Aptitude

1. Numbers

- **Quadratic Identities:**
  - `(a + b)² = a² + b² + 2ab`
  - `(a − b)² = a² + b² − 2ab`
- **Divisibility Rules:**
  - Sum of digits divisible by 3 ⇒ Number divisible by 3
  - Last digit 0 or 5 ⇒ Number divisible by 5
  - Sum of digits divisible by 9 ⇒ Number divisible by 9
  - Difference of sum of alternate digits divisible by 11 ⇒ Number divisible by 11
  - Divisibility of 17 -> - Take the **last digit**, double it, subtract from the rest of the number. Repeat if needed, if result is divisible by 7, original number is divisible by 7
  - Divisibility for 13-> - Take last digit, multiply by 9, subtract from rest of number. Repeat if needed. If result is divisible by 13, original number is divisible by 13

#PROBLEMS:

1. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?
   Let number = `10x + y`
   Reverse = `10y + x`
   Difference: `(10x + y) − (10y + x) = 36` ⇒ `9(x − y) = 36` ⇒ `x − y = 4`
   Difference between digits = |x - y| = 4
   Answer: `4`
2. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is:
   Let number = `10x + y`, Product: `x × y = 8`,

After adding 18: `10x + y + 18 = 10y + x` ⇒ `9x − 9y = −18` ⇒ `x − y = −2` ⇒ `y − x = 2`

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Also, `x × y = 8`, try possible integers:
If `x = 2, y = 4` satisfies both:
Number = 24
Answer: 24
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3. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number?

   Let the ten's digit be x and unit's digit be y.
   Then, $x + y = 15$ and $x - y = 3$ or $y - x = 3$.
   Solving $x + y = 15$ and $x - y = 3$, we get: $x = 9$, $y = 6$.
   Solving $x + y = 15$ and $y - x = 3$, we get: $x = 6$, $y = 9$.
   So, the number is either 96 or 69.
   Hence, the number cannot be determined.
4. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is:
   Let the numbers be a, b and c.
   Then, _a^2 + b^2 + c^2 = 138 and (ab + bc + ca) = 131.
   (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 x 131 = 400.
   ⇒ (a + b + c) = 400 = 20.
5. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by:

   Let the ten's digit be x and unit's digit be y.
   Let the ten's digit be x and unit's digit be y.

   Then, number = $10x + y$.

   Number obtained by interchanging the digits = $10y + x$.
   $(10y + x) = 11(x + y)$, which is divisible by 11.
6. The product of two numbers is 120 and the sum of their squares is 289. The sum of the number is:

Let the numbers be x and y.

Then, x__y = 120 and x_2 + _y_2 = 289.

(_x + y)^2 = x^2 + _y^2 + 2x__y = 289 + (2 x 120) = 529

x + y = 529 = 23.

LCM, HCF:

1. *H.C.F. and L.C.M. of Fractions:*

$$1.\ \text{H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$

$$2.\ \text{L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$

2. *Product of two numbers = Product of their H.C.F. and L.C.M.*
3. *Co-primes:* Two numbers are said to be co-primes if their H.C.F. is 1.
   Common Aptitude Patterns:
   Example 1:
   Find greatest number that divides 105 and 165 exactly → Find HCF

Example 2:
Three bells ring at 4, 6, 8 seconds — after how many seconds will they ring together? → Find LCM(4, 6, 8) = 24 sec

Example 3:
LCM of two numbers is 60, HCF is 5, one number is 15 — Find the other number:
HCF × LCM = Product of numbers
5 × 60 = 15 × X ⇒ X = 20

#PROBLEMS :

1. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.
   Required number = H.C.F. of (91 - 43), (183 - 91) and (183 - 43)
   = H.C.F. of 48, 92 and 140 = 4.

2. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?
L.C.M. of 2, 4, 6, 8, 10, 12 is 120.
So, the bells will toll together after every 120 seconds(2 minutes).

In 30 minutes, they will toll together	30	+ 1 = 16 times.

3. The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:

   Greatest number of 4-digits is 9999.
   L.C.M. of 15, 25, 40 and 75 is 600.
   On dividing 9999 by 600, the remainder is 399.
   Required number (9999 - 399) = 9600.
4. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

   Let the numbers be 37a and 37b.
   Then, 37a x 37b = 4107
   ab = 3.
   Now, co-primes with product 3 are (1, 3).
   So, the required numbers are (37 x 1, 37 x 3) *i.e.,* (37, 111).
   Greater number = 111.
5. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:
   L.C.M. of 5, 6, 4 and 3 = 60.

   On dividing 2497 by 60, the remainder is 37.

   Number to be added = (60 - 37) = 23.
6. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they again at the starting point ?
   L.C.M. of 252, 308 and 198 = 2772.

So, A, B and C will again meet at the starting point in 2772 sec. *i.e.,* 46 min. 12 sec.

7. The H.C.F. of $\frac{9}{10}$, $\frac{12}{25}$, $\frac{18}{35}$ and $\frac{21}{40}$ is:

$$\text{Required H.C.F.} = \frac{\text{H.C.F. of } 9, 12, 18, 21}{\text{L.C.M. of } 10, 25, 35, 40} = \frac{3}{1400}$$