NOTE: This lecture differs significantly from the textbook.

The Uses of Encryption

Jun Li

lijun@cs.uoregon.edu

Learning Objectives

Learn:

- Public Key Encryption Characteristics
- Cryptographic hash functions
- Key exchange
- Digital signature
- Certificates

Key Proliferation Problem in a Symmetric Cryptosystem

- *n* users require *n*(n-1)/2* keys
  - to allow every pair of users to communicate secretly

- In asymmetric cryptosystem, *n* users require *n*2* keys
  - Every user can simply use the public key of the recipient to encrypt a message

Public Key Encryption Characteristics

(1) $P = D\ (k_{PRIV},\ E(k_{PUB},\ P))$

(2) $P = D\ (k_{PUB},\ E(k_{PRIV},\ P))$

Assume $(k_{PUB},\ k_{PRIV})$ belongs to Alice

anyone can know k_{PUB} and do $E(k_{PUB},\ *)$, but

only Alice should know k_{PRIV} and can do $E(k_{PRIV},\ *)$.

From (1): Only Alice can decrypt $E(k_{PUB},\ P)$ to get P.

From (2): Q: Is this a message from Alice?

A: Decrypt it with Alice's k_{PUB}.

Comparing Secret Key and Public Key Encryption

	Secret Key (Symmetric)	Public Key (Asymmetric)
Number of keys	**1**	**2**
Protection of key	**Must be kept secret**	**The private key must be secret; public key can go public**
Best uses	**Cryptographic workhorse**	**Key exchange; authentication**
Key distribution	**Out-of-band**	**Public key can be used**
Speed	**Fast (bit operations)**	**Typically 10,000 times slower (modular exponentiation using * /)**

Four Applications of Encryption

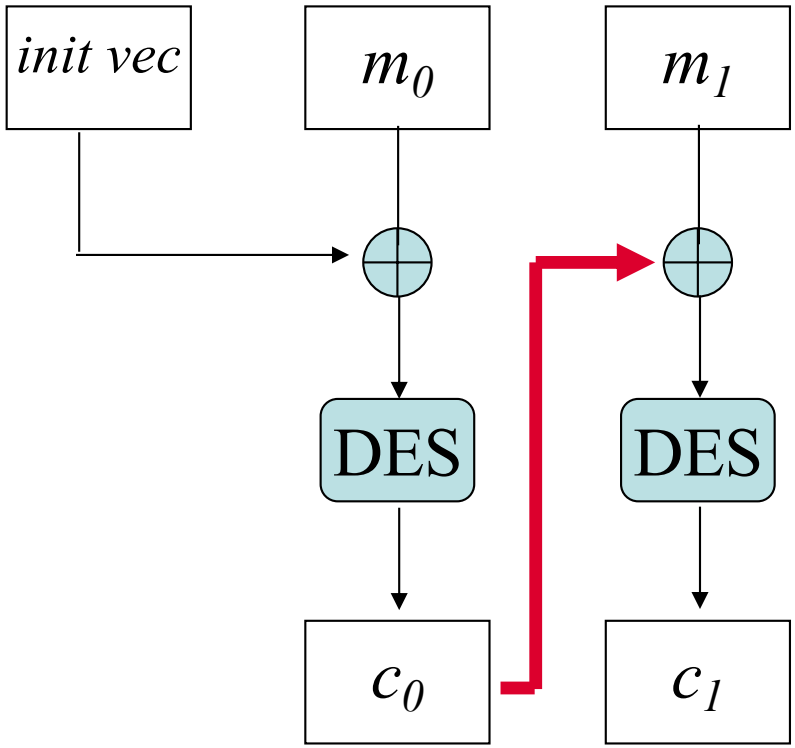
- Encryption is not just for secrecy
  - How about integrity?
- Four brilliant uses
  - Cryptographic hash functions
  - Key exchange
  - Digital signature
  - Certificates

Cryptographic Hash Functions

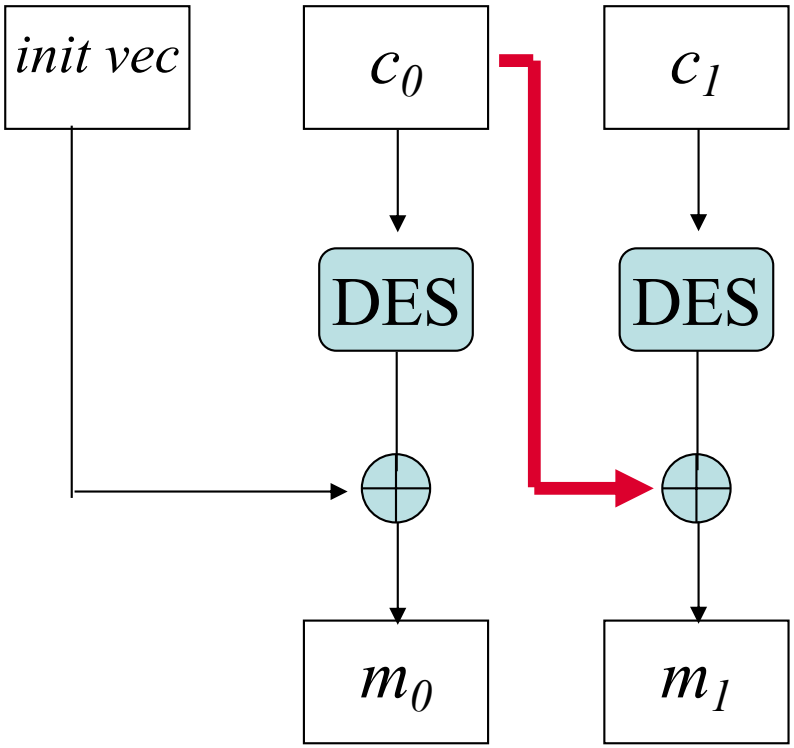
- Hash, checksum, message digest
  - All synonyms
- One-way functions
  - Much easier to compute than their inverses
  - Any change to even a single bit will alter the result
- Cryptographic functions (DES, AES) can make good one-way functions
  - As outsiders do not know the key

Cipher Block Chaining (CBC)

Encipherment



Decipherment



- Use the last cipher block (c_n) as hash
- Can be any block cipher

Popular Cryptographic Hash Functions

- **MD4, MD5** (message digest)
  - By Ron Rivest and RSA Laboratories
  - 128-bit digest
- **SHA/SHS** (Secure Hash Algorithm or Standard)
  - Digest size is between 128 and 512 bits, depending on the version of SHA

Key Exchange

- Problem: S($k_{\text{PRIV-S}}$, $k_{\text{PUB-S}}$), R($k_{\text{PRIV-R}}$, $k_{\text{PUB-R}}$)=> *K?*
- Method 1: S picks *K* and sends E($k_{\text{PRIV-S}}$, *K*) to R
  - Does it work?
- Method 2: E($k_{\text{PUB-R}}$, *K*) to R?
  - Does it work?
- Method 3: E($k_{\text{PUB-R}}$, E($k_{\text{PRIV-S}}$, *K*))

Key Exchange

- Problem: S, R, field size *n*, starting number g => *K?*
- S and R each think up a secret number, *s* and *r*
- S sends R g^s, R sends to S g^r
- S computes $(g^r)^s$, R computes $(g^s)^r$
- Secret $K = (g^r)^s = (g^s)^r$

Is This Method Safe?

- Attacks can intercept g^s, g^r
- If he also knows g, he can get K as:

  $s=\ln(g^s)/\ln(g)$ and $k = g^{rs}$

A Technical Detail: **mod *n***

- Discrete Logarithm Problem: Find a value of x such that

$$Y = g^x \bmod n$$

  for a given Y, g, and prime n.
- Difficulty increases exponentially as n increases
- So what S and R send to each other are $g^s \bmod n$ and $g^r \bmod n$, respectively.
- This is called **Diffie-Hellman key exchange protocol**

Digital Signature

- A **digital signature** is a construct that authenticates both the ***origin*** and ***contents*** of a message in a manner that is provable to a disinterested third party.

- Provides a service of nonrepudiation

Case Study 1: Is $\{m\}\ k$ a Signature?

- Alice sends Bob

  $$m\ \{m\}\ k$$

  k is the shared secret key between Alice and Bob;

  $\{m\}\ k$ is another form of $\mathbf{E}(m, k)$.
- Bob deciphers $\{m\}\ k$ and compares with m
  - Thus verifying that the message is from Alice
  - And the contents are not modified
- But Bob cannot prove to a third party that $m\ \{m\}\ k$ is not created by Bob himself!
  - So $\{m\}\ k$ is not a signature of m

Case Study 2: Is $\{m\}\ d_{Alice}$ a Signature?

- Alice sends Bob

  $$m\ \{m\}\ d_{Alice}$$

  d_{Alice} is the private key of Alice
- Bob deciphers $\{m\}\ d_{Alice}$ and compares with m
  - Thus verifying that the message is from Alice
  - And the contents are not modified
- A judge can verify $\{m\}\ d_{Alice}$ is indeed signed by Alice if:

  $$m = \{\{m\}\ d_{Alice}\}\ e_{Alice}$$

Public Key Signature

- Instead of using $\{\boldsymbol{m}\}\ \boldsymbol{d}_{\boldsymbol{Alice}}$, Alice actually signs the message as

  $$\{\boldsymbol{h}(\boldsymbol{m})\}\ d_{Alice}$$

  where $\boldsymbol{h}$ is a cryptographic hash function
- And sends Bob

  $$\boldsymbol{m}\ \{\boldsymbol{h}(\boldsymbol{m})\}\ d_{Alice}$$
- Q: how does Bob verifies the signature?

Certificates

- A **certificate** is a token that binds an identity to a cryptographic key

$$C_{Alice} = e_{Alice} \,||\, Alice \,||\, T \,||\, \underline{signed\ by\ Cathy}$$
$$\{h\{e_{Alice} \,||\, Alice \,||\, T\}\}\, d_{Cathy}$$

- A **certificate authority** (CA) issues certificates

Certificate Verification

- Suppose Bob knows Cathy's public key e_{Cathy}
- When Bob obtains C_{Alice},
  - Verifies C_{Alice}'s signature using e_{Cathy}
  - Then knows that Cathy is vouching that e_{Alice} is Alice's public key, issued at time T
  - If Bob trusts what Cathy believes
  - Then Bob knows e_{Alice} is Alice's public key

But, Bob Has to Know e_{Cathy}

- Two solutions
  - Chain of certificates
    - There is another certificate for e_{Cathy}
  - Merkle's Tree Authentication Scheme
    - Eliminates Cathy's signature

Chain of Certificates

- *X.509: Centralized trust model*
  - Tree-like CA hierarchy employed
  - Every node has a local CA
  - A local CA has its CA, the parent
  - The parent CA has its parent
  - And there is a root CA
  - Together, a tree of CAs!
- *PGP: Decentralized trust model*
  - Anyone can sign a certificate to form a web of trust
  - Currently also supports X.509

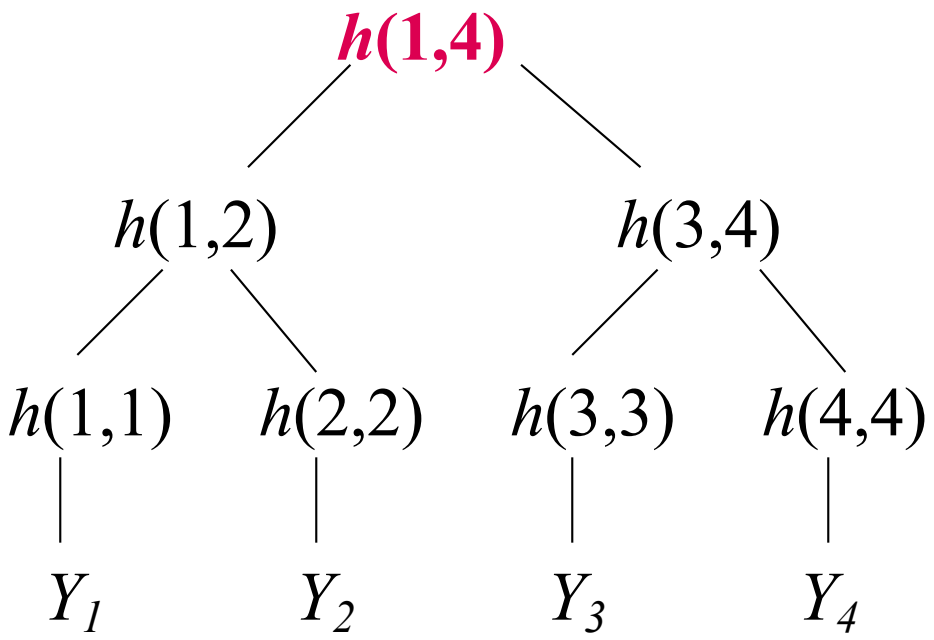
Merkle's Tree Authentication Scheme

- All <id, public key> pairs are stored in a file
- A cryptographic hash function creates a digest of the file
  - The digest is known to the public
- If any pair is changed, it will be detected
  - Since the digest will be different

Digest Algorithm

- A tree-based algorithm

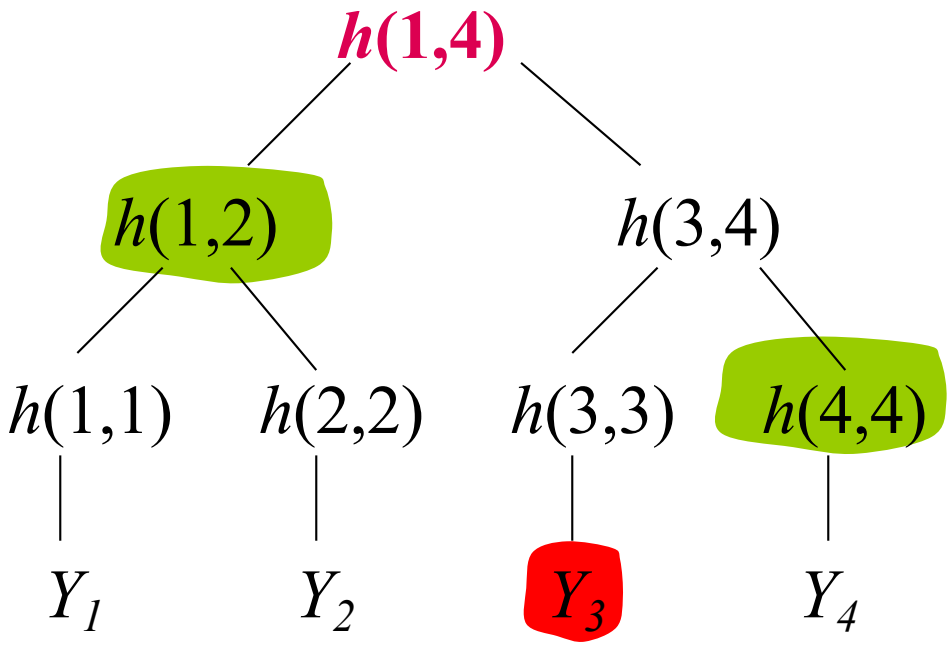
Y_1	Cathy	01389234789357
Y_2	Bob	89230378597823
Y_3	Alice	72384927894027
Y_4	David	32748902378240



Public Key Verification

- How can Bob verify whether or not Alice's public key is 72384927894027.
- Bob will re-compute the digest, and compare that with the publicly known value of the digest
  - If Alice's public key is not 72384927894027, a discrepancy will be detected

Y_1	Cathy	013892347893 57
Y_2	Bob	892303785978 23
Y_3	Alice	723849278940 27
Y_4	David	327489023782 40



Alice, 72384927894027

Authentication Path

- Bob knows Y_3
- Bob needs to know $h(4,4)$ and $h(1,2)$
- Y_3, $h(4,4)$ and $h(1,2)$ is the **authentication path** for Alice's public key
  - They can be put together and used for certifying Alice's public key
- *This is a certificate without a signature!*