Aptitude

1. Numbers

- **Quadratic Identities:**
    - `(a + b)² = a² + b² + 2ab`
    - `(a − b)² = a² + b² − 2ab`
- **Divisibility Rules:**
    - Sum of digits divisible by 3 ⇒ Number divisible by 3
    - Last digit 0 or 5 ⇒ Number divisible by 5
    - Sum of digits divisible by 9 ⇒ Number divisible by 9
    - Difference of sum of alternate digits divisible by 11 ⇒ Number divisible by 11
    - Divisibility of 17 -> - Take the **last digit**, double it, subtract from the rest of the number. Repeat if needed, if result is divisible by 7, original number is divisible by 7
    - Divisibility for 13-> - Take last digit, multiply by 9, subtract from rest of number. Repeat if needed. If result is divisible by 13, original number is divisible by 13

`#PROBLEMS` :

1. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?
   Let number = `10x + y`
   Reverse = `10y + x`
   Difference: `(10x + y) − (10y + x) = 36` ⇒ `9(x − y) = 36` ⇒ `x − y = 4`
   Difference between digits = |x - y| = 4
   Answer: `4`

2. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is:
   Let number = `10x + y` , Product: `x × y = 8` ,

After adding 18: `10x + y + 18 = 10y + x` ⇒ `9x - 9y = -18` ⇒ `x - y = -2` ⇒ `y - x = 2`

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Also, `x × y = 8`, try possible integers:
If `x = 2, y = 4` satisfies both:
Number = 24
Answer: 24
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3. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number?

   Let the ten's digit be x and unit's digit be y.
   Then, $x + y = 15$ and $x - y = 3$ or $y - x = 3$.
   Solving $x + y = 15$ and $x - y = 3$, we get: $x = 9$, $y = 6$.
   Solving $x + y = 15$ and $y - x = 3$, we get: $x = 6$, $y = 9$.
   So, the number is either 96 or 69.
   Hence, the number cannot be determined.

4. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is:
   Let the numbers be a, b and c.
   Then, $\_a^2 + b^2 + c^2 = 138$ and $(ab + bc + ca) = 131$.
   $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 \times 131 = 400$.
   ⇒ $(a + b + c) = \sqrt{400} = 20$.

5. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by:

   Let the ten's digit be x and unit's digit be y.
   Let the ten's digit be x and unit's digit be y.

   Then, number = $10x + y$.

   Number obtained by interchanging the digits = $10y + x$.
   $(10y + x) = 11(x + y)$, which is divisible by 11.

6. The product of two numbers is 120 and the sum of their squares is 289. The sum of the number is:

Let the numbers be *x* and *y*.

Then, $x \cdot y = 120$ and $x^2 + y^2 = 289$.

$(x + y)^2 = x^2 + y^2 + 2x \cdot y = 289 + (2 \times 120) = 529$

$x + y = \sqrt{529} = 23$.

7. find the unit number of (2467)^153 * (341)^72

Focus Only on the Last Digit of Each Base:

- $2467^{153} \rightarrow$ Last digit is **7**, so only 7^{153} matters.
- $341^{72} \rightarrow$ Last digit is **1**, so only 1^{72} matters.
- $(7^{153}) \times (1^{72})->$ only 7^{153} matters
- Let's compute the first few:

Power	7^n	Last Digit
7^1	7	7
7^2	49	9
7^3	343	3
7^4	2401	1

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Pattern repeats every 4 terms: 7, 9, 3, 1
153mod4=1
If remainder is 1 → Unit digit is same as 7^1 → 7
ANS: 7
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8. find the number of zeros at the end of the product 5^5*10^10*15^15*...125^125

LCM, HCF:

1. *H.C.F. and L.C.M. of Fractions:*

1. H.C.F. $= \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$

2. L.C.M. $= \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

2. *Product of two numbers = Product of their H.C.F. and L.C.M.*
3. *Co-primes:* Two numbers are said to be co-primes if their H.C.F. is 1.
   Common Aptitude Patterns:
   Example 1:
   Find greatest number that divides 105 and 165 exactly → Find HCF

Example 2:
Three bells ring at 4, 6, 8 seconds — after how many seconds will they ring together? → Find LCM(4, 6, 8) = 24 sec

Example 3:
LCM of two numbers is 60, HCF is 5, one number is 15 — Find the other number:
HCF × LCM = Product of numbers
5 × 60 = 15 × X ⇒ X = 20

#PROBLEMS :

1. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.
   Required number = H.C.F. of (91 - 43), (183 - 91) and (183 - 43)
   = H.C.F. of 48, 92 and 140 = 4.
2. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?
   L.C.M. of 2, 4, 6, 8, 10, 12 is 120.
   So, the bells will toll together after every 120 seconds(2 minutes).

In 30 minutes, they will toll together	30	+ 1 = 16 times.

3. The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:

   Greatest number of 4-digits is 9999.
   L.C.M. of 15, 25, 40 and 75 is 600.
   On dividing 9999 by 600, the remainder is 399.
   Required number (9999 - 399) = 9600.
4. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

Let the numbers be 37a and 37b.

Then, 37a x 37b = 4107

ab = 3.

Now, co-primes with product 3 are (1, 3).

So, the required numbers are (37 x 1, 37 x 3) *i.e.,* (37, 111).

Greater number = 111.

5. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:

L.C.M. of 5, 6, 4 and 3 = 60.

On dividing 2497 by 60, the remainder is 37.

 Number to be added = (60 - 37) = 23.

6. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they again at the starting point ?

L.C.M. of 252, 308 and 198 = 2772.

So, A, B and C will again meet at the starting point in 2772 sec. *i.e.,* 46 min. 12 sec.

7. The H.C.F. of $\dfrac{9}{10}$, $\dfrac{12}{25}$, $\dfrac{18}{35}$ and $\dfrac{21}{40}$ is:

Required H.C.F. = $\dfrac{\text{H.C.F. of 9, 12, 18, 21}}{\text{L.C.M. of 10, 25, 35, 40}}$ = $\dfrac{3}{1400}$

Percentage:

Percentage Increase/Decrease:

If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is:

$$\left[\frac{R}{(100+R)} \times 100 \right]\%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

$$\left[\frac{R}{(100-R)} \times 100 \right]\%$$

Results on Population:

Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

$$1. \text{ Population after } n \text{ years} = P\left(1 + \frac{R}{100}\right)^n$$

$$2. \text{ Population } n \text{ years ago} = \frac{P}{\left(1 + \frac{R}{100}\right)^n}$$

If it is decrease instead of increase in rate, then we subtract the R/100 from 1

Fraction Conversions

%	Fraction
10%	1/10
12.5%	1/8
20%	1/5
25%	1/4

%	Fraction
33.33%	1/3
50%	1/2
66.66%	2/3
75%	3/4
80%	4/5
90%	9/10

Successive Increase/Decrease:

If a price increases by 20% and then decreases by 10%:

Net % Change=(20−10)+(20×(−10))/100=10−2=8% increase

#PROBLEMS :

1. A fruit seller had some apples. He sells 40% apples and still has 420 apples. Originally, he had:

   Suppose originally he had x apples.
   Then, (100 - 40)% of x = 420.
   60/100 $x = 420$
   $x= (420$ 100 $)/60$ =700

2. In a certain school, 20% of students are below 8 years of age. The number of students above 8 years of age is $\frac{2}{3}$ of the number of students of 8 years of age which is 48. What is the total number of students in the school?

   80% of x = 48 +2/3 of 48
   x = 100.

3. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio of A : B.
   5A + 4B = 2/3(6A+8B).
   15A + 12B = 12A + 16B.
   3A = 4B.
   A/B = 4/3.
   A:B = 4:3.

4. A student multiplied a number by 3/5 instead of 5/3. What is the percentage error in the calculation?

   error=5/3 x- 3/5 x = 16/15 x
   error% = (16x/15 *3/5x* 100)% = 64%

5. Rajeev buys good worth Rs. 6650. He gets a rebate of 6% on it. After getting the rebate, he pays sales tax @ 10%. Find the amount he will have to pay for the goods.

   Rebate = 6% of Rs. 6650 = Rs.(6 x 6650)/100 = 399
   Sales tax = 10% of Rs. (6650 - 399) = Rs.(10/100* 6251) =Rs. 625.10
   Final amount = Rs. (6251 + 625.10) = Rs. 6876.10

6. The population of a town increased from 1,75,000 to 2,62,500 in a decade. The average percent increase of population per year is:

   Increase in 10 years = (262500 - 175000) = 87500.
   Increase% =((87500/175000) x 100))% = 50%.
   Required average = 50/10 = 5%

7. An agent gets a commission of 2.5% on the sales of cloth. If on a certain day, he gets Rs. 12.50 as commission, the cloth sold through him on that day is worth

   2.5% of total sales of cloth = Rs.12.50
   total sales of cloth = (Rs.12.50×100)/2.5 = Rs. 500

8. The price of shirt is increased by 15% and then reduced by 15%. The final price of the shirt is:

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   Let's say the original price of the shirt is x.
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   After increasing by 15%, the price becomes x + 15% of x = x+0.15x = 1.15x
   Then after reducing by 15%, the price becomes
   1.15x−15% of 1.15x = 1.15x−0.15×1.15x
   = 1.15x−0.1725 = 0.9775
   Therefore, the final price of the shirt is 97.75% of the original price.

RATIO & PROPORTION:

1. Mean proportional between a and b is Sqrt(*ab*.)
2. If a/b = c/d then, (a+b)/(a-b) =(c+d)/(c-d)
3. **Finding New Ratio after Increase/Decrease**

   Example: Income of A : B = 5 : 6
   A's income increases by 20%, B's by 25%
   New ratio:
   A → 5 × 1.2(got from 100+20%) = 6
   B → 6 × 1.25(got from 100+25%) = 7.5
   So, New Ratio = 6 : 7.5 = 12 : 15 = **4 : 5**
4. **Inverse Ratio for Work/Time Problems**

   If time taken by A : B = 2 : 3
   Their work rates are inverse → Work done ratio = 3 : 2
5. **Shortcut for Replacement Type Problems**

   Mixture problems like replacing milk with water:
   Remaining Quantity Formula:
   `Final Quantity = Initial Quantity × ( (Total - Replaced)/Total )^n`
   Where **n** = number of times replaced.

#PROBLEMS :

1. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is:

Let the third number be *x*.
Then, first number = 120% of *x* = 120x/100=6x/5
Second number = 150% of *x* = 15-x/100=3x/2
Ratio of first two numbers =12 x : 15 x = 4 : 5

2. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

   Find New Quantities

Math = 5 × 1.4 = 7
Physics = 7 × 1.5 = 10.5
Biology = 8 × 1.75 = 14
New Ratio = **7 : 10.5 : 14**
To eliminate the decimal, multiply all by **2**:

7 × 2 : 10.5 × 2 : 14 × 2 = 14 : 21 : 28 = 2 : 3 : 4.

3. In a mixture 60 litres, the ratio of milk and water 2 : 1. If this ratio is to be 1 : 2, then the quantity of water to be further added is:

Given:

- Total Mixture = 60 litres
- **Milk : Water = 2 : 1**
  So, Total parts = 2 + 1 = 3 parts
  Milk = (2/3)×60=40 litres
  Water = (1/3)×60=20 litres

After adding water, new ratio = **1 : 2**

New Water Quantity = 20+x litres, Milk still 40(not added,removed)
Milk/Water =1/2
40/(20+x) = 1/2
80=20+x
x=80−20=60 litres

4. The sum of three numbers is 98. If the ratio of the first to second is 2 :3 and that of the second to the third is 5 : 8, then the second number is:

To combine these, express both with a common "Second" term.
- From First : Second = 2 : 3
  Multiply both by 5 → **10 : 15**
- From Second : Third = 5 : 8
  Multiply both by 3 → **15 : 24**
  So, combined ratio:
  First : Second : Third = 10 : 15 : 24
  Express with Total Sum
  Total parts = 10+15+24=49 parts

Total sum = 98

Value of one part:

One part=98/49=2

Second number = 15×2=30

ANS: 30

5. If Rs. 782 be divided into three parts, proportional to $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$, then the first part is:

Denominators are: 2, 3, 4

LCM of 2, 3, 4 = 12

Multiply each term by LCM (12):

1/2 *12 = 6 (2/3)* 12 =8 3/4 *12 = 9*

Final ratio: 6 : 8 : 9

1st part = 782 6/23 = 204

ANS: 204

6. The fourth proportional to 5, 8, 15 is:

Let the fourth proportional to 5, 8, 15 be *x*.
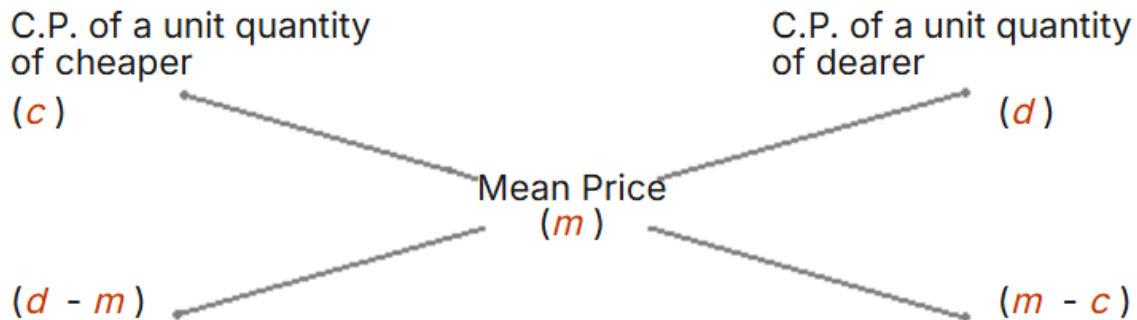
Then, 5 : 8 : 15 : *x*

5x =(8 * 15)

x =24

Alligation or Mixture:

1. Rule of Alligation:

If two ingredients are mixed, then:

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \left(\frac{\text{C.P. of dearer - Mean Price}}{\text{Mean price - C.P. of cheaper}}\right)$$

We present as under:

C.P. of a unit quantity
of cheaper
(c)

C.P. of a unit quantity
of dearer
(d)

Mean Price
(m)

(d - m)

(m - c)

∴ (Cheaper quantity) : (Dearer quantity) = (d - m) : (m - c).

2. Suppose a container contains x of liquid from which y units are taken out and replaced by water.

After n operations, the quantity of pure liquid $=(x(1-(y/x)^n))$ units

#PROBLEMS :

1. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

Suppose the vessel initially contains 8 litres of liquid.
Let x litres of this liquid be replaced with water.
Quantity of water in new mixture =(3-3x/8(water to be removed) +x(water to be added))
Quantity of syrup in new mixture =(5-5x/8(syrup to be removed))

Half water, half syrup = 1:1

(3-3x/8 +x) = (5-5x/8)
5x + 24 = 40 - 5x
10x= 16
x = 8/5
Part of mixture replaced = 8/5 * 1/8(Cause total parts =8) = 1/5