CS 315 Intermediate Algorithms
Winter 2023

Assignment 3

Due 11:59 PM, Sunday, February 5, 2023

1. Consider the graph below. You will be building a MST for this graph in two ways. When there is a tie on the edge weights, consider the edges or nodes in alphabetical order. For example, for an edge, this means (A,F) before (B,C), and (C,D) before (C,G). Show all intermediate steps, and the set of edges contained in the MST.

   (a) Prim's method.

   (b) Kruskal's method.

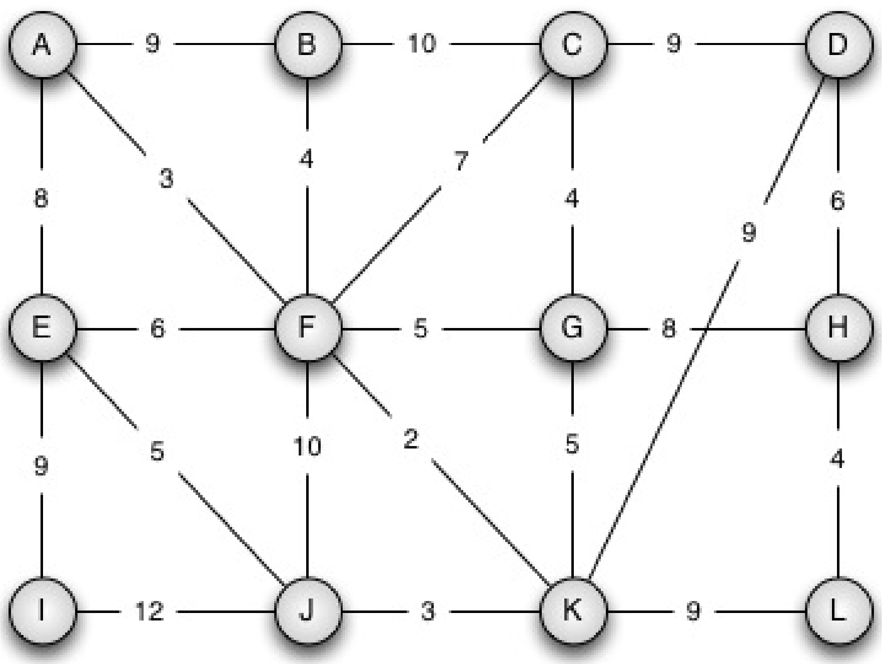

Figure 1: for question 1

[8 points]

2. We are given a weighted graph $G = (V, E)$ with weights given by W. The nodes represent cities and the edges are (positive) distances between the cities. We want to get a car that can travel from a start node s and reach all other cities. Gas can be purchased at any city, and the gas-tank capacity needed to travel between cities u and v is the distance $W[u, v]$. Give an algorithm by modifying Edge Relaxation to determine the *minimum* gas-tank capacity required of a car that can travel from s to any other city. For this problem, just show your modified version of the Edge Relaxation operation or function; do not provide anything else (e.g., pseudocodes of your entire algorithm). **[6 points]**

3. We are given a graph $G = (V, E)$ where V represents a set of locations and E represents a communications channel between two points. We are also given locations $s, t \in V$, and a reliability function $r : V \times V \to [0, 1]$. You need to give an efficient algorithm which will output the reliability of the most reliable path from s to t in G.

   For any points $u, v \in V$, $r(u, v)$ is the probability that the communication link (u, v) will not fail: $0 \leq r(u, v) \leq 1$. Note that if there is a path with two edges, for example, from u to v to w, then the reliability of that path is $r(u, v) \cdot r(v, w)$.

   For this problem, you modify EDGE RELAXATION. Just show your modified version of the EDGE RELAXATION operation or function; do not provide anything else (e.g., pseudocodes of your entire algorithm). **[6 points]**

4. Exercise 25.2-4 from the CLRS textbook.
   More specifically, show and explain that $O(n^2)$ space can be achieved without sacrificing correctness by dropping the superscripts in the Floyd-Warshall algorithm by computing the distance matrices $\mathcal{D}^{(k)}$ in place using a single matrix $\mathcal{D}$. **[5 points]**

Total: 25 points