

Aptitude

1. Numbers

- **Quadratic Identities:**

- $(a + b)^2 = a^2 + b^2 + 2ab$

- $(a - b)^2 = a^2 + b^2 - 2ab$

- **Divisibility Rules:**

- Sum of digits divisible by 3 \Rightarrow Number divisible by 3
 - Last digit 0 or 5 \Rightarrow Number divisible by 5
 - Sum of digits divisible by 9 \Rightarrow Number divisible by 9
 - Difference of sum of alternate digits divisible by 11 \Rightarrow Number divisible by 11
 - Divisibility of 17 \rightarrow - Take the **last digit**, double it, subtract from the rest of the number. Repeat if needed, if result is divisible by 7, original number is divisible by 7
 - Divisibility for 13 \rightarrow - Take last digit, multiply by 9, subtract from rest of number. Repeat if needed. If result is divisible by 13, original number is divisible by 13

#PROBLEMS :

1. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 36. What is the difference between the two digits of that number?

Let number = $10x + y$

Reverse = $10y + x$

Difference: $(10x + y) - (10y + x) = 36 \Rightarrow 9(x - y) = 36 \Rightarrow x - y = 4$

Difference between digits = $|x - y| = 4$

Answer: 4

2. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is:

Let number = $10x + y$, Product: $x \times y = 8$,

After adding 18: $10x + y + 18 = 10y + x \Rightarrow 9x - 9y = -18 \Rightarrow x - y = -2 \Rightarrow y - x = 2$

Also, ' $x \times y = 8$ ', try possible integers:

If ' $x = 2, y = 4$ ' satisfies both:

Number = 24

Answer: 24

3. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number?

Let the ten's digit be x and unit's digit be y .

Then, $x + y = 15$ and $x - y = 3$ or $y - x = 3$.

Solving $x + y = 15$ and $x - y = 3$, we get: $x = 9, y = 6$.

Solving $x + y = 15$ and $y - x = 3$, we get: $x = 6, y = 9$.

So, the number is either 96 or 69.

Hence, the number cannot be determined.

4. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is:

Let the numbers be a, b and c .

Then, $a^2 + b^2 + c^2 = 138$ and $(ab + bc + ca) = 131$.

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 \times 131 = 400$.

$\Rightarrow (a + b + c) = 400 = 20$.

5. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be divisible by:

Let the ten's digit be x and unit's digit be y .

Let the ten's digit be x and unit's digit be y .

Then, number = $10x + y$.

Number obtained by interchanging the digits = $10y + x$.

$(10y + x) = 11(x + y)$, which is divisible by 11.

6. The product of two numbers is 120 and the sum of their squares is 289. The sum of the number is:

Let the numbers be x and y .

Then, $x_y = 120$ and $x_2 + y_2 = 289$.

$$(_x + y)^2 = x^2 + y^2 + 2x_y = 289 + (2 \times 120) = 529$$

$$x + y = 529 = 23.$$

7. find the unit number of $(2467)^{153} \times (341)^{72}$

Focus Only on the Last Digit of Each Base:

- $2467^{153} \rightarrow$ Last digit is **7**, so only 7^{153} matters.
- $341^{72} \rightarrow$ Last digit is **1**, so only 1^{72} matters.
- $(7^{153}) \times (1^{72}) \rightarrow$ only 7^{153} matters
- Let's compute the first few:

Power	7^n	Last Digit
7^1	7	7
7^2	49	9
7^3	343	3
7^4	2401	1

Pattern repeats every 4 terms: 7, 9, 3, 1

$$153 \bmod 4 = 1$$

If remainder is 1 \rightarrow Unit digit is same as $7^1 \rightarrow 7$

ANS: 7

8. find the number of zeros at the end of the product $5^5 10^{10} 15^{15} \dots 125^{125}$

LCM, HCF:

1. *H.C.F. and L.C.M. of Fractions:*

$$1. \text{ H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$

$$2. \text{ L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$

2. *Product of two numbers = Product of their H.C.F. and L.C.M.*
3. **Co-primes:** Two numbers are said to be co-primes if their H.C.F. is 1.

Common Aptitude Patterns:

Example 1:

Find greatest number that divides 105 and 165 exactly → Find HCF

Example 2:

Three bells ring at 4, 6, 8 seconds — after how many seconds will they ring together? → Find LCM(4, 6, 8) = 24 sec

Example 3:

LCM of two numbers is 60, HCF is 5, one number is 15 — Find the other number:

HCF × LCM = Product of numbers

$$5 \times 60 = 15 \times X \Rightarrow X = 20$$

#PROBLEMS :

1. Find the **greatest number** that will divide 43, 91 and 183 so as to leave the **same remainder** in each case.

Required number = H.C.F. of (91 - 43), (183 - 91) and (183 - 43)

= H.C.F. of 48, 92 and 140 = 4.

2. Six bells commence **tolling together** and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

L.C.M. of 2, 4, 6, 8, 10, 12 is 120.

So, the bells will toll together after every 120 seconds(2 minutes).

In 30 minutes, they will toll together	30	+ 1 = 16 times.
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3. The greatest number of four digits which is divisible by 15, 25, 40 and 75 is:

Greatest number of 4-digits is 9999.

L.C.M. of 15, 25, 40 and 75 is 600.

On dividing 9999 by 600, the remainder is 399.

Required number (9999 - 399) = 9600.

4. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

Let the numbers be $37a$ and $37b$.

Then, $37a \times 37b = 4107$

$ab = 3$.

Now, co-primes with product 3 are (1, 3).

So, the required numbers are $(37 \times 1, 37 \times 3)$ *i.e.*, (37, 111).

Greater number = 111.

5. The **least number** which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:
L.C.M. of 5, 6, 4 and 3 = 60.

On dividing 2497 by 60, the remainder is 37.

Number to be added = $(60 - 37) = 23$.

6. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they again at the starting point ?
L.C.M. of 252, 308 and 198 = 2772.

So, A, B and C will again meet at the starting point in 2772 sec. *i.e.*, 46 min. 12 sec.

The H.C.F. of $\frac{9}{10}$, $\frac{12}{25}$, $\frac{18}{35}$ and $\frac{21}{40}$ is:

7.

$$\text{Required H.C.F.} = \frac{\text{H.C.F. of } 9, 12, 18, 21}{\text{L.C.M. of } 10, 25, 35, 40} = \frac{3}{1400}$$

Percentage:

Percentage Increase/Decrease:

If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is:

$$\left[\frac{R}{(100 + R)} \times 100 \right] \%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

$$\left[\frac{R}{(100 - R)} \times 100 \right] \%$$

Results on Population:

Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

$$1. \text{ Population after } n \text{ years} = P \left(1 + \frac{R}{100} \right)^n$$

$$2. \text{ Population } n \text{ years ago} = \frac{P}{\left(1 + \frac{R}{100} \right)^n}$$

If it is decrease instead of increase in rate, then we subtract the R/100 from 1

Fraction Conversions

%	Fraction
10%	1/10
12.5%	1/8
20%	1/5
25%	1/4

%	Fraction
33.33%	1/3
50%	1/2
66.66%	2/3
75%	3/4
80%	4/5
90%	9/10

Successive Increase/Decrease:

If a price increases by 20% and then decreases by 10%:

Net % Change = $(20 - 10) + (20 \times (-10)) / 100 = 10 - 2 = 8\%$ increase

#PROBLEMS :

1. A fruit seller had some apples. He sells 40% apples and still has 420 apples. Originally, he had:

Suppose originally he had x apples.

Then, $(100 - 40)\%$ of $x = 420$.

$$60/100 x = 420$$

$$x = (420 \times 100) / 60 = 700$$

2. In a certain school, 20% of students are below 8 years of age. The number of students above 8 years of age is $\frac{2}{3}$ of the number of students of 8 years of age which is 48. What is the total number of students in the school?

$$80\% \text{ of } x = 48 + \frac{2}{3} \text{ of } 48$$

$$x = 100.$$

3. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B. Find the ratio of A : B.

$$5A + 4B = \frac{2}{3}(6A + 8B).$$

$$15A + 12B = 12A + 16B.$$

$$3A = 4B.$$

$$A/B = 4/3.$$

$$A:B = 4:3.$$

4. A student multiplied a number by $\frac{3}{5}$ instead of $\frac{5}{3}$. What is the percentage error in the calculation?

$$\text{error} = \frac{5}{3}x - \frac{3}{5}x = \frac{16}{15}x$$

$$\text{error}\% = \left(\frac{16x}{15} \cdot \frac{3}{5} \cdot 100\right)\% = 64\%$$

5. Rajeev buys good worth Rs. 6650. He gets a rebate of 6% on it. After getting the rebate, he pays sales tax @ 10%. Find the amount he will have to pay for the goods.

$$\text{Rebate} = 6\% \text{ of Rs. } 6650 = \text{Rs.} (6 \times 6650)/100 = 399$$

$$\text{Sales tax} = 10\% \text{ of Rs. } (6650 - 399) = \text{Rs.} (10/100 \times 6251) = \text{Rs. } 625.10$$

$$\text{Final amount} = \text{Rs. } (6251 + 625.10) = \text{Rs. } 6876.10$$

6. The population of a town increased from 1,75,000 to 2,62,500 in a decade. The average percent increase of population per year is:

$$\text{Increase in 10 years} = (262500 - 175000) = 87500.$$

$$\text{Increase}\% = \left(\frac{87500}{175000} \times 100\right)\% = 50\%.$$

$$\text{Required average} = 50/10 = 5\%$$

7. An agent gets a commission of 2.5% on the sales of cloth. If on a certain day, he gets Rs. 12.50 as commission, the cloth sold through him on that day is worth

$$2.5\% \text{ of total sales of cloth} = \text{Rs. } 12.50$$

$$\text{total sales of cloth} = (\text{Rs. } 12.50 \times 100)/2.5 = \text{Rs. } 500$$

8. The price of shirt is increased by 15% and then reduced by 15%. The final price of the shirt is:

Let's say the original price of the shirt is x .

After increasing by 15%, the price becomes $x + 15\% \text{ of } x = x + 0.15x = 1.15x$

Then after reducing by 15%, the price becomes

$$1.15x - 15\% \text{ of } 1.15x = 1.15x - 0.15 \times 1.15x$$

$$= 1.15x - 0.1725 = 0.9775$$

Therefore, the final price of the shirt is 97.75% of the original price.