

## Assignment 1

*Due 11:59 PM Friday January 20, 2023*

Problem 1 (5 points): For the following tree in Figure 1, write the adjacency list representation and the adjacency matrix representation, respectively.

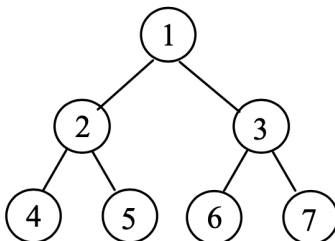


Figure 1: A tree

Problem 2 (10 points): For the following graph in Figure 2, demonstrate how the Breadth-First Search (BFS) and the Depth-First Search (DFS) work, respectively. Use the node “q” as the starting node. During the search, when you have multiple choices regarding which node to visit next (i.e., when enumerating the direct neighbors of a node as in BFS, or when going to another branch or “sibling” node of a common “parent” node as in DFS), please use the alphabetical order to choose the next node. Show the intermediate steps, and the final output sequence of the nodes.

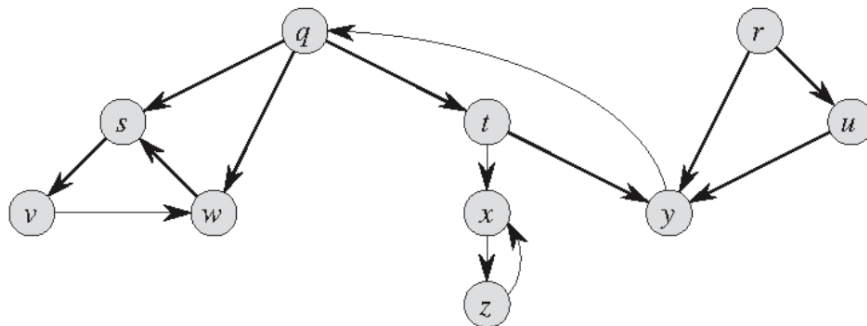


Figure 2: A directed graph

Problem 3 (5 points): Suppose you work for a lab which is studying butterflies. It has a sample of  $n$  butterflies,  $L_1, L_2, \dots, L_n$ . It is believed that all these butterflies belong to two different species. The researchers have made a series of  $r$  (note: the value of  $r$  can be different from that of  $n$ ) “determinations” in order to try to find out whether or not these butterflies indeed belong to two different species. A determination is of the form  $(L_i, L_j)$ , which indicates  $L_i$  and  $L_j$  belong to different species. Your job is to give an  $O(n + r)$  time algorithm to decide whether all the determinations are “consistent”. Please describe your algorithm. Note that being “consistent” means there is no contradiction across all the determinations. For example, “ $(L_1, L_2)$ ”, “ $(L_1, L_3)$ ”, and “ $(L_2, L_3)$ ” are not consistent.

Problem 4 (5 points): Exercise 3-16 in the DPV textbook.

In addition to the problem statement of 3-16 as in the textbook, we add the following clarification. A set of courses can be taken in one semester if and only if there is no direct and indirect “prerequisite” relationship among any of the courses in this set. “Direct” means a directed edge; “indirect” means a path (following the directions of the edges). Besides, a course can be taken if and only if all of its prerequisite courses were taken in a previous semester.