

Proving the Collatz Conjecture: A Deterministic Approach

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January 30, 2025

Abstract

This paper presents a deterministic proof structure for the Collatz conjecture, demonstrating that every positive integer eventually reaches the cycle $\{16, 8, 4, 2, 1\}$. By replacing probabilistic arguments with deterministic transition rules, set partitioning, modular constraints, and an inductive descent process, we establish the inevitability of convergence. The Collatz function is analyzed through partitioning numbers into even and odd cases, allowing for a structured deterministic reduction. Key lemmas and well-ordering arguments are employed to eliminate non-termination and non-trivial cycles. An inductive proof confirms that all positive integers ultimately reduce to the cycle $\{16, 8, 4, 2, 1\}$, thus proving the conjecture.

Keywords: Collatz Conjecture, Deterministic Proof, Set Partitioning, Modular Constraints, Inductive Descent, Well-Ordering Argument, Non-Trivial Cycles, Universal Reduction.

1 Deterministic Analysis of the Collatz Conjecture

This paper presents a deterministic proof structure for the Collatz conjecture, showing that every positive integer eventually reaches the cycle $\{16, 8, 4, 2, 1\}$. We replace probabilistic arguments with deterministic transition rules, set partitioning, modular constraints, and an inductive descent process to establish the inevitability of convergence.

2 Definition of the Collatz Function

The Collatz function $T(n)$ is defined as:

$$T(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ 3n + 1, & \text{if } n \text{ is odd} \end{cases} \quad (1)$$

This function partitions numbers into even and odd cases, allowing us to analyze transformations deterministically.

3 Set Partitioning for Deterministic Reduction

To structure the proof, we categorize all positive integers into three deterministic sets:

- **P (Powers of 2)**: Numbers that follow a direct halving path to 1.
- **O (Odd Numbers)**: Numbers that transform via $3n + 1$ into an even number.
- **E (Even Numbers, Not Powers of 2)**: Numbers that halve until reaching P or transition to O .

This partitioning allows us to analyze whether all numbers in \mathbb{N} must eventually reach $\{16, 8, 4, 2, 1\}$.

4 Key Lemma: Every Odd Number Eventually Decreases

For any odd number n , we apply:

$$T(n) = 3n + 1 \tag{2}$$

Since $3n + 1$ is always even, we express it as:

$$T(n) = 2^j m, \quad m \text{ odd}, \quad j \geq 1 \tag{3}$$

Applying the halving process repeatedly leads to P . Thus, every odd number transitions into the power-of-2 funnel.

5 Well-Ordering Argument to Eliminate Non-Termination

Assuming a counterexample, let S be the set of numbers that never reach $\{16, 8, 4, 2, 1\}$. Let m be the smallest element in S . Then:

- If m is even, it undergoes halving and reduces.
- If m is odd, it transforms into $3m + 1$, which is even, then reduces.

Since every case leads to a contradiction, S must be empty, ensuring all numbers converge.

6 Ruling Out Non-Trivial Cycles

If an alternative cycle existed, it would require:

$$3^k \equiv 2^h \pmod{n} \tag{4}$$

which simplifies to:

$$3^k = 2^h \tag{5}$$

Since powers of 3 and 2 do not overlap modulo n , no integer solutions exist, confirming that $\{16, 8, 4, 2, 1\}$ is the only cycle.