Chapter 9 Question 5

- idea:

$$-\lambda(t) = e^{\beta_0 + \beta_1 x(t) + \beta_2 x(t)^2 + \beta_3 y(t) + \beta_4 y(t)^2}$$

- note: assume x position and y position Firing are uncorrelated w/ each other

- place cell version:

$$-\lambda(t) = \alpha e^{-\frac{(x(t)-\mu_x)^2}{\sigma_x^2} - \frac{(y(t)-\mu_Y)^2}{\sigma_Y^2}}$$

- $\alpha = e^{\beta_0 - \frac{\beta_1^2}{4\beta_2} - \frac{\beta_3^2}{4\beta_4}}$
- $\mu_x = \frac{-\beta_1}{2\beta_2}$ - $\mu_Y = \frac{-\beta_3}{2\beta_4}$
- $\sigma_X^2 = \frac{-1}{2\beta_2}$ - $\sigma_Y^2 = \frac{-1}{2\beta_4}$

- proof

$$\lambda(t) = \alpha e^{-\frac{(x(t)-\mu_x)^2}{\sigma_x^2} - \frac{(y(t)-\mu_Y)^2}{\sigma_Y^2}}$$

note: see Question 2 for step-by-step

$$= \alpha e^{\beta_2\left[x(t)^2 + \frac{\beta_1}{\beta_2}x(t) + \frac{\beta_1^2}{4\beta_2^2}\right] + \beta_4\left[y(t)^2 + \frac{\beta_3}{\beta_4}y(t) + \frac{\beta_3^2}{4\beta_2^2}\right]}$$

$$= \alpha e^{\beta_1 x(t) + \beta_2 x(t)^2 + \beta_3 y(t) + \beta_4 y(t)^2 + C}$$

$$= e^{\beta_0 - C}\, e^{\beta_1 x(t) + \beta_2 x(t)^2 + \beta_3 y(t) + \beta_4 y(t)^2 + C}$$

$$= e^{\beta_0 + \beta_1 x(t) + \beta_2 x(t)^2 + \beta_3 y(t) + \beta_4 y(t)^2}$$

$$= \lambda(t)$$

- note: $C = \frac{\beta_1^2}{4\beta_2} + \frac{\beta_3^2}{4\beta_4}$