Sampling from a quasi-stationary distribution

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Overview

1. About my thesis
2. Where does the Blanchet's method fit?
3. An application

About my thesis

My work

- My work
  - Generating samples from an intractable distribution of the form

$$\pi(\theta) \propto \prod_{i=1}^{N} f_i(\theta) \tag{1}$$

  - Info on unbiased estimators of $\phi = \frac{||\nabla \log(\pi)||^2 + \nabla^\top \nabla \log(\pi)}{2} - l$ is available .
  - Application to big data problems $(N >> 1)$.
- How do I approach ?
  - Realize that the quasi-stationary density of a Brownian motion which is killed at rate ϕ is given by π.
  - Employ an extension of Blanchet's approach to simulate QSD of Brownian motion.
  - We address the big data problem by employing sub-sampling approach on ϕ which leads to sub-linear cost with respect to data size.

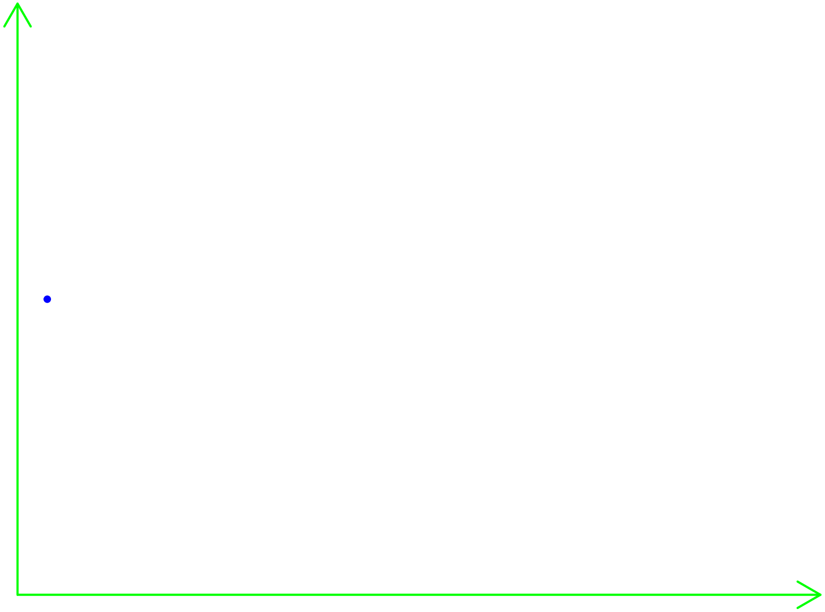
In our last meeting

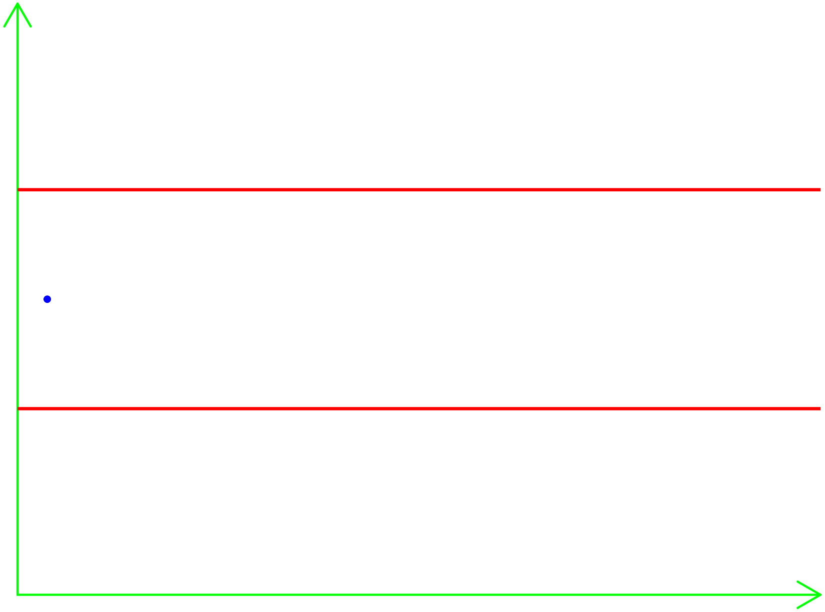
- Presented the ReScaLE method when the rate is kill function ϕ is bounded.
- Presented some examples built on artificial data.

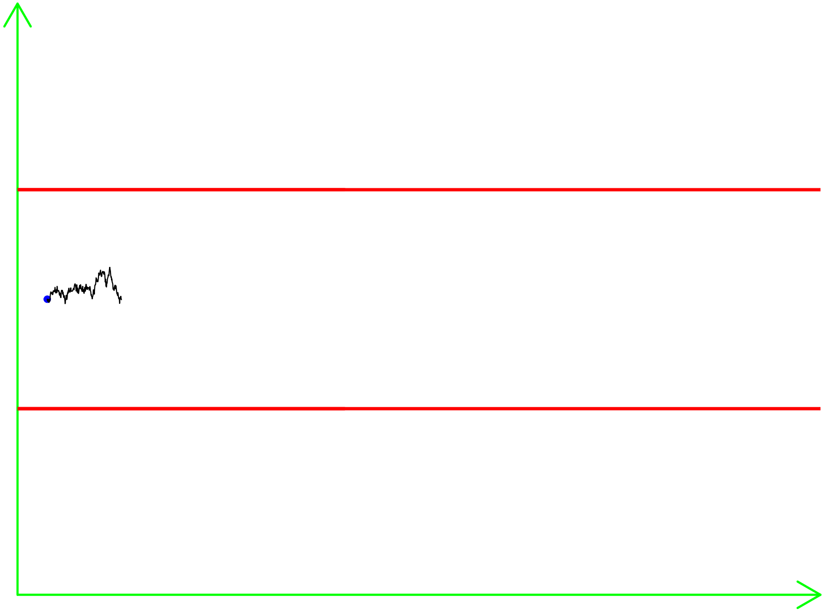
Progress so far...

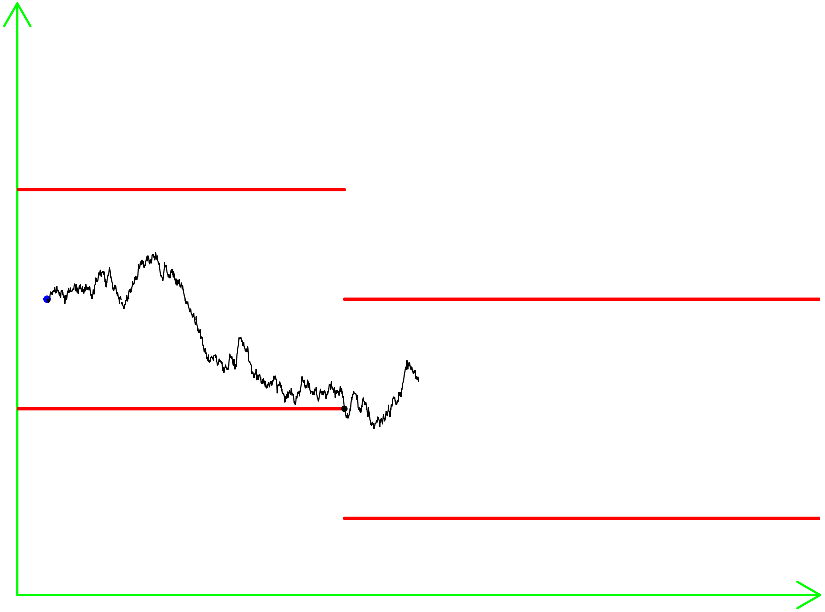
- The general version of the ReScaLE method is coded, which can be applied to an unbounded rate of kill (ϕ). function. This uses
  - path space simulation of Brownian motion,
  - an unbiased estimator of ϕ using sub-sampling.
- An application of the ReScaLE method which simulates according to the posterior distribution of parameters of a logistic regression applied on Menarche data.
- A new Monte Carlo method to simulate the exit time of $d-$dimensional Brownian motion, which has lower computational cost.
  - The new method simulates the exit time of d-dimensional Brownian motion, by directly using the density of exit time of $d-$ dimensional Brownian motion.

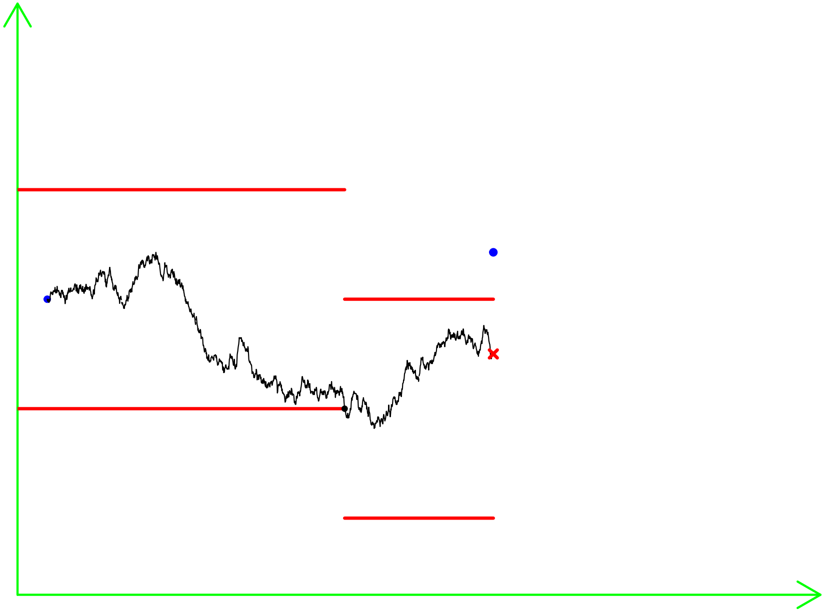
Where does the Blanchet's method fit?

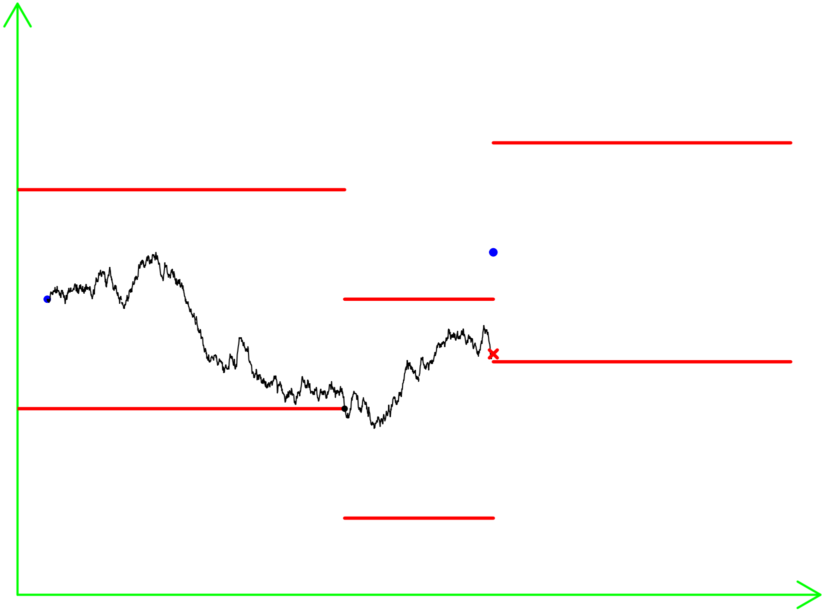


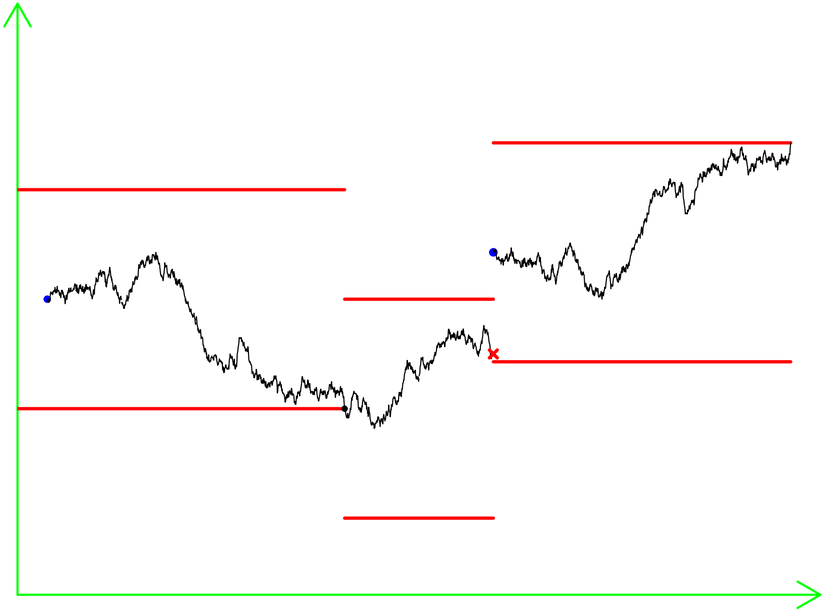












An application

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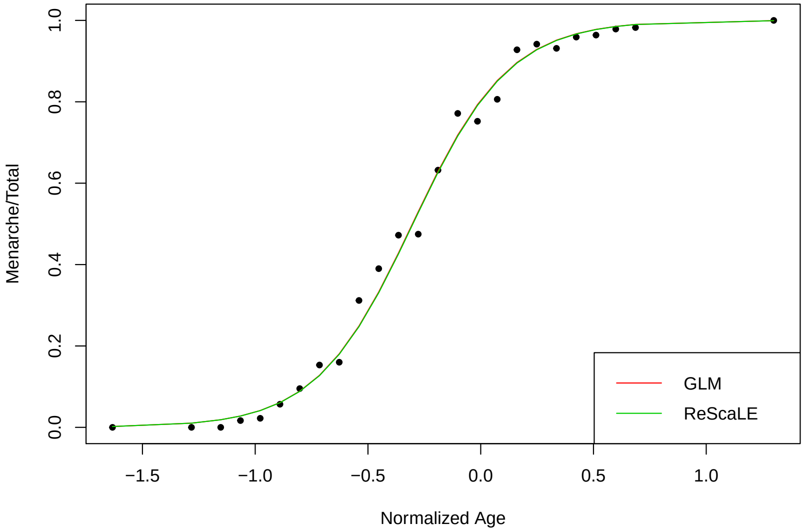
Aim is to sample from the posterior distribution

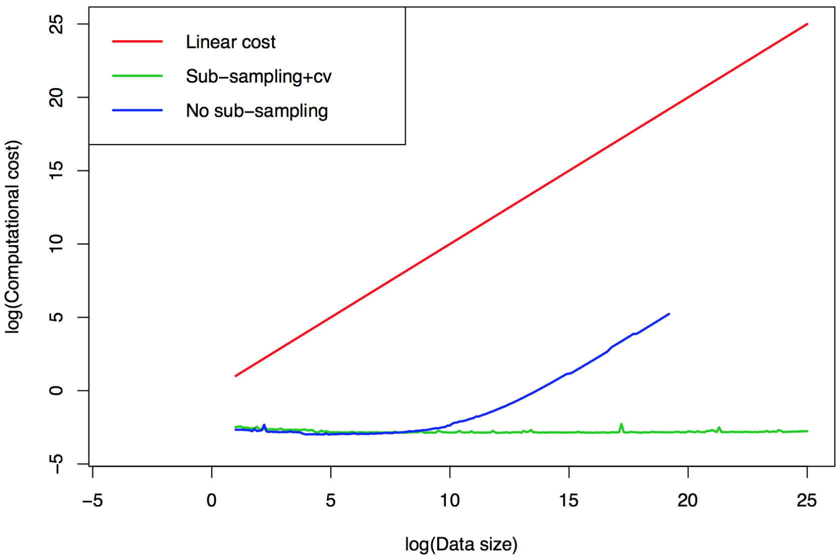
$$\pi(\alpha, \beta|...) \propto \prod_{i=1}^{25} p_i^{Menarche_i}(1-p_i)^{Total_i - Menarche_i}, \tag{2}$$

$$p_i = 1/(1+\exp(-\alpha - \beta Age_i)). \tag{3}$$



GLM
ReScaLE
Menarche/Total
Normalized Age
1.0
0.8
0.6
0.4
0.2
0.0
−1.5
−1.0
−0.5
0.0
0.5
1.0

Cost vs data size



Future works

- A new Monte Carlo method to simulate the exit times of multi-dimensional Brownian motion.
- Comparison of the run-time of old and new simulation method to simulate the exit time of multi-dimensional Brownian motion.
- A second application of the ReScaLE method to a high-dimensional posterior problem in the big data setting. We would specifically question
  - how does the cost vary with data size ?
  - how does the running cost vary with the dimensions ?
  - quality of the samples obtained using ReScaLE.
- Comparison of different rebirth strategies in a multi-dimensional setting which uses a general version of the ReScaLE method. Specifically we compare
  - run-time,
  - rate of convergence,
  - quality of simulation outputs.
- Comparison of the run-time, rate of convergence of the ReScaLE method with respect to dimensions.
- Comparison of simulation outputs between the ReScaLE method and Markov Chain.

Bibliography

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