Jane Street Puzzle Solution

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Problem Statement :

There are 10 castles, numbered 1, 2, 3, . . . , 10, and worth 1, 2, 3, . . . , 10 points respectively. You have 100 soldiers, which you can allocate between the castles however you wish. Your opponent also (independently) does the same. The number of soldiers on each castle is then compared, and for each castle, whoever has the most soldiers on that castle wins its points (in the case of a tie, no one gets points).

Castle	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Alice	10	10	10	10	10	10	10	10	10	10
Carol	20	0	10	5	10	5	10	20	0	20

In this match, Alice wins castles 2, 4, 6, 9, for a total of 21 points, and Carol wins castles 1, 8, 10, for a total of 19 points (no one wins castles 3, 5, 7). We're going to play a tournament. You get one entry and your final score is the average of your scores playing head-to-head against entries from several hundred Jane Streeters. An entry should be submitted as a list of 10 non-negative integers, adding up to 100, where the nth element is the number of units of resources being sent to region n. What's your entry? How did you go about coming up with it? How would your entry change if you were only allowed 90 soldiers? What about if you received 110 resources? (In both cases everyone else still gets 100).

My approach and solution

Maximum possible points won by a player and winning threshold

A player can win maximum number of points, if her opponent arranges soldiers like

Castle	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Opponent	100	0	0	0	0	0	0	0	0	0
Given player	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10

This player wins the maximum number of points as long as

$$a_i \geq 1, \text{ for all } i \in \{1, 2...10\}$$

such that they sum to 100. Under this restriction, the player loses in C1 while wins in the remaining ones. This results into a total of

$$(1 + ... + 10) - 1 = 54$$

points. Therefore, the player will be able to win if she scores 28 or more points.

My winning strategy

It can be observed that if a player wins the following cobinations of castles $\{C2, C3, C4, C5, C6, C8\}$; this accumulates them a total of $2+3+5+4+6+8 = 28$ points. (**Note: There can be other combinations of**

castles which sums to 28 for example $\{C1, C2, C3, C4, C5, C6, C7\}$ or $\{C3, C4, C5, C6, C10\}$, however my choice of $\{C2, C3, C4, C5, C6, C8\}$ was made keeping in mind that castles with very large and very low points are ignored.) Moreover, I allocate soldiers to castles $\{C2, C3, C4, C5, C6, C8\}$ proportinal to the points contributed towards the total. I plan to give more weightage to castles carrying more points, therefore I distribute points to the castles according to the function $f(x) \propto x^2$ where x is the number of points that a castle carry. However, my strategy is to use only 90% of the resources for castles $\{C2, C3, C4, C5, C6, C8\}$. Therefore, as per above restrictions I allocate 90 soldiers according to the following formula

$$\text{Soldiers in Castle } i = \frac{i^2}{2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 8^2} \times 90$$

for $i \in \{2, 3, 4, 5, 6, 8\}$. The following function in `R` carries the task of resource allocation according to the above strategy:

```r
square_strategy = function(castles, resource = 90){
  round(castles^2/(sum(castles^2))*resource)
}
# soldiers sent to castles 2,3,4,5,6 and 8
square_strategy(c(2,3,4,5,6,8),90)
```

```
## [1]  2  5  9 15 21 37
```

It can be observed that due to rounding of numbers the total number of soldiers is 89 thus I allocate the remaining 1 soldier to castle $C2$. Next, remaining 10 soldiers are allocated to castles $\{C1, C7, C9, C10\}$ by sending $1, 3, 3, 3$ soldiers respectively. The last allocation gives an edge in case opponent ignores these cells or allocates very-very few soldiers.

Castle	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Total
My strategy 100	1	3	5	9	15	21	3	37	3	3	100

When the total number of soldiers is 110

When I am given 110 soldiers to distribute among the castles, I would send 1 extra soldier to each castle. Therefore the soldier allocation can be done as follows:

Castle	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Total
My strategy 110	2	4	6	10	16	22	4	38	4	4	110

When the total number of soldiers is 90

At the same time when I have 90 soldiers to work with, I would use the same strategy as in the case of 100 soldiers except for the fact that the castle using highest number of soldiers receives 10 less soldiers this time. In this case I would send only 27 soldiers to castle $C8$ while keeping others fixed. This has been listed in the table below.

Castle	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Total
My strategy 90	1	3	5	9	15	21	3	27	3	3	90