Team Number:	90046
Problem Chosen:	B

2019 APMCM summary sheet

Regional economic vitality is an important part of regional comprehensive competitiveness and in order to improve it, some regions have launched many preferential policies. Due to the contribution of different resources, however, these policies have different impacts in different regions. This paper will discuss multiple factors that may influence economic vitality to establish an economic vitality measurement model. We also provide some suggestions for regional economic development based on models.

To solve question *1*, we select 6 influencing factors and find the values of them in Eastern region during the past decade. We can obtain the relational model of the 6 factors by *R-type Cluster Analysis*. Then we use *Principal Component Analysis* to rank the factors and obtain their weight coefficients. The economic influencing factor model based on *R-type Cluster Analysis* and *Principal Component Analysis* is established.

To solve question *2*, we continue to use the factors in question *1* and establish a short-term prediction model of resident population based on *Logistic Method* and a short-term prediction model of the quantity of surviving enterprises based on *Polynomial Fitting Method*. We classify factors like CPI, FAI, GDP and household consumption as economic category. To predict them, we adopt a short-term prediction model based on *Quadratic Exponential Smoothing Method*. For the long-term prediction model, we modify the accuracy of weighted coefficients in short-term prediction and use the actual and short-term prediction data to make long-term prediction for the 6 factors. Combined with the economic vitality prediction model built in question *1*, the short-term and long-term evaluation values are obtained.

To solve question *3*, we build an evaluation model of economic vitality based on *Principal Component Analysis* and extract highly representative principal components based on the contribution rate of each principal component. Combined with feature vector of each influencing index, we give the appraisal and ranking of each city given in *Attachment 3*.

At last, based on conclusions of question *1,2 and 3* and the rankings of cities, we give practical suggestions and proposals to further improve the economic vitality in Eastern region.

Keywords: Economic Vitality PCA Cluster Analysis Exponential Smoothing

Contents

I. Introduction

1.1 Background

Nowadays, leaders and authorities of both countries or regions of a country are seeking ways to improve and boost their regional comprehensive competitiveness [2]. However, the regional comprehensive competitiveness is determined by multiple factors such as economic vitality, living condition and culture depth, which makes it to be a hot topic [1].

The regional economic vitality is one of the most important part of regional comprehensive competitiveness. Recently, in order to boost the economic vitality, some regions have implemented series of preferential policies to spur the economy vitality. For example, Soochow reduces the investment attraction approval steps and supports the start-ups by providing financial assistance and Wuhan launches policy to lower the settlement threshold and living cost by up to 20% to attract talents. However, due to the existing regional disparity, these policies effect differently. How to seize the key factors and effectively improve the regional economic vitality is really worth studying [4].

1.2 General Description of Solutions to the Problem

In an effort to research the factors influencing economic vitality and how to improve the regional economic vitality, we have acquired some sources along with the given data to build suitable model and solve the problems.

- To begin our work, we take Eastern region as an example. We build a relational model involving influencing factors of economic vitality such as the total number of surviving enterprises, the amount of residents, regional GDP and so on. Then we study the program of action to improve the regional economic vitality and analyse the effects on the change of regional economic vitality from the perspective of changing trend of population and enterprise vitality.
- We continue to use the relational model built in question *1* and the economic vitality measurement index. Then we analyse the short-term and long-term effects on the economic vitality of Eastern region using prediction model when economic policy transformation is implemented. For the population forecast, we adopt the *Logistic* model. For the quantity of enterprises, we primarily use gray forecast model *GM (1, 1)* and apply *Polynomial Fitting Method* to improve the performance. The

household consumption, gross domestic production, CPI and FAI can be classified as economic categories. Therefore, we use time series prediction model based on exponential smoothing to address them.

- Evaluating economic vitality is a complicated issue. Therefore, we select a system of 10 indexes that to a large extent can act as measurement of economic vitality and build a measurement model using PCA. Then we rank the economic vitality of cities in *Attachment 3*.
- According to the conclusions for Problems *1-3*, we provide an economic development proposal for Eastern region discussed in Problem *2* so that the economic vitality in this region presents the benign sustainable development and its competitiveness is stronger.

II. Assumptions and Symbols

2.1 Assumptions and Justifications

- We assume that the given data and surveyed data are valid and authentic.
- We assume that the error or deviation in the data process is within the acceptable range.
- We assume that the selection of factors or indexes that may influence the economic vitality is objective.
- We assume that the factors or indexes we did not choose will not affect the economic vitality to a large extent.
- We assume that the cancellation rate of enterprises in a region is represented by the average rate of each city in that region.

2.2 Symbols and Notations

Serial Number	Symbol	Description
1	A	The matrix of influencing factors in Eastern region from year 2009 to 2018.
2	R	The matrix of correlation coefficients.
3	Z	The appraisal index.

4	IN	The matrix of influencing indexes on which we rank cities in *Attachment 3*based.
5	$S^{(1)}$	The exponential smoothing value.
6	$S^{(2)}$	The second-exponential smoothing value.

III. Establishment of Model

3.1 Data Process

We abstract 6 factors that may affect the economic vitality in Eastern region from the given and surveyed data as follows.

- The amount of residents in Eastern region from year 2009 to year 2018.
- The amount of enterprises in Eastern region from year 2009 to year 2018.
- CPI (consumer price index) in Eastern region from year 2009 to year 2018.
- GDP (gross domestic product) in Eastern region from year 2009 to year 2018.
- FAI (fixed-asset investment) in Eastern region from year 2009 to year 2018.
- Household consumption in Eastern region from year 2009 to year 2018.

3.1.1 *The Amount of Residents*

The amount of residents from 2009 to 2018 in each province of Eastern region and the total number in Eastern region are shown in Table 2. The data is surveyed from *National Bureau of Statistics* [7]. (unit: 10000)

We also draw the bar chart of the total number changing from 2009 to 2018 in Figure 1. We can see from the figure that the total number is generally rising throughout 10 years.

3.1.2 *The Amount of Enterprises*

Cancellation rate can be defined as the ratio of cancelled enterprises to the whole quantity of enterprises. Through the cancellation rate we can easily obtain the data of cancelled enterprises. Therefore, based on the whole quantity and increment of enterprises in *Attachment 2* and *Attachment 3*, the number of surviving enterprises in each year from 2009 to 2018 can be obtained. The amount of surviving enterprises in each year and changing trend from 2009 to 2018 are shown in Figure 2. We can see from the figure that the total number of enterprises is rising during the decade.

Table 2 The amount of residents from 2009 to 2018 in each province of Eastern region and the total number.

Year	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
Beijing	2154	2171	2173	2171	2152	2115	2069	2019	1962	1860
Tianjin	1560	1557	1562	1547	1517	1472	1413	1355	1299	1228
Hebei	7556	7520	7470	7425	7384	7333	7288	7241	7194	7034
Shandong	10047	10006	9947	9847	9789	9733	9685	9637	9588	9470
Jiangsu	8051	8029	7999	7976	7960	7939	7920	7899	7869	7810
Shanghai	2424	2418	2420	2415	2426	2415	2380	2347	2303	2210
Zhejiang	5737	5657	5590	5539	5508	5498	5477	5463	5447	5276
Fujian	3941	3911	3874	3839	3806	3774	3748	3720	3693	3666
Guangdong	11346	11169	10999	10849	10724	10644	10594	10505	10441	10130
Hainan	934	926	917	911	903	895	887	877	869	864
Total	53750	53364	52951	52519	52169	51818	51461	51063	50665	49548

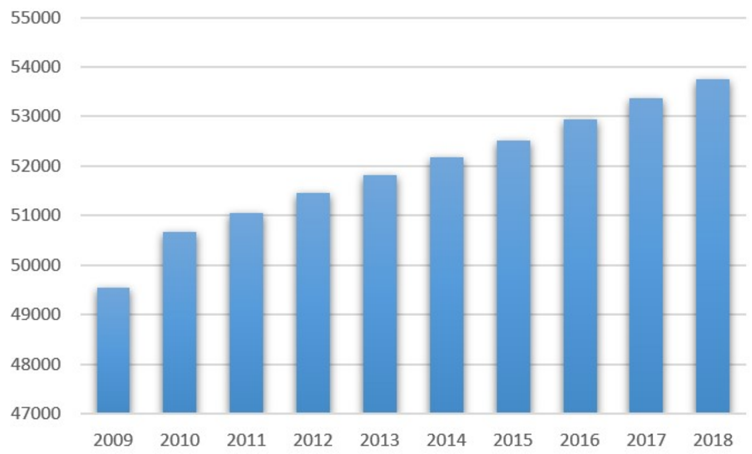

Figure 1 The bar chart of the total number of residents changing from 2009 to 2018.

3.1.3 *Consumer Price Index*

Consumer price index from 2009 to 2018 in each province of Eastern region and the average number are shown in Table 3. The data is surveyed from *National Bureau of Statistics*. (last year: 100)

We also draw the line chart of the average number of CPI changing from 2009 to 2018 in Figure 3. We can see from the figure that the average number fluctuates throughout the 10 years.

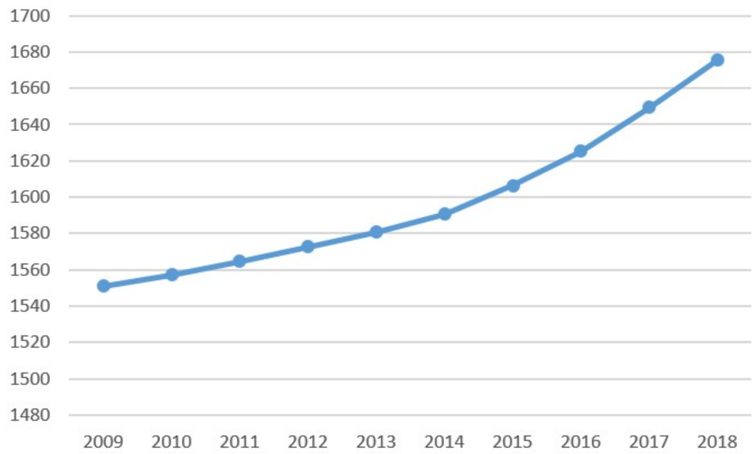

Figure 2 The line chart of the total number of enterprises changing from 2009 to 2018.

Table 3 Consumer price index from 2009 to 2018 in each province of Eastern region and the average number.

Year	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
Beijing	102.50	101.90	101.40	101.80	101.60	103.30	103.30	105.60	102.40	98.50
Tianjin	102.00	102.10	102.10	101.70	101.90	103.10	102.70	104.90	103.50	99.00
Hebei	102.40	101.70	101.50	100.90	101.70	103.00	102.60	105.70	103.10	99.30
Shandong	102.50	101.50	102.10	101.20	101.90	102.20	102.10	105.00	102.90	100.00
Jiangsu	102.30	101.70	102.30	101.70	102.20	102.30	102.60	105.30	103.80	99.60
Shanghai	101.60	101.70	103.20	102.40	102.70	102.30	102.80	105.20	103.10	99.60
Zhejiang	102.30	102.10	101.90	101.40	102.10	102.30	102.20	105.40	103.80	98.50
Fujian	101.50	101.20	101.70	101.70	102.00	102.50	102.40	105.30	103.20	98.20
Guangdong	102.20	101.50	102.30	101.50	102.30	102.50	102.80	105.30	103.10	97.70
Hainan	102.50	102.80	102.80	101.00	102.40	102.80	103.20	106.10	104.80	99.30
Average	102.18	101.82	102.13	101.53	102.08	102.63	102.67	105.38	103.37	98.97

3.1.4 *Gross Domestic Product*

Gross domestic product from 2009 to 2018 in each province of Eastern region and the total number are shown in Table 4. The data is surveyed from *National Bureau of Statistics*. (unit: 100 million yuan)

We also draw the bar chart of the total number of GDP changing from 2009 to 2018 in Figure 4. We can see from the figure that GDP is steadily increasing in Eastern region during the decade.

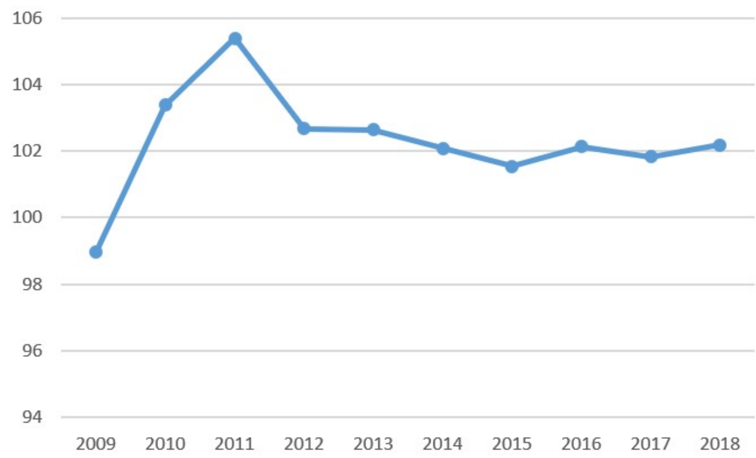

Figure 3 The line chart of the avarage number of CPI changing from 2009 to 2018.

Table 4 Gross Domestic Product from 2009 to 2018 in each province of Eastern region and the total number.

Year	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
Beijing	30320	28015	25669	23015	21331	19801	17879	16252	14114	12153
Tianjin	18810	18550	17885	16538	15727	14442	12894	11307	9224	7522
Hebei	36010	34016	32070	29806	29421	28443	26575	24516	20394	17235
Shandong	76470	72634	68024	63002	59427	55230	50013	45362	39170	33897
Jiangsu	92595	85870	77388	70116	65088	59753	54058	49110	41425	34457
Shanghai	32680	30633	28179	25123	23568	21818	20182	19196	17166	15046
Zhejiang	56197	51768	47251	42886	40173	37757	34665	32319	27722	22990
Fujian	35804	32182	28811	25980	24056	21868	19702	17560	14737	12237
Guangdong	97278	89705	80855	72813	67810	62475	57068	53210	46013	39483
Hainan	4832	4463	4053	3703	3501	3178	2856	2523	2065	1654
Total	480996	447835	410186	372983	350101	324765	295892	271355	232031	196674

3.1.5 *Fixed-Asset Investment*

Fixed-asset investment from 2009 to 2018 in each province of Eastern region and the total number are shown in Table 5. The data of 2018 is predicted with data of previous years with an error within 5% using fitting method. The data is surveyed from *National Bureau of Statistics*. (unit: 100 million yuan)

We also draw the line chart of the total number of FAI changing from 2009 to 2018 in Figure 5. We can see from the line chart that the total number of FAI is rising during the decade.

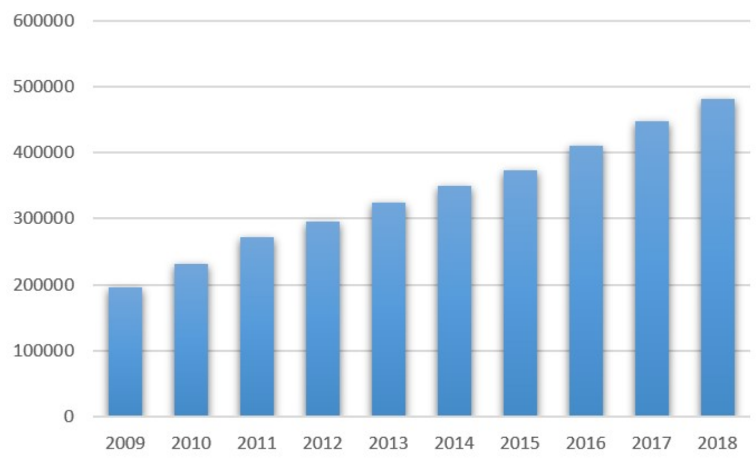

Figure 4 The bar chart of the total number of GDP changing from 2009 to 2018.

Table 5 Fixed-asset investment from 2009 to 2018 in each province of Eastern region and the total number.

Year	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
Beijing	8862	8370	7944	7496	6924	6847	6112	5579	5403	4617
Tianjin	13881	11289	12779	11832	10518	9130	7935	7068	6278	4738
Hebei	37070	33407	31750	29448	26672	23194	19661	16389	15083	12270
Shandong	61480	55203	53323	48312	42496	36789	31256	26750	23281	19035
Jiangsu	58598	53277	49663	46247	41939	36373	30854	26693	23184	18950
Shanghai	7600	7247	6756	6353	6016	5648	5118	4962	5109	5044
Zhejiang	35233	31696	30276	27323	24263	20782	17649	14185	12376	10742
Fujian	35233	26416	23237	21301	18178	15327	12440	9911	8199	6231
Guangdong	39358	37762	33304	30343	26294	22308	18752	17069	15624	12933
Hainan	4720	4244	3890	3451	3112	2698	2145	1657	1317	988
Total	301693	268911	252923	232107	206412	179098	151923	130263	115854	95548

3.1.6 *Household Consumption*

Household Consumption from 2009 to 2018 in each province of Eastern region and the total number are shown in Table 6. The data of 2018 is predicted with data of previous years with an error within 3% using fitting method. The data is surveyed from *National Bureau of Statistics*. (unit: yuan)

We also draw the bar chart of the total number changing from 2009 to 2018 in Figure 6. We can see from the bar chart that the total number of household consumption is increasing throughout 10 years.

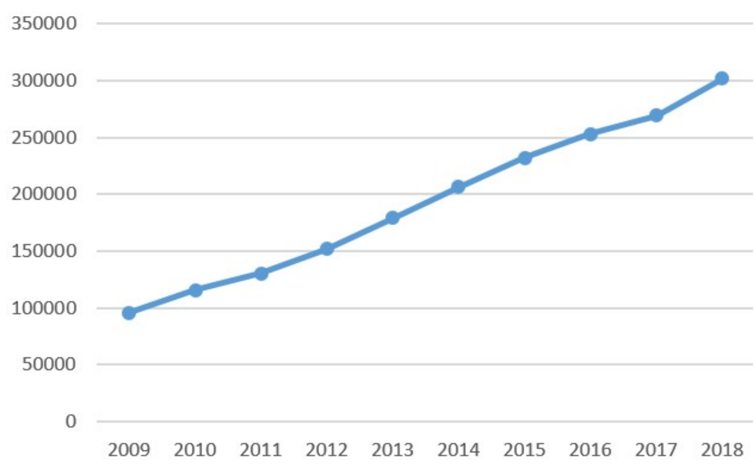

Figure 5 The line chart of the total number of FAI changing from 2009 to 2018.

Table 6 Household consumption from 2009 to 2018 in each province of Eastern region and the total number.

Year	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
Beijing	56766	52912	48883	39200	36057	33337	30350	27760	24982	22023
Tianjin	41563	38975	36257	32595	28492	26261	22984	20624	17852	15200
Hebei	16502	15893	14328	12829	12171	11557	10749	9551	8057	7193
Shandong	30602	28353	25860	20684	19184	16728	15095	13524	11606	10494
Jiangsu	42541	39796	35875	31682	28316	23585	19452	17167	14035	11993
Shanghai	55861	53617	49617	45816	43007	39223	36893	35439	32271	26582
Zhejiang	35487	33851	30743	28712	26885	24771	22845	21346	18274	15867
Fujian	26643	25969	23355	20828	19099	17115	16144	14958	13187	11336
Guangdong	32444	30762	28495	26365	24582	23739	21823	19578	17211	15243
Hainan	21747	20939	18431	17019	12915	11712	10634	9238	7553	6695
Total	360155	341067	311844	275730	250708	228028	206969	189185	165028	142626

3.2 Model Built to Solve Question *1*

3.2.1 *The Relational Model of Influencing Factors of Economic Vitality Based on R-type Cluster Analysis and Principal Component Analysis*

According to question *1* and the results of data process, in order to research the similarity between influencing factors, we adopt the method *R-type Cluster Analysis* to build the relational model to measure the economic vitality in Eastern region. We give the details of model construction by steps using *R-type Cluster Analysis* as follows [5].

1) Firstly, we measure the similarity between influencing factors. The value of each

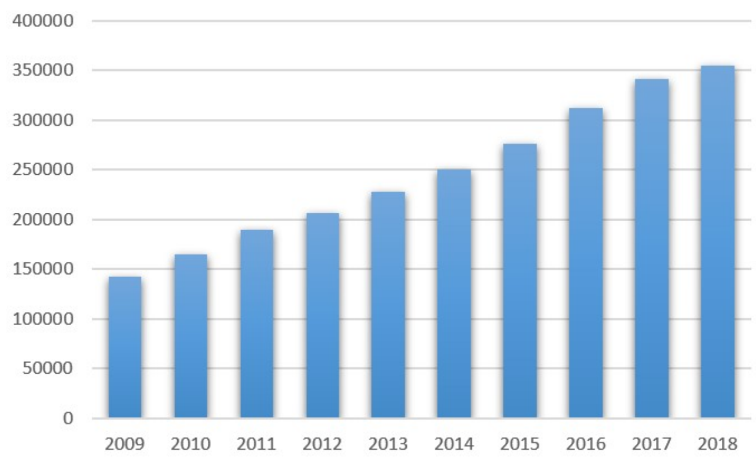

Figure 6 The bar chart of the total number of household consumption changing from 2009 to 2018.

factor in ith year can be described as

$$\left[x_{1j}, x_{2j}, \ldots\ldots, x_{ij}\right]^T \in R^n \tag{1}$$

where the integer i ranges from 1 to 10 representing year from 2009 to 2018 and the integer j ranges from 1 to 6 representing factors *household consumption, FAI, GDP, CPI, the amount of residents and the amount of enterprises* respectively. Therefore, the similarity between x_{im} and x_{ik} can be measured by correlation coefficient r_{mk} as follows.

$$r_{mk} = \frac{\sum\limits_{i=1}^{10}(x_{im} - \bar{x}_{im})(x_{ik} - \bar{x}_{ik})}{\left[\sum\limits_{i=1}^{10}(x_{im} - \bar{x}_{im})^2 \sum\limits_{i=1}^{10}(x_{ik} - \bar{x}_{ik})^2\right]^{\frac{1}{2}}} \tag{2}$$

Then we can create a correlation coefficient matrix $R_{6\times6}$ as follows.

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1k} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mk} \end{bmatrix} \tag{3}$$

where $\forall m, k \in [1, 6]$, $|r_{mk}| \leq 1$ and r_{mk} equals r_{km}. If $|r_{mk}|$ approximates 1, the similarity between x_{im} and x_{ik} is high. If $|r_{mk}|$ approximates 0, the similarity between x_{im} and x_{ik} is low.

2) Secondly, we cluster the factors by average distance D formulated as follows.

$$D(G_1, G_2) = \frac{1}{n_1 n_2} \sum_{x_{im}\in G_1} \sum_{x_{ik}\in G_2} d(x_{im}, x_{ik}) \tag{4}$$

where d is the distance between sample points in G_1 and G_2 and n_1 and n_2 are the numbers of sample points in G_1 and G_2.

Since we obtain the relationship of these factors through *R-type Cluster Analysis*, we adopt *Principal Component Analysis* to measure the weight of each factor. We give details of model construction by steps.

1) Firstly, we use the results of 6 factor variables from data process to create the factor matrix $A_{6\times 10}$ which is defined in the following equation.

$$A = (a_{ij})_{6\times 10} \tag{5}$$

where the integer i ranges from 1 to 6 representing factors *household consumption, FAI, GDP, CPI, the amount of residents and the amount of enterprises* respectively and the integer j ranges from 1 to 10 representing year from 2009 to 2018. Therefore, the factor variables can be symbolized as x_i.

2) Secondly, we normalize each element value in the factor matrix using the following equation.

$$\tilde{a}_{ij} = \frac{a_{ij} - \mu_i}{s_i} \tag{6}$$

$$\mu_i = \frac{1}{10}\sum_{j=1}^{10} a_{ij} \tag{7}$$

$$s_i = \sqrt{\frac{1}{9}\sum_{j=1}^{10} (a_{ij} - \mu_i)^2} \tag{8}$$

μ_i and s_i are the sample mean value and the sample standard deviation of ith factor respectively. Accordingly, the factor variables x_i can be normalized as $\tilde{x}_i$ with the following equation.

$$\tilde{x}_i = \frac{x_i - \mu_i}{s_i} \tag{9}$$

3) Thirdly, we formulate the correlation coefficient matrix R with the following equation.

$$R = (r_{i_1 i_2})_{6\times 6} \tag{10}$$

$$r_{i_1 i_2} = \frac{\sum\limits_{j=1}^{10} \tilde{a}_{i_1 j} \cdot \tilde{a}_{i_2 j}}{9} \tag{11}$$

where i_1 and i_2, with different value from 1 to 6, are the same as i. $r_{i_1 i_2}$ is the correlation coefficient of i_1th factor and i_2th factor and also the element of correlation coefficient matrix.

4) Fourthly, we calculate the feature values λ_i ($\lambda_i \geq 0$) and corresponding normalized feature vectors $[u_{1j}, u_{2j}, \ldots, u_{6j}]^T$ of correlation coefficient matrix R. Then the

principal components formed by 6 normalized feature vectors are formulated as follows.

$$\begin{cases} y_1 = u_{11}\tilde{x}_1 + u_{21}\tilde{x}_2 + \ldots + u_{61}\tilde{x}_6 \\ y_2 = u_{12}\tilde{x}_1 + u_{22}\tilde{x}_2 + \ldots + u_{62}\tilde{x}_6 \\ y_3 = u_{13}\tilde{x}_1 + u_{23}\tilde{x}_2 + \ldots + u_{63}\tilde{x}_6 \\ y_4 = u_{14}\tilde{x}_1 + u_{24}\tilde{x}_2 + \ldots + u_{64}\tilde{x}_6 \\ y_5 = u_{15}\tilde{x}_1 + u_{25}\tilde{x}_2 + \ldots + u_{65}\tilde{x}_6 \\ y_6 = u_{16}\tilde{x}_1 + u_{26}\tilde{x}_2 + \ldots + u_{66}\tilde{x}_6 \end{cases} \tag{12}$$

where y_i denotes the ith principal component.

5) Fifthly, we select p out of 6 principal components to measure their comprehensive appraisal value. Contribution rate b_i of each feature value λ_i is also the contribution rate of corresponding principal component y_i, which is formulated as follows.

$$b_i = \frac{\lambda_i}{\sum\limits_{i=1}^{6} \lambda_i} \tag{13}$$

Then the accumulation contribution rate of p principal components can be formulated as follows.

$$\partial_p = \frac{\sum\limits_{i=1}^{p} \lambda_i}{\sum\limits_{i=1}^{6} \lambda_i} \tag{14}$$

If ∂_p approximates 1, it means that we can choose these p principal components to substitute for the original 6 principal components. The comprehensive appraise of principal components is then simplified.

6) Lastly, the result of comprehensive appraise Z can be acquired through computation. The following equation gives the formulation of Z.

$$Z = \sum_{i=1}^{p} b_i y_i \tag{15}$$

3.2.2 *Model Results*

The result of *R-type Cluster Analysis* is shown in Figure 7. The number 1 to 6 denote to *household consumption, FAI, GDP, CPI, the amount of residents and the quantity of enterprises* respectively.

The appraisal model obtained through *Principal Component Analysis* is then applied *T-transformation* to get the following result.

$$Z = 64.255\tilde{x}_3 + 57.629\tilde{x}_1 + 53.559\tilde{x}_2 + 44.113\tilde{x}_5 + 40.243\tilde{x}_6 + 40.198\tilde{x}_4 \tag{16}$$

The rankings and the appraisal of factors are shown in Table 7.

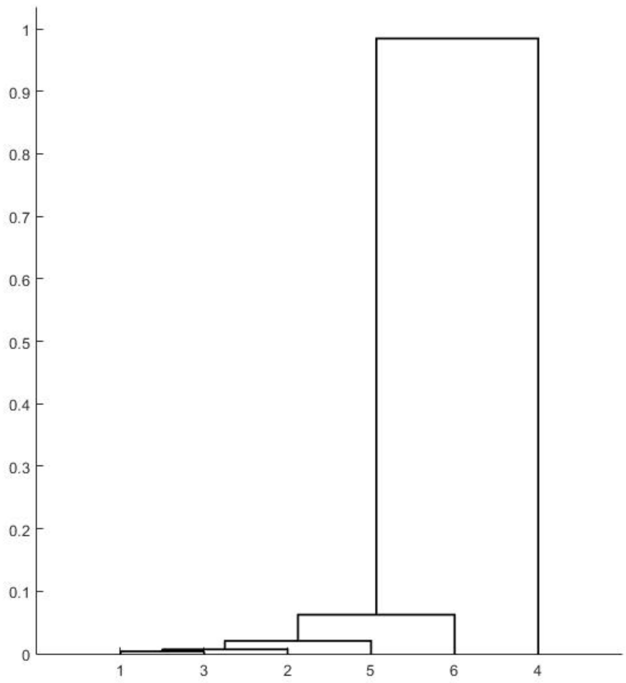

Figure 7 The cluster diagram.

Table 7 The rankings and the appraisal of factors.

Index	Ranking	Appraisal
GDP	1	4.472392974
Household Consumption	2	2.393639109
FAI	3	1.11678447
Residents	4	-1.84694511
Enterprises	5	-3.06107799
CPI	6	-3.07479345

3.3 Model Built to Solve Question *2*

3.3.1 *Short-term and Long-term Prediction of Residents Based on Logistic Model*

In a short term, the growth of residents will benefit the economy in terms of labour resource and skilled talents. However, the restriction from limited living room and finite natural resources will constrain the resident growth. Therefore, we propose to use *Logistic Model* to predict the population growth of residents and give the detailed steps [4].

1) The population is a function of year i and can be formed as $n(i)$. The growth rate

Table 8 The results of actual numbers of residents and prediction numbers.

Year	Actual Value	Predictive Value
2009	49548	50124.31
2010	50665	50502.82
2011	51063	50887.09
2012	51461	51277.25
2013	51818	51673.45
2014	52169	52075.82
2015	52519	52484.50
2016	52951	52899.66
2017	53364	53321.44
2018	53750	53750.00
2019	Unkown	54185.51
2020	Unkown	54628.16
2021	Unkown	55078.11
2022	Unkown	55535.55
2023	Unkown	56000.47
2024	Unkown	56473.58
2025	Unkown	56954.74
2026	Unkown	57444.09
2027	Unkown	57941.79
2028	Unkown	58447.71

r is a function of population and can then be described as a decrement function $r(n)$. We suppose $r(n)$ is a linear function and is formulated as follows.

$$r(n) = r - sn \tag{17}$$

where s is a coefficient. Assumption is made that the maximum population which Eastern region can hold is n_{max}. That means when the population reaches maximum, the growth rate reduces to nearly 0.

2) Based on the above model assumption, we further formulate the growth rate as follows.

$$r(n) = r(1 - \frac{n}{n_{max}}) \tag{18}$$

Logistic model is built and shown as follows.

$$\begin{cases} \frac{dn}{di} = n \cdot r(1 - \frac{n}{n_{\max}}) \\ n(i_0) = n_0 \end{cases} \tag{19}$$

where i_0 and n_0 are the initial value of year and population respectively. The solution is

$$n(i) = \frac{n_{\max}}{1 + (\frac{n_{\max}}{n_0} - 1)e^{-r(i-i_0)}} \tag{20}$$

Based on the solution given above, we import data in *Excel* to *Matlab* and get the prediction results of resident population in next years shown in Table 8.

3.3.2 *Short-term Prediction of Quantity of Surviving Enterprises Based on GM(1,1) and Polynomial Fitting Method*

Based on the background, we plan to apply commonly used *GM(1,1)* to predict the quantity of enterprises in next years. We give the steps of model construction.

1) Firstly, we test the data of enterprise quantity. The primary data sequence is $m^{(0)}$ and

$$m^{(0)} = [m^{(0)}(1), m^{(0)}(2), m^{(0)}(3), ..., m^{(0)}(i)] \tag{21}$$

where i is year. The sequence ratio $\lambda(k)$ is define as follows.

$$\lambda(k) = \frac{m^{(0)}(k-1)}{m^{(0)}(k)} \tag{22}$$

where k equals i and starts from 2. The range of sequence ratio $(1.0040, 1.0158)$ is obtained through *Matlab* and justifies the use of *GM(1,1)* by covering the following range.

$$\Theta = (e^{-\frac{2}{\pi}}, e^{\frac{1}{6}}) = (0.834, 1.181) \tag{23}$$

2) Secondly, we obtain $m^{(1)}$ through the following equation.

$$m^{(1)} = [m^{(0)}(1), \sum_{k=1}^{2} m^{(0)}(k), \sum_{k=1}^{3} m^{(0)}(k), ..., \sum_{k=1}^{10} m^{(0)}(k)] \tag{24}$$

Then the data matrix B and the data column vector Y are formulated as

$$B = \begin{bmatrix} -\frac{1}{2}[m^{(1)}(1) + m^{(1)}(2)] & 1 \\ -\frac{1}{2}[m^{(1)}(2) + m^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[m^{(1)}(9) + m^{(1)}(10)] & 1 \end{bmatrix}, Y = \begin{bmatrix} m^{(0)}(2) \\ m^{(0)}(3) \\ \vdots \\ m^{(0)}(10) \end{bmatrix} \tag{25}$$

3) Thirdly, *GM(1,1)* is built and shown as follows.

$$\begin{cases} \frac{dm^{(1)}}{di} + \hat{a}m^{(1)} = \hat{b} \\ \hat{u} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (B^T B)^{-1} B^T Y \end{cases} \tag{26}$$

The solution is

$$\hat{m}^{(1)}(k+1) = [m^{(0)}(1) - \frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \tag{27}$$

4) Fourthly, We import the data of enterprises to *Matlab* and get the fitting results.

5) Finally, we test our model using sequence ratio deviation $p(k)$. We calculate the sequence ratio $\lambda(k)$ with reference data $m^{(0)}(k-1)$ and $m^{(0)}(k)$. Therefore, the sequence ratio deviation is formulated as follows.

$$p(k) = 1 - (\frac{1-0.5a}{1+0.5a}) \cdot \lambda(k) \tag{28}$$

The final sequence ratio deviation is 0.18261 and less than 0.2. The prediction results meet the requirement.

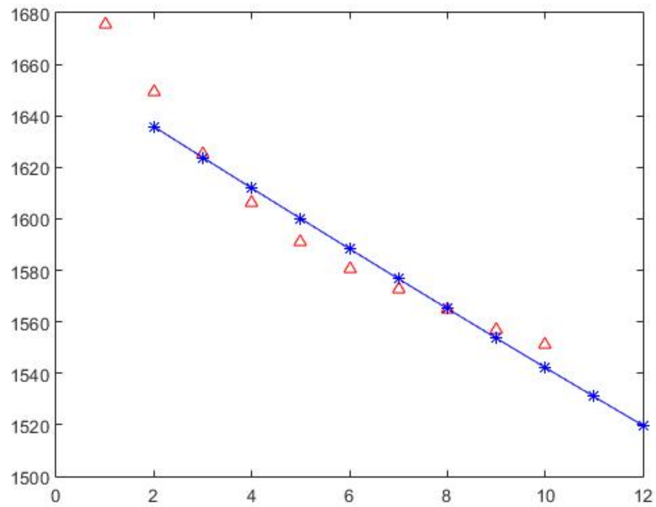

Figure 8 The fitting result of enterprises using grey model.

The fitting result of enterprises using grey model is shown in Figure 8. The data in red is the actual data and the data in blue is the predictive data.

We can see clearly from the fitting result that the deviation between actual data and prediction data is large. That means the performance of *GM(1,1)* is not satisfying. Therefore, we propose an improved model using function fitting.

We apply *Linear Least Square* to improve our model and the details of it is discussed by steps.

1) We regulate the fitting function to be

$$f(x) = C_1 r_1(x) + C_2 r_2(x) + \ldots\ldots + C_m r_m(x) \tag{29}$$

where $r_m(x)$ is a set of linearly independent functions and C_m are coefficients. m is positive integer.

2) Through observation, we choose $r_m(x)$ to be the cubic exponent of x. The fitting function is then formulated to be

$$f(x) = C_1 + C_2 x + ... + C_m x^{m-1} \tag{30}$$

where m equals 4.

3) We define the equation

$$J(C_1, C_2, ..., C_m) = \sum_{i=1}^{10} {\delta_i}^2 = \sum_{i=1}^{10} [f(x_i) - y_i]^2 \tag{31}$$

where

$$\frac{dJ}{dC_j} = 0 \tag{32}$$

and obtain

$$\sum_{i=1}^{10} r_j(x_i)[\sum_{k=1}^{m} C_k r_k(x_i) - y_i] = 0 \tag{33}$$

also

$$\sum_{k=1}^{m} C_k[\sum_{i=1}^{10} r_j(x_i) r_k(x_i)] = \sum_{i=1}^{10} r_j(x_i) y_i \tag{34}$$

which can be further described by the matrix

$$R = \begin{pmatrix} r_1(x_1) & \dots & r_m(x_1) \\ \vdots & \ddots & \vdots \\ r_1(x_{10}) & \cdots & r_m(x_{10}) \end{pmatrix} \tag{35}$$

to be

$$R^T R A = R^T Y \tag{36}$$

where

$$A = [C_1, C_2, ..., C_m]^T \tag{37}$$

and

$$Y = [y_1, y_2, ..., y_n]^T \tag{38}$$

The unique solution is as follows.

$$A = (R^T R)^{-1} R^T Y \tag{39}$$

The fitting result using *Linear Least Square* is shown in Figure 9.

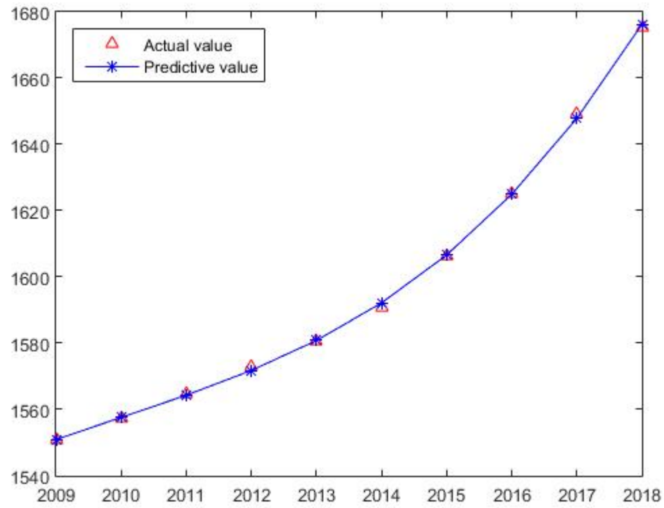

Figure 9 The fitting result of enterprises using Linear Least Square.

3.3.3 *Short-term Prediction of Other factors Based on Second-exponential Smoothing Model*

Based on previous analysis, we classify CPI, GDP, FAI and household consumption as economic category, compared with the other two factors. Commonly, the effect of history data on prediction usually fades away as time proceeds. Therefore, a better and more practical way to predict the value of these factors is applying weight which decreases with time to history data in the prediction work. We choose to apply second-exponential smoothing model and discuss the details in model construction by steps.

1) Firstly, the exponential smoothing prediction model is formulated as

$$S_i^{(1)} = \alpha e_i + (1-\alpha)e_{i-1}^{(1)} = e_{i-1}^{(1)} + \alpha(e_i - S_{i-1}^{(1)}) \tag{40}$$

where e_i and α are time sequence of factors and weighted coefficient respectively. The integer i ranges from 1 to 10 representing year from 2009 to 2018 and α is from 0 to 1. The iteration equation of moving average $\mu_i^{(1)}$ is

$$\mu_i^{(1)} = \mu_{i-1}^{(1)} + \frac{e_i - e_{i-N}}{N} \tag{41}$$

where α equals $\frac{1}{N}$. We then choose $\mu_{i-1}^{(1)}$ to be the best estimation of e_{i-N} and get the following equation.

$$\mu_i^{(1)} = \mu_{i-1}^{(1)} + \frac{e_i - \mu_{i-1}^{(1)}}{N} = \frac{e_i}{N} + (1 - \frac{1}{N})\mu_{i-1}^{(1)} \tag{42}$$

We replace $\mu_i^{(1)}$ with S_i and get the following equation.

$$S_i^{(1)} = \alpha e_i + (1-\alpha)S_{i-1}^{(1)} \tag{43}$$

That means $S_i^{(1)}$ is the weighted average of all history data with the weighted coefficient to be α, $\alpha(1-\alpha)$, $\alpha(1-\alpha)^2$ and so on. Obviously, the following equation is established.

$$\sum_{k=0}^{\infty} \alpha(1-\alpha)^k = \frac{\alpha}{1-(1-\alpha)} = 1 \tag{44}$$

Then the exponential smoothing prediction model can be further formulated as

$$\hat{e}_{i+1} = S_i^{(1)} = \alpha e_i + (1-\alpha)\hat{e}_i \tag{45}$$

2) Secondly, we transform the model equation to

$$\hat{e}_{i+1} = \hat{e}_i + \alpha(e_i - \hat{e}_i) \tag{46}$$

We can see from the above equation that the selection of weighted coefficient α directly affects the prediction results. Therefore, the selection principle that we stick to is to minimize the prediction error.

3) Based on exponential smoothing model, we make improvement to obtain second-exponential smoothing model with following equations.

$$\begin{cases} S_i^{(1)} = \alpha e_i + (1-\alpha)e_{i-1}^{(1)} \\ S_i^{(2)} = \alpha S_i^{(1)} + (1-\alpha)S_{i-1}^{(2)} \end{cases} \tag{47}$$

where $S_i^{(1)}$ and $S_i^{(2)}$ are the value of exponential smoothing and second-exponential smoothing respectively. Furthermore, the time sequence $\{e_i\}$ can be predicted through linear model formulated as follows.

$$\hat{e}_{i+m} = a_i + b_i m \tag{48}$$

where m is a positive integer coefficient and a_i and b_i are

$$\begin{cases} a_i = 2S_i^{(1)} - S_i^{(2)} \\ b_i = \frac{\alpha}{1-\alpha}(S_i^{(1)} - S_i^{(2)}) \end{cases} \tag{49}$$

Since the second-exponential smoothing prediction model is applied, we give the prediction results of the factors CPI, FAI, GDP and household consumption.

1) For CPI, the fitting result is shown in Figure 10. The short-term prediction results are shown in Table 9. In order to obtain the minimum prediction error, α is chosen to be 0.5 with prediction error to be 0.0002194.

2) For FAI, the fitting result is shown in Figure 11. The short-term prediction results are shown in Table 10. In order to obtain the minimum prediction error, α is chosen to be 0.9 with prediction error to be 0.095353.

Table 9 The short-term prediction results of CPI.

Year	Prediction
2019	1020.742
2020	1020.816
2021	1020.889

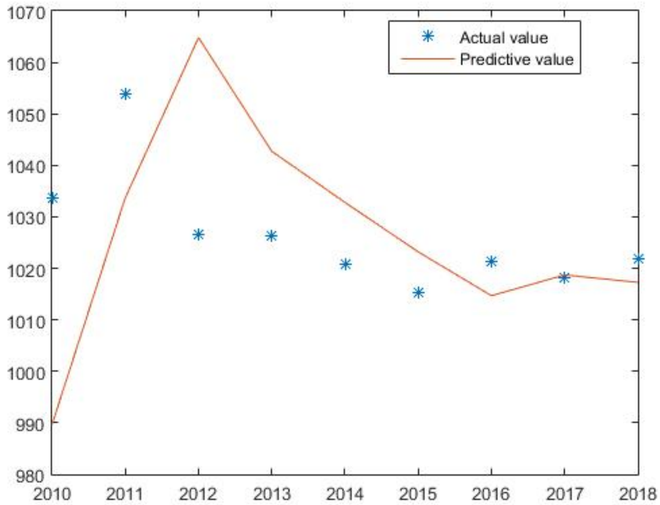

Figure 10 The fitting result of CPI.

3) For GDP, the fitting result is shown in Figure 12. The short-term prediction results are shown in Table 11. In order to obtain the minimum prediction error, α is chosen to be 0.9 with prediction error to be 0.14047.

4) For household consumption, the fitting result is shown in Figure 13. The short-term prediction results are shown in Table 12. In order to obtain the minimum prediction error, α is chosen to be 0.9 with prediction error to be 0.10502.

Table 10 The short-term prediction results of FAI.

Year	Prediction
2019	331281.8013
2020	361027.1149
2021	390772.4285

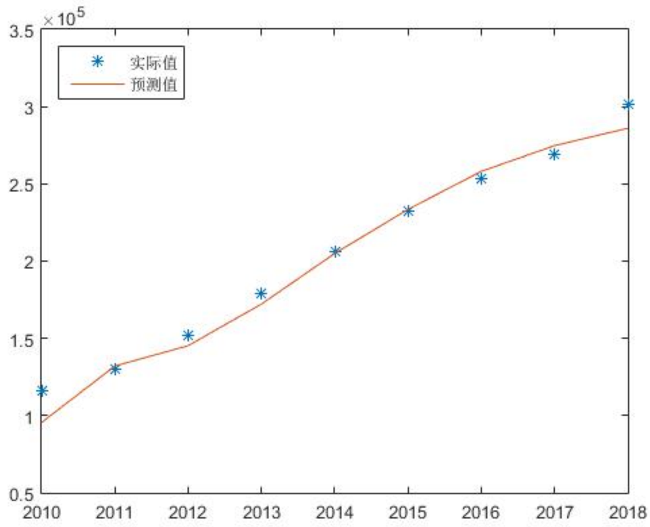

Figure 11 The fitting result of FAI.

Table 11 The short-term prediction results of GDP.

Year	Prediction
2019	514984.7163
2020	548933.7789
2021	582882.8415

3.4 Model Built to Solve Question *3*

3.4.1 *Economic Vitality Measurement and Ranking Model Based on PCA*

According to question *3* and the surveyed data from *National Bureau of Statistics* and *Local Bureau of Statistics*, we adopt the method PCA to build the economic vitality measurement and ranking model. We give the details of model construction using PCA by steps as follows.

1) Firstly, we use the data of 10 indexes to build the index system to create the index

Table 12 The short-term prediction results of household consumption.

Year	Prediction
2019	373045.2115
2020	390435.9472
2021	407826.6829

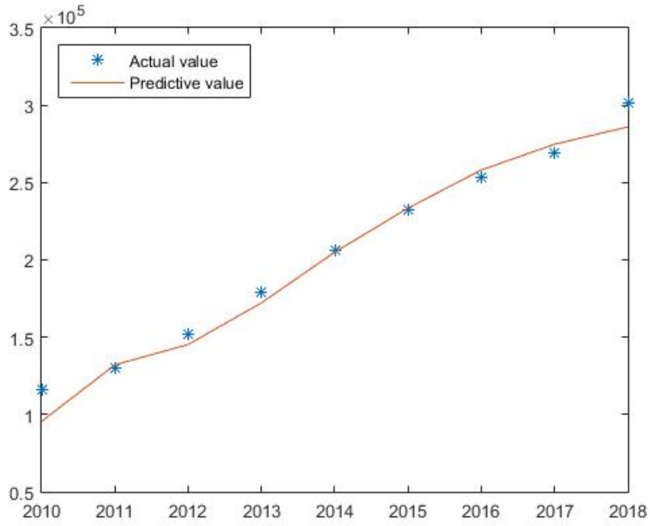

Figure 12 The fitting result of GDP.

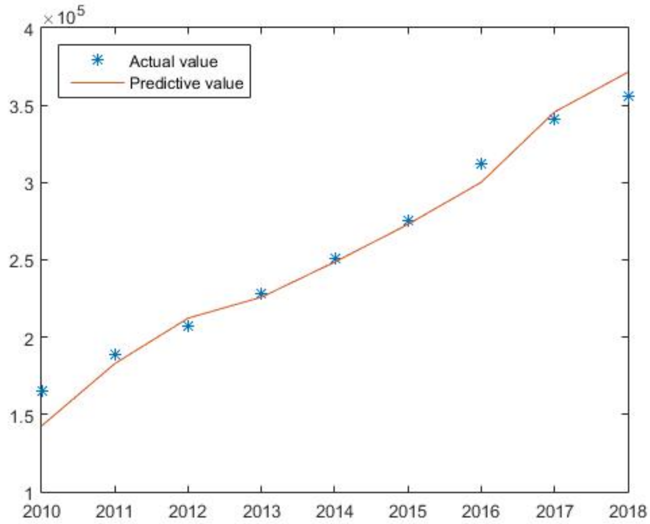

Figure 13 The fitting result of household consumption.

matrix $IN_{19\times10}$ which is defined in the following equation.

$$IN = (a_{ij})_{19\times10} \tag{50}$$

where the integer i ranges from 1 to 19 representing each city in *Attachment 3* and the integer j ranges from 1 to 10 representing indexes *GDP (100 million), gross value of the primary industry (100 million), gross value of the secondary industry (100 million), gross value of the tertiary industry (100 million), the amount of residents (10000), average salary of employees (yuan), household saving balance at the end of the year (100 million), total retail sales of goods (100 million), total imports and exports (million dollars) and the quantity of enterprises (10000)* respectively. Therefore, the indexes can be symbolized as x_j.

2) Secondly, we normalize each element in the index matrix using the following equation.

$$\tilde{a}_{ij} = \frac{a_{ij} - \mu_j}{s_j} \tag{51}$$

$$\mu_j = \frac{1}{19}\sum_{i=1}^{19} a_{ij} \tag{52}$$

$$s_j = \sqrt{\frac{1}{18}\sum_{i=1}^{19} (a_{ij} - \mu_j)^2} \tag{53}$$

where μ_j and s_j are the sample mean value and the sample standard deviation of jth index respectively. Accordingly, the indexes x_j can be normalized as $\tilde{x_j}$ with the following equation.

$$\tilde{x}_j = \frac{x_j - \mu_j}{s_j} \tag{54}$$

3) Thirdly, we formulate the correlation coefficient matrix R with the following equation.

$$R = (r_{j_1 j_2})_{10\times 10} \tag{55}$$

$$r_{j_1 j_2} = \frac{\sum_{i=1}^{19} \tilde{a}_{ij_1} \cdot \tilde{a}_{ij_2}}{18} \tag{56}$$

where j_1 and j_2, with different value from 1 to 10, are the same as j. $r_{j_1 j_2}$ is the correlation coefficient of j_1th index and j_2th index and also the element of correlation coefficient matrix.

4) Fourthly, we calculate the feature values λ_j ($\lambda_j \geq 0$) and corresponding normalized feature vectors $[u_{1j}, u_{2j}, \ldots, u_{10j}]^T$ of correlation coefficient matrix R. Then the principal components formed by 10 normalized feature vectors are formulated as follows.

$$\begin{cases} y_1 = u_{11}\tilde{x}_1 + u_{21}\tilde{x}_2 + \ldots + u_{101}\tilde{x}_{10} \\ y_2 = u_{12}\tilde{x}_1 + u_{22}\tilde{x}_2 + \ldots + u_{102}\tilde{x}_{10} \\ y_3 = u_{13}\tilde{x}_1 + u_{23}\tilde{x}_2 + \ldots + u_{103}\tilde{x}_{10} \\ y_4 = u_{14}\tilde{x}_1 + u_{24}\tilde{x}_2 + \ldots + u_{104}\tilde{x}_{10} \\ y_5 = u_{15}\tilde{x}_1 + u_{25}\tilde{x}_2 + \ldots + u_{105}\tilde{x}_{10} \\ y_6 = u_{16}\tilde{x}_1 + u_{26}\tilde{x}_2 + \ldots + u_{106}\tilde{x}_{10} \\ y_7 = u_{17}\tilde{x}_1 + u_{27}\tilde{x}_2 + \ldots + u_{107}\tilde{x}_{10} \\ y_8 = u_{18}\tilde{x}_1 + u_{28}\tilde{x}_2 + \ldots + u_{108}\tilde{x}_{10} \\ y_9 = u_{19}\tilde{x}_1 + u_{29}\tilde{x}_2 + \ldots + u_{109}\tilde{x}_{10} \\ y_{10} = u_{110}\tilde{x}_1 + u_{210}\tilde{x}_2 + \ldots + u_{1010}\tilde{x}_{10} \end{cases} \tag{57}$$

where y_j denotes the jth principal component.

5) Fifthly, we select p out of 10 principal components to measure their comprehensive appraisal value. Contribution rate b_j of each feature value λ_j is also the contribution rate of corresponding principal component y_j, which is formulated as follows.

$$b_j = \frac{\lambda_j}{\sum\limits_{j=1}^{10} \lambda_j} \tag{58}$$

Then the accumulation contribution rate of p principal components can be formulated as follows.

$$\partial_p = \frac{\sum\limits_{j=1}^{p} \lambda_j}{\sum\limits_{j=1}^{10} \lambda_j} \tag{59}$$

If ∂_p approximates 1, it means that we can choose these p principal components to substitute for the original 10 principal components to simplify the appraise.

6) Lastly, the result of comprehensive appraise Z can be acquired through computation. The following equation gives the formulation of z.

$$Z = \sum_{j=1}^{p} b_j y_j \tag{60}$$

3.4.2 *Data and Model Results*

Table 13 The surveyed data of 10 indexes for city ranking.

City	Shanghai	Shenzhen	Beijing	Guangzhou	Chongqing	Chengdu	Nanjing	Hangzhou	Suzhou
GDP	30633	22490	28015	21503	19425	13889	11715	12603	18597
Gross value of the primary industry	111	20	120	233	1276	501	263	311	214
Gross value of the secondary industry	9331	9320	5327	6015	8585	5998	4455	4362	8933
Gross value of the tertiary industry	21192	13150	22568	15254	9564	7390	6997	7930	9450
The amount of residents	1455	563	1359	898	3390	1435	681	754	1068
Average salary of employees	130765	100173	134994	98612	73272	79292	101502	96670	87350
Household saving balance at the end of the year	6642	10838	5431	1537	2252	1276	1272	1567	8166
Total retail sales of goods	11830	6016	11575	9403	8068	6404	5605	5717	4956
Total imports and exports	476197	414146	324017	143250	66601	58316	61187	69013	316079
The quantity of enterprises	157	174	118	90	70	61	56	49	54

Based on the model constructed using PCA, we apply *Matlab* to process surveyed data and produce model results. The surveyed data of the 10 indexes for question *3* is

City	Tianjin	Qingdao	Dongguan	Zhengzhou	Wuhan	Xi'an	Ningbo	Changsha	Shengyang	Kunming
GDP	18549	11037	7582	9130	13410	7470	9842	10536	5865	4858
Gross value of the primary industry	169	381	24	159	408	281	306	379	268	210
Gross value of the secondary industry	7594	4546	3594	4248	5861	2596	5119	4998	2261	1866
Gross value of the tertiary industry	10787	6110	3965	4724	7141	4593	4417	5158	3335	2782
The amount of residents	1050	803	834	842	854	906	597	709	737	563
Average salary of employees	96965	83539	53446	70486	79684	77774	91705	85187	74181	76350
Household saving balance at the end of the year	2310	1157	5103	1057	1403	655	1245	800	656	561
Total retail sales of goods	5730	4541	2688	4057	6196	4330	4048	4548	3990	2591
Total imports and exports	112919	74124	12264	59635	28597	37700	112197	13886	12846	7818
The quantity of enterprises	44	41	43	43	40	38	31	29	22	24

shown in Table 13 [7]-[10]. (*Sources: National Bureau of Statistics and local bureau of statistics*)

Primarily we select all of the 10 normalized indexes deviated from the original indexes and obtain the 10 principal components. Then we figure out the feature values of each normalized index to the 10 principal components through computation and the numbers in Table 14 show them clearly.

Table 14 Feature values of each normalized index to the principal components.

Principal Component	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	$\tilde{x}_7$	$\tilde{x}_8$	$\tilde{x}_9$	$\tilde{x}_{10}$
y_1	0.392416	0.084282	-0.06727	0.09987	-0.28189	0.081519	-0.13994	-0.0376	-0.35948	0.766864
y_2	-0.05294	0.66503	0.051769	0.1299	0.496243	-0.01219	-0.34922	0.338802	-0.2268	-0.02809
y_3	0.315398	0.17511	0.417432	0.664158	-0.17558	0.324296	0.098946	-0.15247	0.163802	-0.24216
y_4	0.380725	0.005971	-0.25961	-0.14805	-0.31597	-0.0264	-0.20459	-0.00241	-0.5204	-0.59371
y_5	0.128161	0.62569	0.1067	-0.3409	-0.17424	-0.36302	0.333918	-0.41397	0.139556	6.79E-07
y_6	0.325207	-0.1309	-0.46454	0.352311	0.412176	-0.38635	-0.14953	-0.39561	0.197891	-1.1E-06
y_7	0.293202	-0.20703	0.568556	-0.23893	-0.02374	-0.34884	-0.55047	0.075506	0.244643	7.24E-07
y_8	0.345296	0.17889	-0.39818	-0.14414	-0.18538	0.20288	-0.01918	0.480908	0.601894	2.84E-07
y_9	0.37035	-0.17199	0.173384	0.060711	0.237163	-0.34733	0.600566	0.471349	-0.19945	-2.3E-06
y_{10}	0.366386	-0.08286	0.109983	-0.43387	0.502184	0.568158	0.094276	-0.27116	-0.03388	5.02E-07

Afterwards, the contribution rates and the accumulation contribution rates of each principal component to the comprehensive appraise Z are shown in Table 16

Based on the results shown in Table 14 and Table 16, we finally select the first 4 principal components y_1, y_2, y_3 and y_4 instead of the original 10 principal components according to their contribution rates and accumulation contribution rates. The result of

Table 15 Ranking and appraisal of cities.

City	Rankings	Appraisal Index
Shanghai	1	3.56886
Beijing	2	2.505138
Shenzhen	3	2.22517
Chongqing	4	1.774264
Suzhou	5	1.074082
Guangzhou	6	0.896069
Tianjin	7	0.326718
Chengdu	8	-0.02949
Wuhan	9	-0.45961
Hangzhou	10	-0.52865
Nanjing	11	-0.59889
Qingdao	12	-0.824
Ningbo	13	-0.9253
Changsha	14	-1.01348
Zhengzhou	15	-1.21076
Xi'an	16	-1.42262
Dongguan	17	-1.47376
Shengyang	18	-1.79621
Kunming	19	-2.08752

measurement and ranking model is formulated as follows.

$$Z = 0.6252\tilde{y}_1 + 0.2099\tilde{y}_2 + 0.1009\tilde{y}_3 + 0.0267\tilde{y}_4 \tag{61}$$

Finally, we can easily rank the 19 cities in *Attachment 3* according to appraisal index Z and the ranking results are shown in Table 15.

Table 16 Contribution rates and accumulation contribution rates of each principal component to the comprehensive appraise index.

Principal Component	Contribution Rate (%)	Accumulation Contribution Rate (%)
y_1	62.51726	62.517263
y_2	20.98678	83.5040424
y_3	10.08538	93.5894205
y_4	2.671435	96.260856
y_5	1.50563	97.7664857
y_6	1.334661	99.1011462
y_7	0.468751	99.5698969
y_8	0.326133	99.8960302
y_9	0.10397	99.9999999
y_{10}	4.12E-11	100

IV. Conclusions

4.1 Conclusions of Question *1*

- From the results of *R-type Cluster Analysis*, the most similar factors are household consumption, FAL and GDP.
- We classify household consumption, FAL and GDP as a category and the other factors are each in their own category.
- From the results of *Principal Component Analysis*, the ranking order is GDP, household consumption, fixed-asset investment, resident population, quantity of surviving enterprises and CPI.

4.2 Conclusions of Question *2*

- The annual change of CPI is relatively small and irregular , and the predicted values is stable to a constant number. The other 5 factors increase linearly, similar to the change law of the obtained data.
- The implementation of existing economic policies has a relatively stable effect in promoting short-term and long-term economic development.
- In a short term, our economic policies can play a relatively stable role. However,

in order to improve economic competitiveness, we can increase GDP, household consumption and fixed-asset investment in economic policy transformation.

4.3 Conclusions of Question *3*

- We select 10 indexes to especially measure economic vitality of cities. The results show that these 10 indexes perform well.
- The reason why we choose the first 4 principal components is that their accumulation contribution rates reach 96.26% and go beyond 95%. Besides, each contribution rate of them is higher than the other.
- The appraisal index Z is the mathematical model that analyses and measures economic vitality.
- From the ranking and appraisal result, Shanghai ranks 1st, followed by Beijing and Shenzhen. Shanghai is approximately 1.06 higher than the second Beijing and the following cities differ from the previous one by 0.2-0.5. Cities that rank middle like Wuhan, Hangzhou, Nanjing and so on have little difference in terms of economic vitality. Cities like Xi'an, Dongguan, Shengyang and Kunming have a low ranking, which means they have relatively low economic vitality and need improvement.
- It is worth noting that Chongqing and Wuhan rank high in the results. That is probably because Chongqing has a large population and benefits from national policies and Wuhan is attracting the talents in recent years for better development.
- As a whole, the rankings and appraisal largely share similarity with our common sense and reality, which means our proposed model using PCA has satisfying performance.

4.4 The Development Proposal for Eastern region

Among the selected factors that may influence the economic vitality of Eastern region, GDP and household consumption has a greater effect on economic vitality while CPI and quantity of enterprises have relatively smaller influence on economic vitality. Therefore, we gain a preliminary understanding of different economic policy transition directions.

In order to improve cities with low economic vitality in Eastern region, such as Dongguan, we would better focus on GDP and total retail sales of goods while maintaining a certain total amount of imports and exports, instead of setting up a large number of enterprises to increase the number of enterprises.

V. Strength and Weakness

5.1 Strength

1. We build the relational model and the prediction model based on comparison with many existing models, methods and algorithms to finally choose the one with best performance.
2. We choose 6 factors to measure the economic vitality in Eastern region and 10 indexes to evaluate and rank the given 19 cities. The volume of factors and indexes is superb. These factors and indexes cover many aspects in economy and are comprehensive.

5.2 Weakness

1. The long-term prediction accuracy of policy transformation effects is relatively low.

VI. References

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VII. Appendix

Listing 1: The Matlab Source code of Algorithm

```
kk=2;[mdd,ndd]=size(dd);
while ~isempty(V)
[tmpd,j]=min(W(i,V));tmpj=V(j);
for k=2:ndd
[tmp1,jj]=min(dd(1,k)+W(dd(2,k),V));
tmp2=V(jj);tt(k-1,:)=[tmp1,tmp2,jj];
end
tmp=[tmpd,tmpj,j;tt];[tmp3,tmp4]=min(tmp(:,1));
if tmp3==tmpd, ss(1:2,kk)=[i;tmp(tmp4,2)];
else,tmp5=find(ss(:,tmp4)~=0);tmp6=length(tmp5);
if dd(2,tmp4)==ss(tmp6,tmp4)
ss(1:tmp6+1,kk)=[ss(tmp5,tmp4);tmp(tmp4,2)];
else, ss(1:3,kk)=[i;dd(2,tmp4);tmp(tmp4,2)];
end;end
dd=[dd,[tmp3;tmp(tmp4,2)]];V(tmp(tmp4,3))=[];
[mdd,ndd]=size(dd);kk=kk+1;
end; S=ss; D=dd(1,:);
```