean(stamp$Thickness),sd=sd(stamp$Thickness))
  lines(density(newThick),col="grey",lwd=3)
}
```



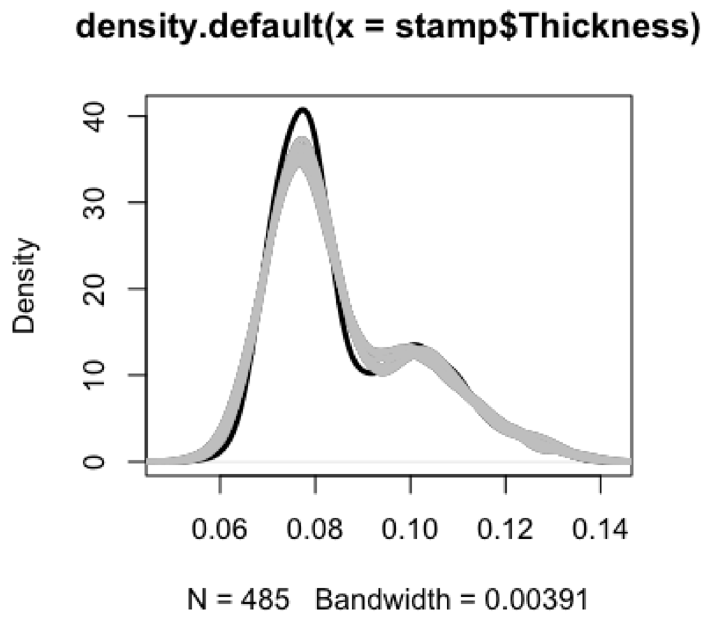
How density estimation works



http://en.wikipedia.org/wiki/File:Comparison_of_1D_histogram_and_KDE.png

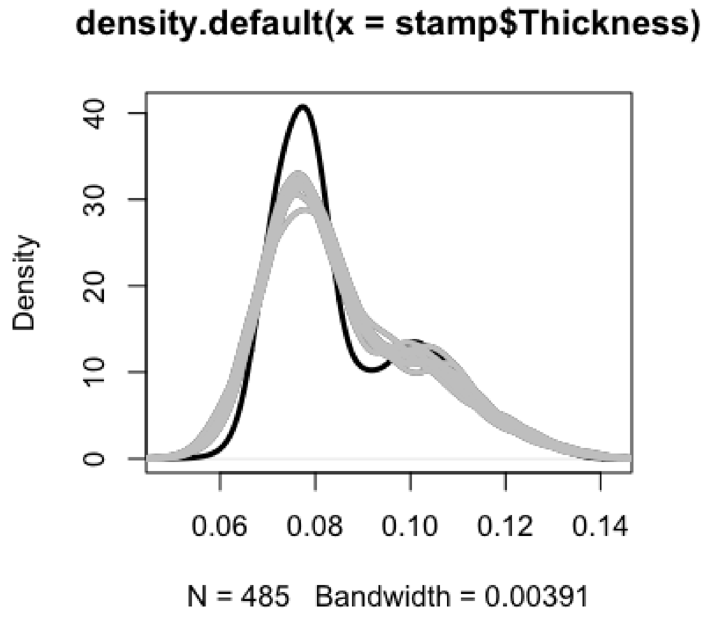
Simulating from the density estimate

```r
plot(density(stamp$Thickness),col="black",lwd=3)
for(i in 1:10){
  newThick <- rnorm(nobs,mean=stamp$Thickness,sd=dens$bw)
  lines(density(newThick),col="grey",lwd=3)
}
```



Increasing variability

```
plot(density(stamp$Thickness),col="black",lwd=3)
for(i in 1:10){
  newThick <- rnorm(nobs,mean=stamp$Thickness,sd=dens$bw*1.5)
  lines(density(newThick,bw=dens$bw),col="grey",lwd=3)
}
```



Notes and further resources

Notes

- Simulation can be applied to missing data problems - simulate what missing data might be
- Simulation values are often drawn from standard distributions, but this may not be appropriate
- Sensitivity analysis means trying different simulations with different assumptions and seeing how estimates change

Further resources

- Advanced Data Analysis From An Elementary Point of View
- The design of simulation studies in medical statistics
- Simulation studies in statistics

15/15