Regression in the real world

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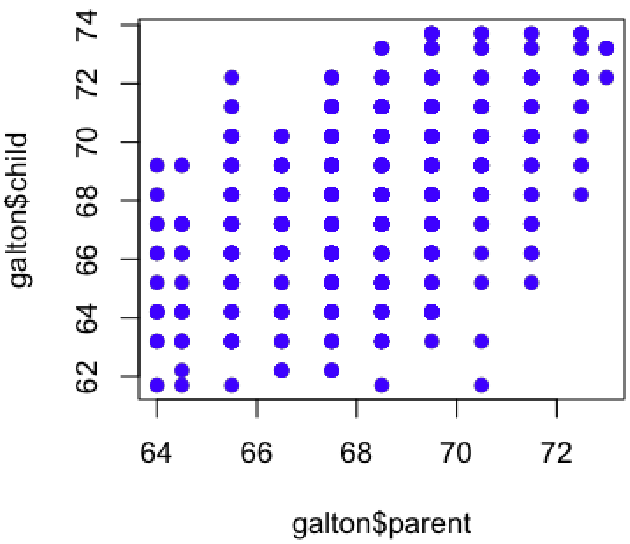
Things to pay attention to

- Confounders
- Complicated interactions
- Skewness
- Outliers
- Non-linear patterns
- Variance changes
- Units/scale issues
- Overloading regression
- Correlation and causation

The ideal data for regression

```
library(UsingR); data(galton)
plot(galton$parent,galton$child,col="blue",pch=19)
```



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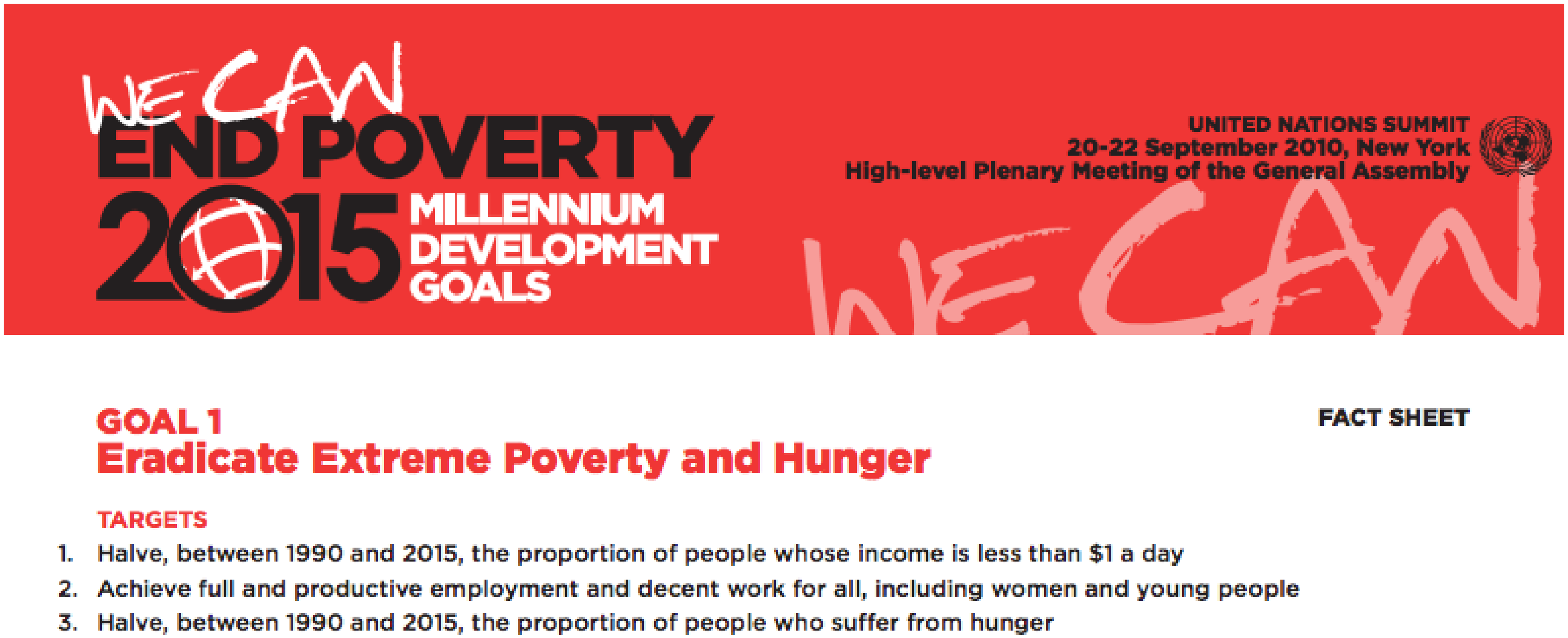
Confounders

Confounder: A variable that is correlated with both the outcome and the covariates

- Confounders can change the regression line
- They can even change the sign of the line
- They can sometimes be detected by careful exploration

Example - Millenium Development Goal 1



http://www.un.org/millenniumgoals/pdf/MDG_FS_1_EN.pdf

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WHO childhood hunger data

```
download.file("http://apps.who.int/gho/athena/data/GHO/WHOSIS_000008.csv?profile=text&filt
              er=COUNTRY:*;SEX:*","./data/hunger.csv",method="curl")
hunger <- read.csv("./data/hunger.csv")
hunger <- hunger[hunger$Sex!="Both sexes",]
head(hunger)
```

```
                               Indicator Data.Source PUBLISH.STATES Year            WHO.region
2  Children aged <5 years underweight (%) NLIS_312819      Published 2004 Eastern Mediterranean
3  Children aged <5 years underweight (%) NLIS_312819      Published 2004 Eastern Mediterranean
6  Children aged <5 years underweight (%) NLIS_312361      Published 2000                Europe
7  Children aged <5 years underweight (%) NLIS_312361      Published 2000                Europe
9  Children aged <5 years underweight (%) NLIS_312879      Published 2005                Europe
10 Children aged <5 years underweight (%) NLIS_312879      Published 2005                Europe
       Country    Sex Display.Value Numeric Low High Comments
2  Afghanistan Female          33.0    33.0  NA   NA       NA
3  Afghanistan   Male          32.7    32.7  NA   NA       NA
6      Albania   Male          19.6    19.6  NA   NA       NA
7      Albania Female          14.2    14.2  NA   NA       NA
9      Albania   Male           7.3     7.3  NA   NA       NA
10     Albania Female           5.8     5.8  NA   NA       NA
```

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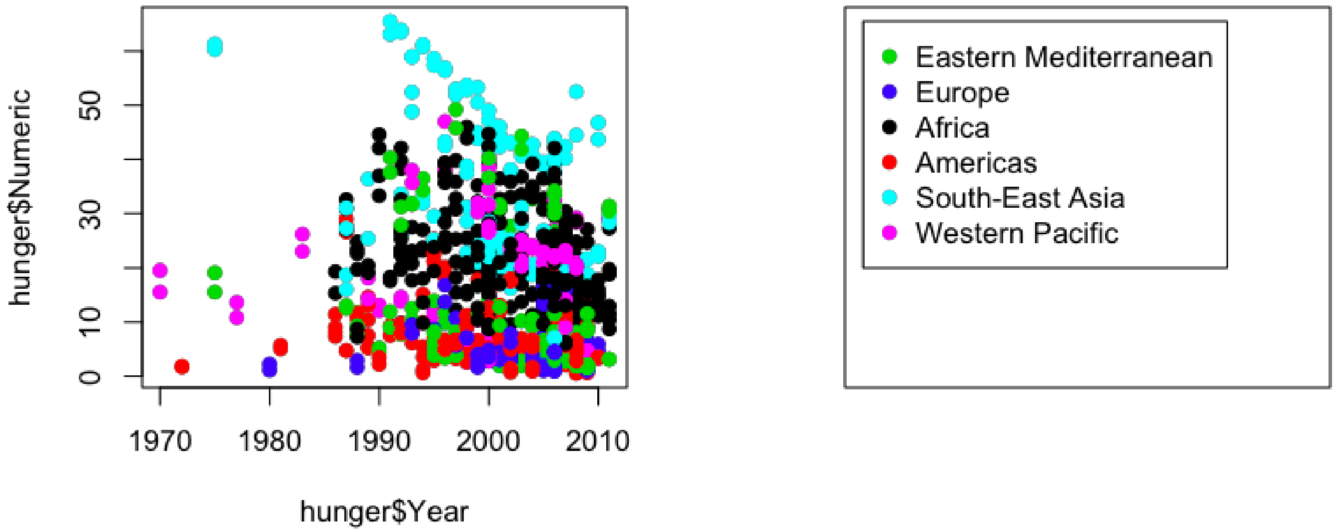
Hunger over time by region

```
par(mfrow=c(1,2))
plot(hunger$Year,hunger$Numeric,col=as.numeric(hunger$WHO.region),pch=19)
plot(1:10,type="n",xaxt="n",yaxt="n",xlab="",ylab="")
legend(1,10,col=unique(as.numeric(hunger$WHO.region)),legend=unique(hunger$WHO.region),pch=19)
```



Region correlated with year

```
anova(lm(hunger$Year ~ hunger$WHO.region))
```

```
Analysis of Variance Table

Response: hunger$Year
                   Df Sum Sq Mean Sq F value Pr(>F)
hunger$WHO.region   5    538   107.5    2.33  0.041 *
Residuals         852  39316    46.1
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Region correlated with hunger

```
anova(lm(hunger$Numeric ~ hunger$WHO.region))
```

```
Analysis of Variance Table

Response: hunger$Numeric
                   Df Sum Sq Mean Sq F value Pr(>F)
hunger$WHO.region   5  76032   15206     154 <2e-16 ***
Residuals         852  84211      99
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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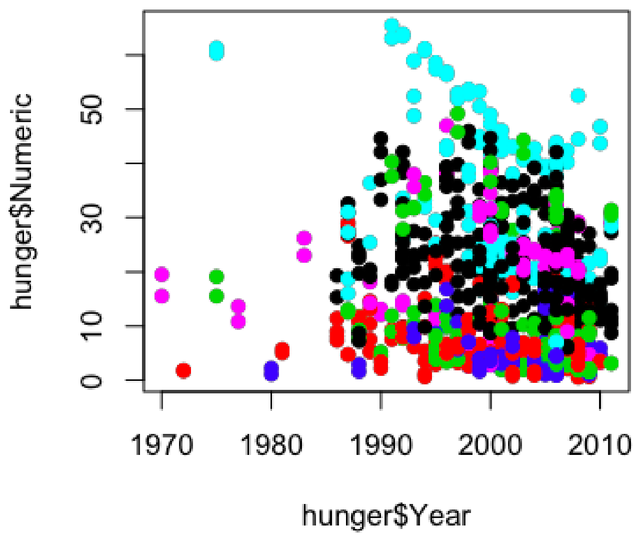
Including region - a complicated interaction

```
plot(hunger$Year,hunger$Numeric,pch=19,col=as.numeric(hunger$WHO.region))
lmRegion <- lm(hunger$Numeric ~ hunger$Year + hunger$WHO.region + hunger$Year*hunger$WHO.region )
abline(c(lmRegion$coeff[1] + lmRegion$coeff[6],lmRegion$coeff[2]+ lmRegion$coeff[12]),col=5,lwd=3)
```



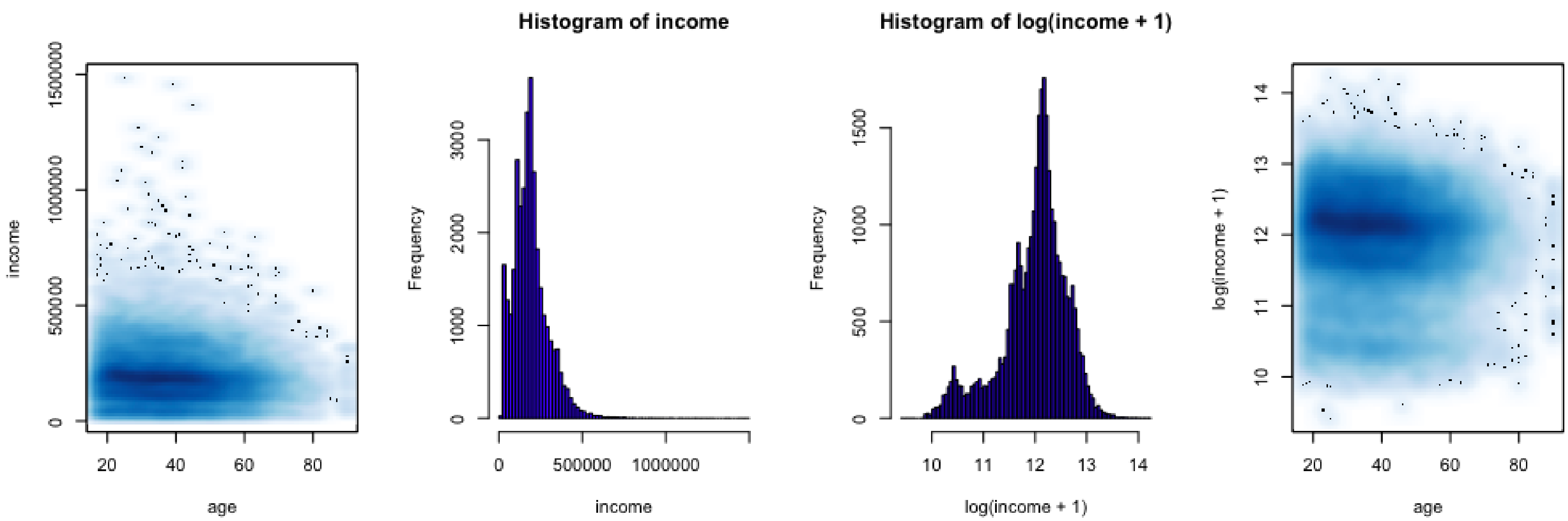
Income data

```
download.file("http://archive.ics.uci.edu/ml/machine-learning-databases/adult/adult.data","./data/incom
incomeData <- read.csv("./data/income.csv",header=FALSE)
income <- incomeData[,3]
age <- incomeData[,1]
```

http://archive.ics.uci.edu/ml/datasets/Census+Income

Logs to address right-skew

```
par(mfrow=c(1,4))
smoothScatter(age,income)
hist(income,col="blue",breaks=100)
hist(log(income+1),col="blue",breaks=100)
smoothScatter(age,log(income+1))
```



(Data transforms)

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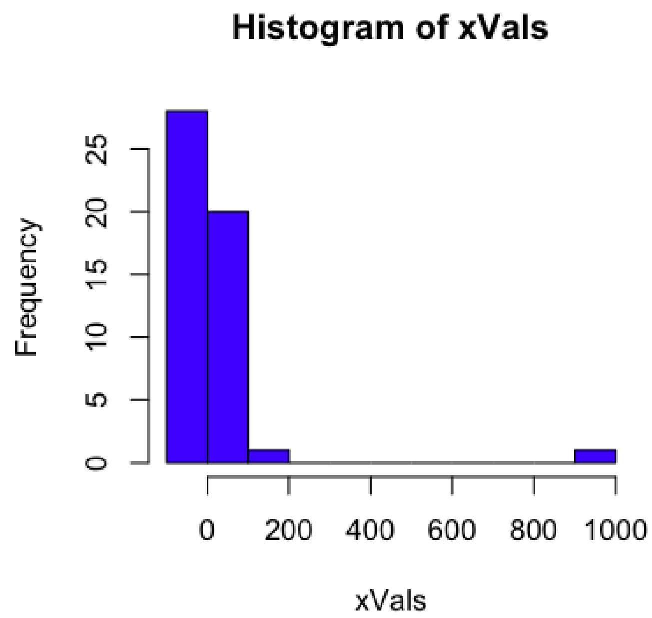
Outliers

"outliers" ... are data points that do not appear to follow the pattern of the other data points.

[A dataset that is 44% outliers]

Example - extreme points

```
set.seed(1235)
xVals <- rcauchy(50)
hist(xVals,col="blue")
```



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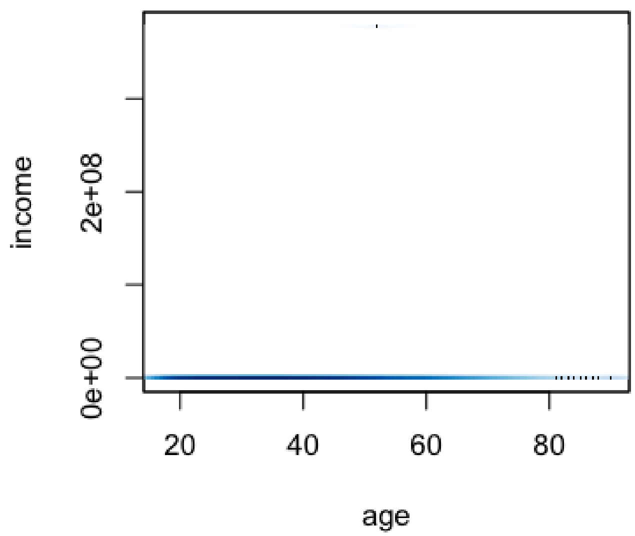
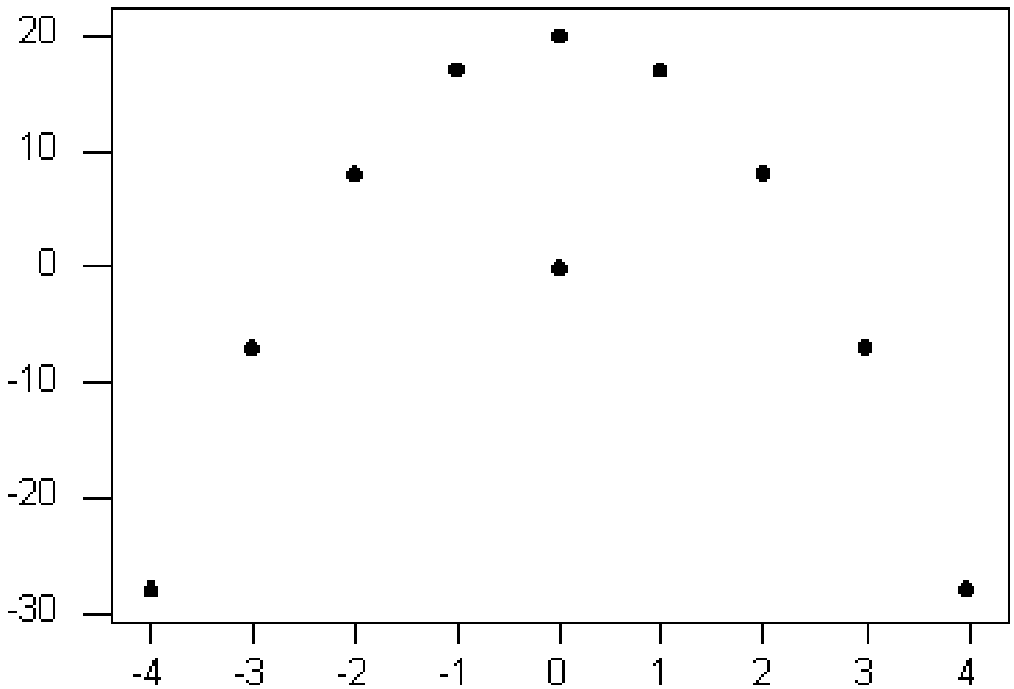
Example - Outliers may be real

```
# Add Tim Cook, CEO of Apple 2011 income
age <- c(age,52)
income <- c(income,378e6)
smoothScatter(age,income)
```



http://www.macworld.com/article/2023491/apple-gives-tim-cook-51-percent-salary-increase.html

Example - Does not fit the trend



A dataset that is 44% outliers

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Outliers - what you can do

- If you know they aren't real/of interest, remove them (but changes question!)
- Alternatively
  - Sensitivity analysis - is it a big difference if you leave it in/take it out?
  - Logs - if the data are right skewed (lots of outliers)
  - Robust methods - we've been doing averages, but there are more robust approaches (Robust,rlm)

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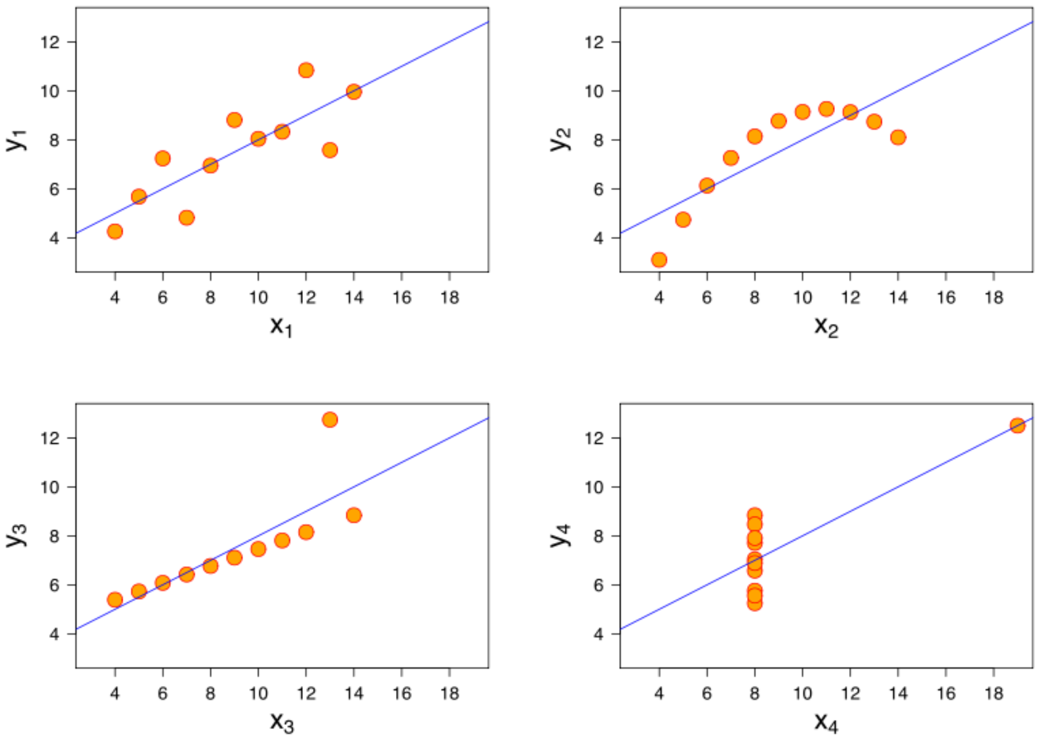
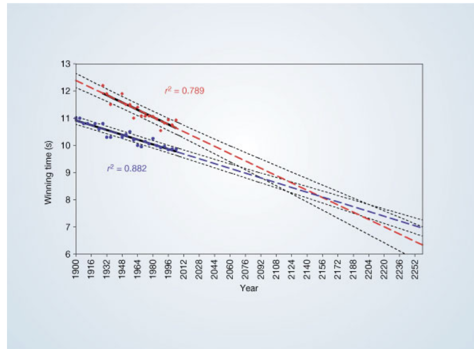
A line isn't always the best summary



http://en.wikipedia.org/wiki/Linear_regression

You can end up saying some pretty silly stuff



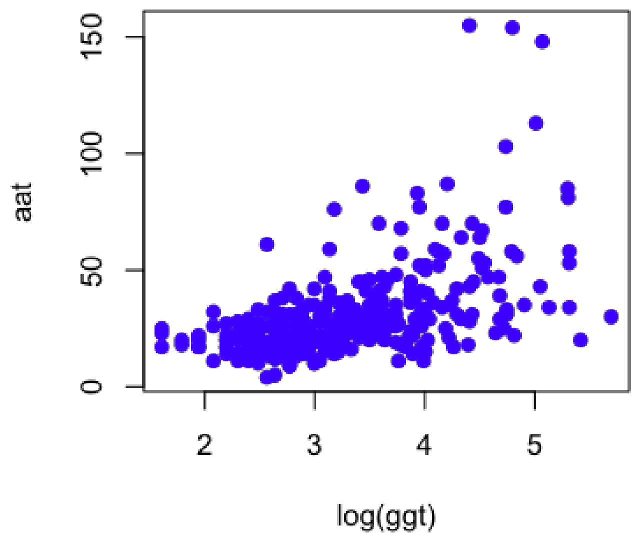
http://www.nature.com/nature/journal/v431/n7008/fig_tab/431525a_F1.html

"We are students aged 16–18 in a Texas high school. Our biology teacher Vidya Rajan asked us to comment on the paper by A. J. Tatem and colleagues (Nature 431, 525; 2004); we believe the projection on which it is based is riddled with flaws..." http://www.nature.com/nature/journal/v432/n7014/full/432147c.html

"They omit to mention, however, that (according to their analysis) a far more interesting race should occur in about 2636, when times of less than zero seconds will be recorded" http://www.nature.com/nature/journal/v432/n7014/full/432147b.html

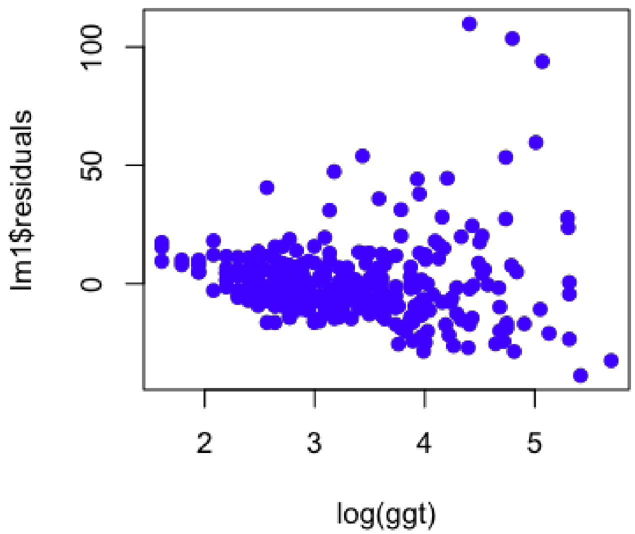
Variance changes

```
bupaData <- read.csv("ftp://ftp.ics.uci.edu/pub/machine-learning-databases/liver-disorders/bupa.data",h
ggt <- bupaData[,5]; aat <- bupaData[,3]
plot(log(ggt),aat,col="blue",pch=19)
```



Plot the residuals

```
lm1 <- lm(aat ~ log(ggt))
plot(log(ggt),lm1$residuals,col="blue",pch=19)
```

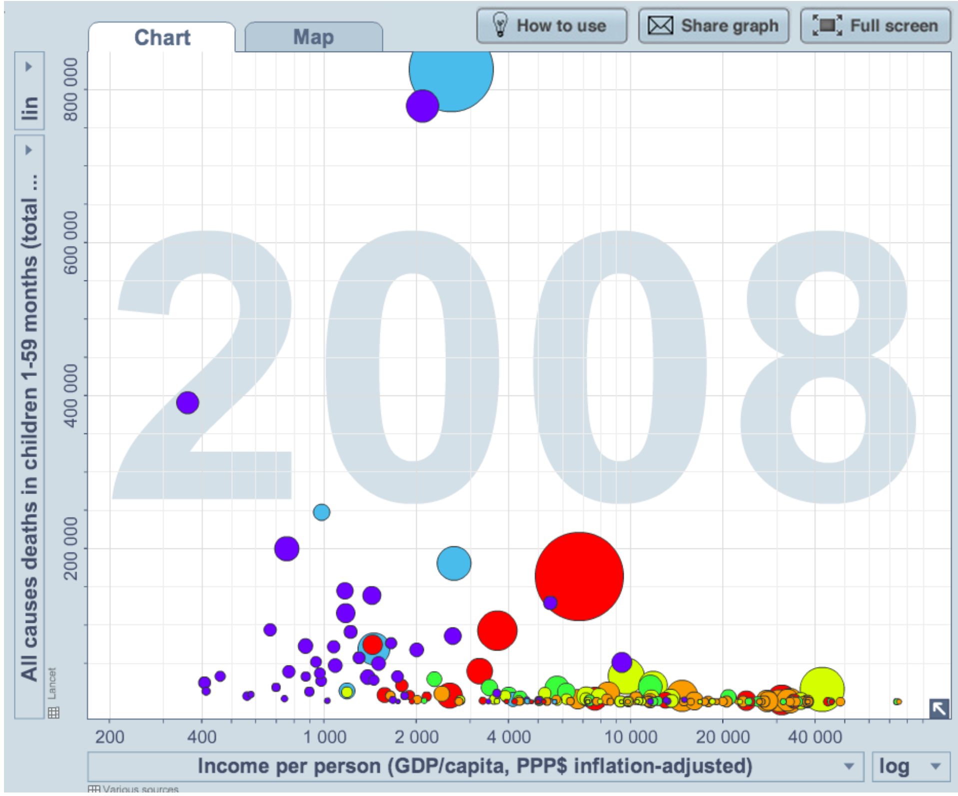


[Power (a.k.a. Box-Cox) transform]

Changing variance - what you can do

- There is a long literature on this problem (heteroskedasticity)
- A few examples
  - Box-Cox Transform
  - Variance stabilizing transform
  - Weighted least squares
  - Huber-white standard errors

Variation in units



All Deaths

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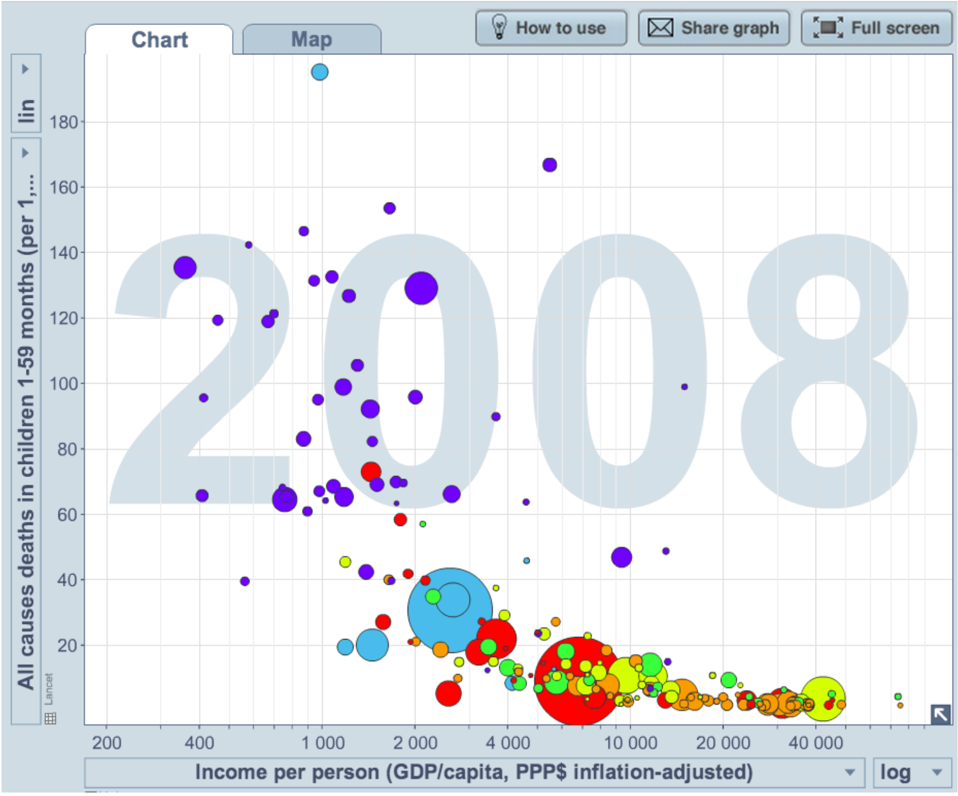
Relative units



[Per 1000 Deaths]

When there is variation in units

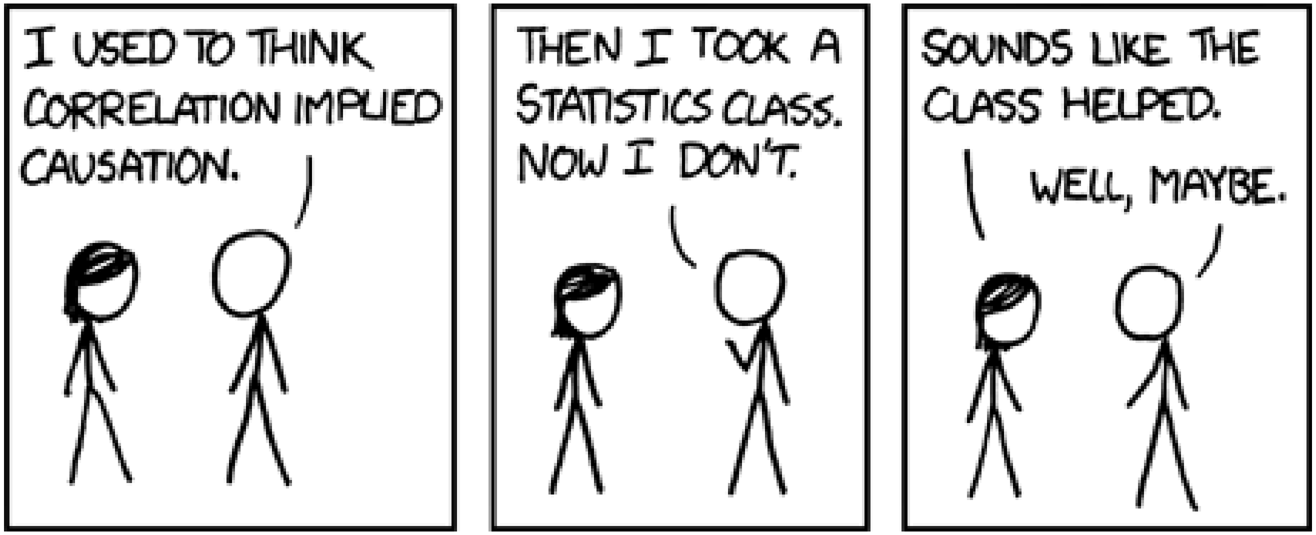
- Standardize, but keep track
  - Affects model fits
  - Affects interpretation
  - Affects inference

Overloading regression

$$AMR_{it}=\alpha+\beta_1 PRIV_{it}+\beta_2 GDP_{it}+\beta_3 LIB_{it}+\beta_4 TRADE_{it}+$$
$$\beta_5 DEM_{it}+\beta_6 WAR_{it}+\beta_7 DEP_{it}+\beta_8 URBAN_{it}+\beta_9 EDUC_{it}+\mu_i+\varepsilon_{it}$$

http://bit.ly/YiB5Um http://wmbriggs.com/blog/?p=7026

Correlation and Causation



http://xkcd.com/552/

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Even when looking for associations

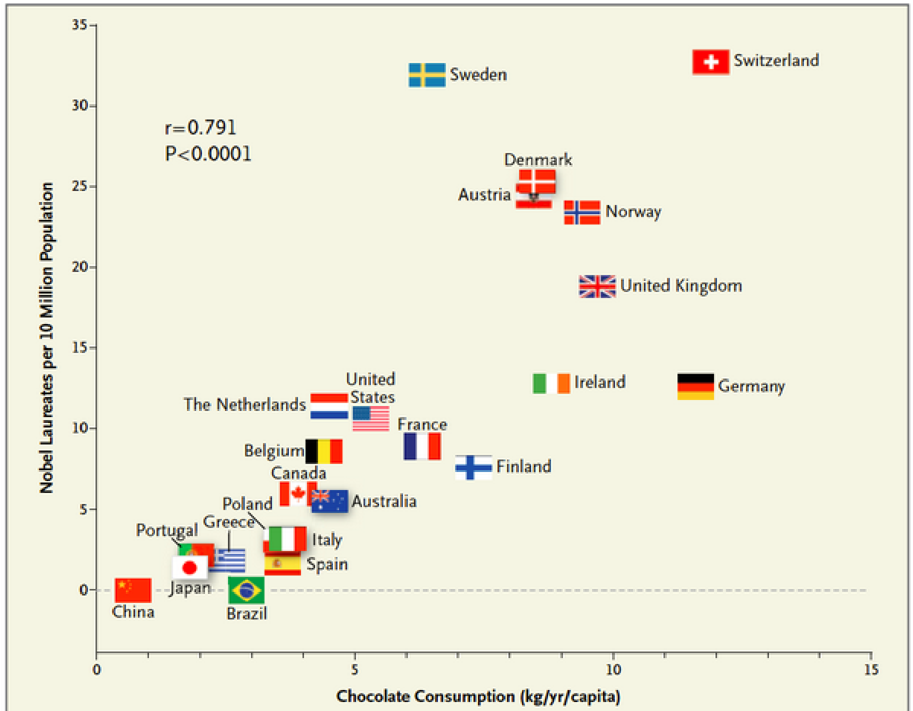
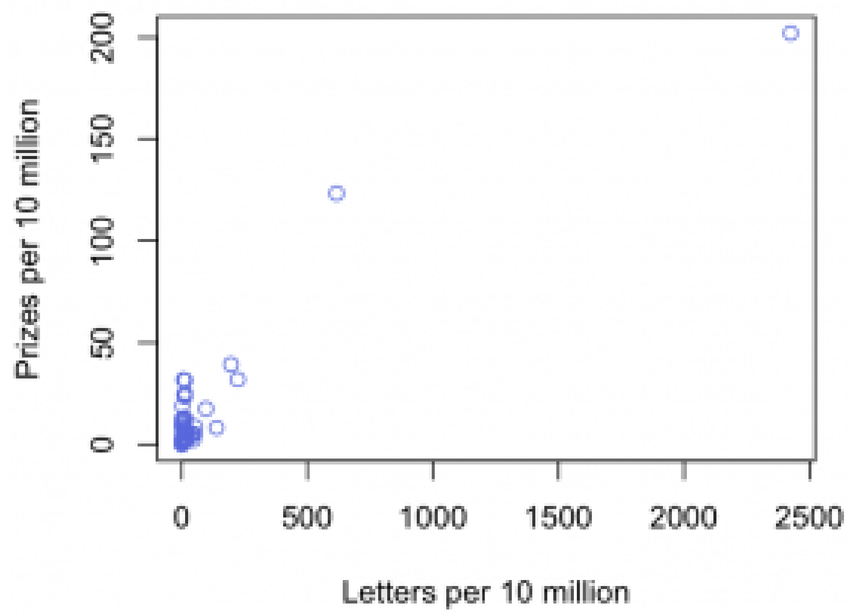


Figure 1. Correlation between Countries' Annual Per Capita Chocolate Consumption and the Number of Nobel Laureates per 10 Million Population.

http://www.nejm.org/doi/full/10.1056/NEJMon1211064

Again, it can get silly



http://www.statschat.org.nz/2012/10/12/even-better-than-chocolate/

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Correlation vs. Causation

- Use caution when interpreting regression results
- Be critical of surprising associations
- Consider alternative explanations