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# CS 471/571 (Fall 2023): Introduction to Artificial Intelligence

## Lecture 23: Bayes Nets – Sampling

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Source: <http://ai.berkeley.edu/home.html>



# Reminders

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- Written assignment 4:
  - Deadline: November 29<sup>th</sup>, 2023

# Bayes' Nets

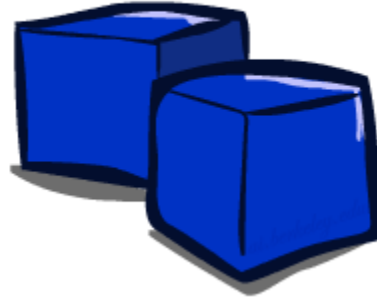
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- ✓ Representation
- ✓ Conditional Independences
- Probabilistic Inference
  - ✓ Enumeration (exact, exponential complexity)
  - ✓ Variable elimination (exact, worst-case exponential complexity, often better)
  - ✓ Inference is NP-complete
    - Sampling (approximate)
- Learning Bayes' Nets from Data



# Approximate Inference: Sampling

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# Sampling in Bayes' Nets

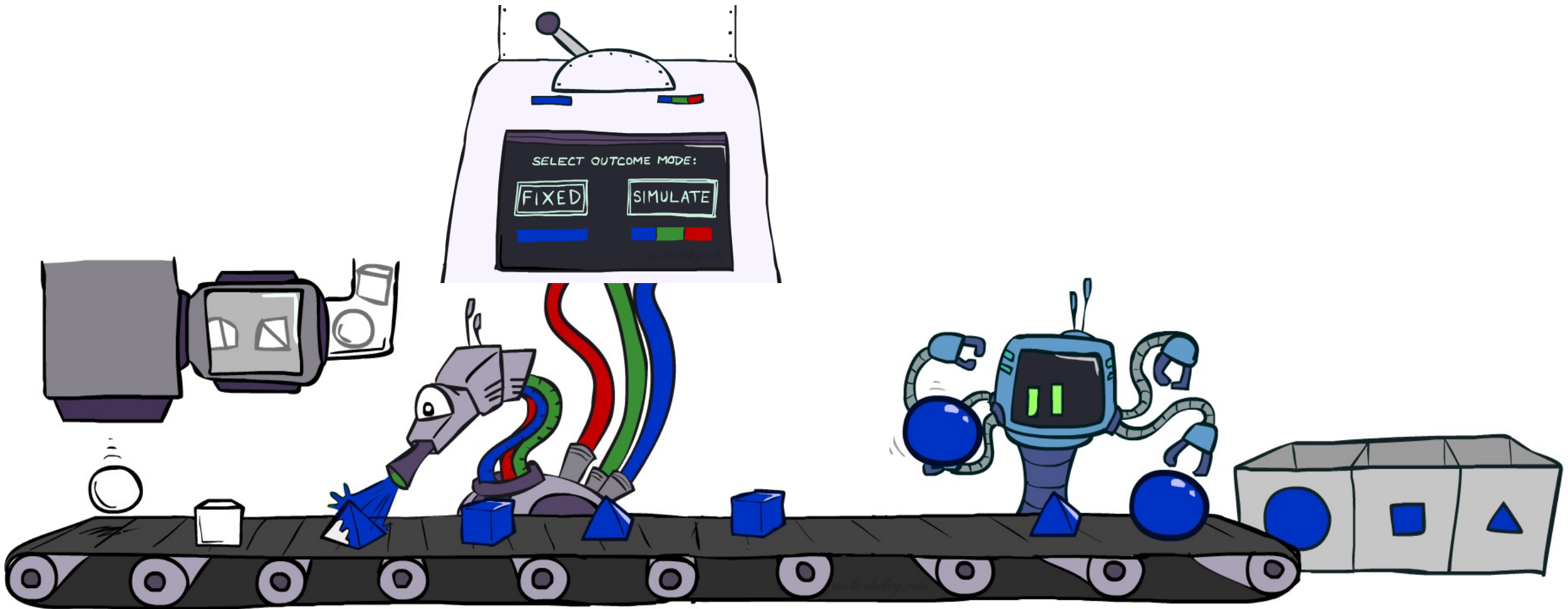
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- Prior Sampling
- Rejection Sampling
- Likelihood Weighting
- Gibbs Sampling

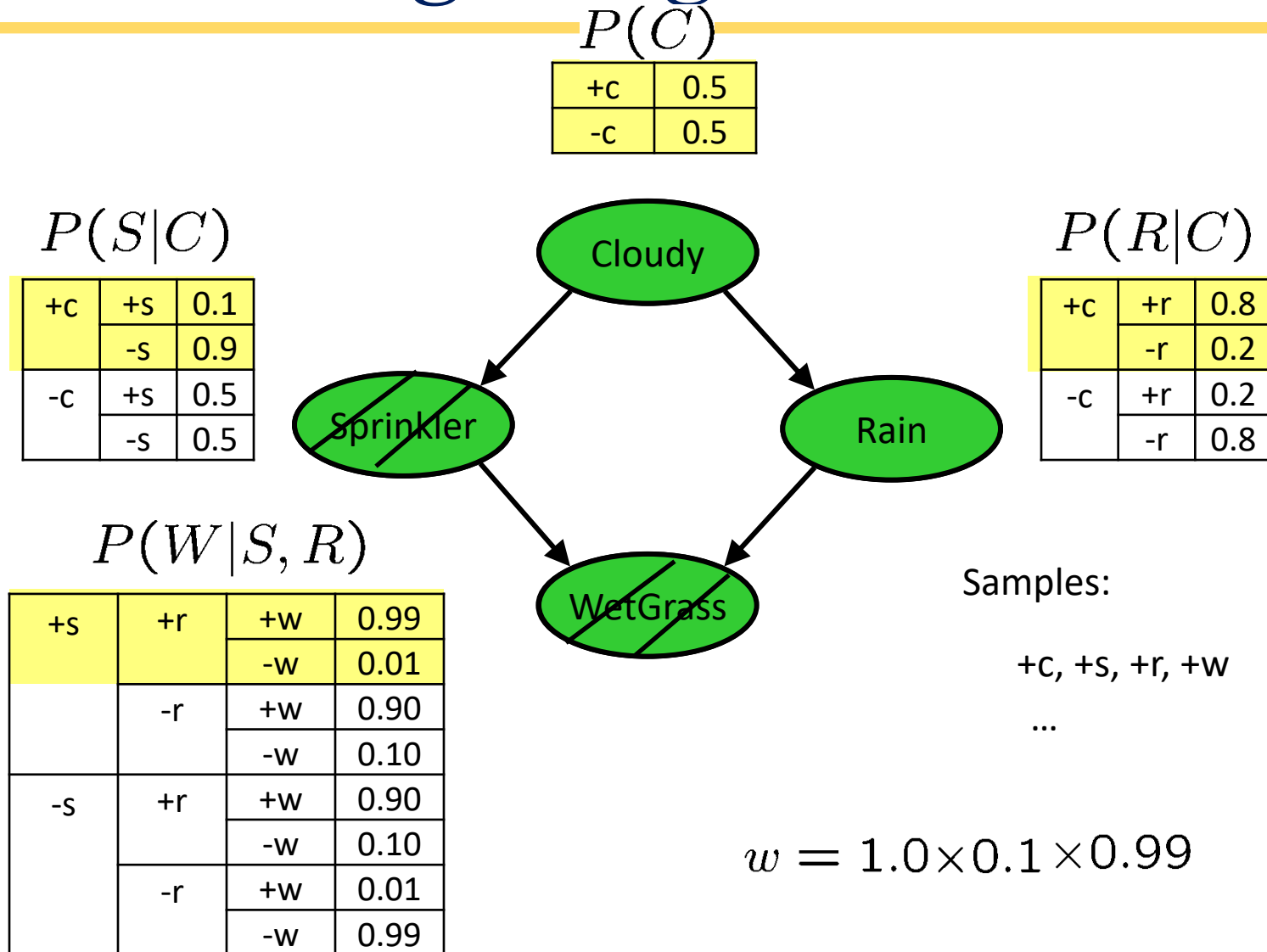


# Likelihood Weighting

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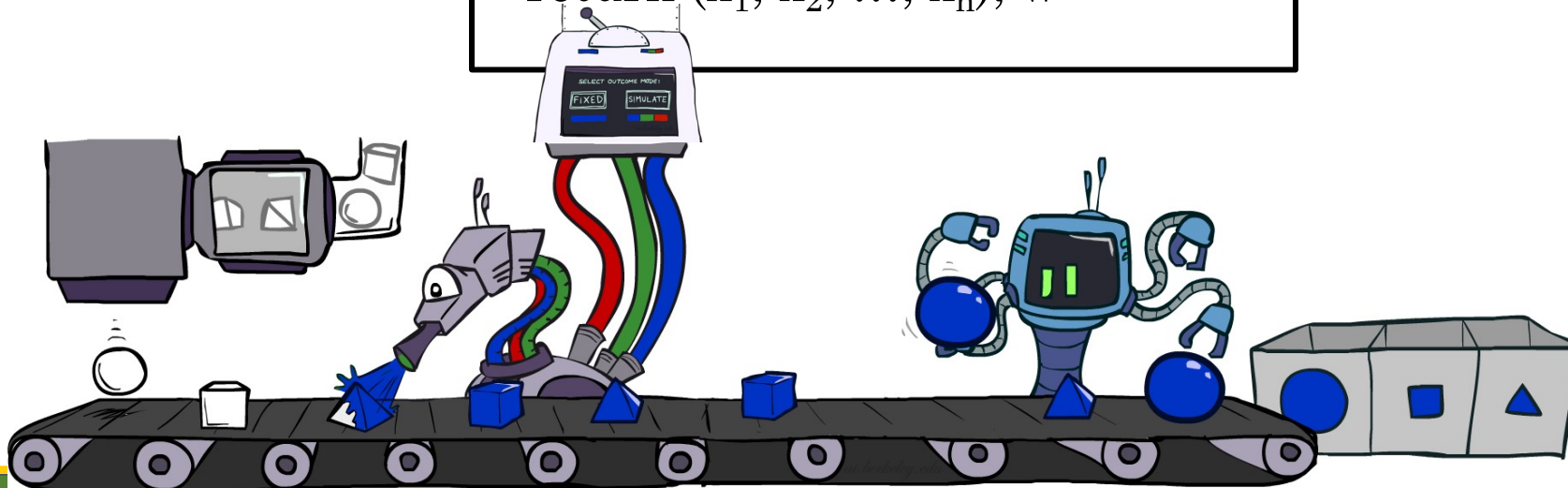


# Likelihood Weighting



# Likelihood Weighting

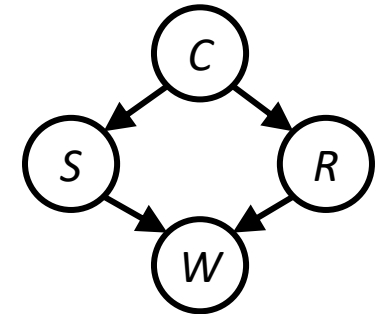
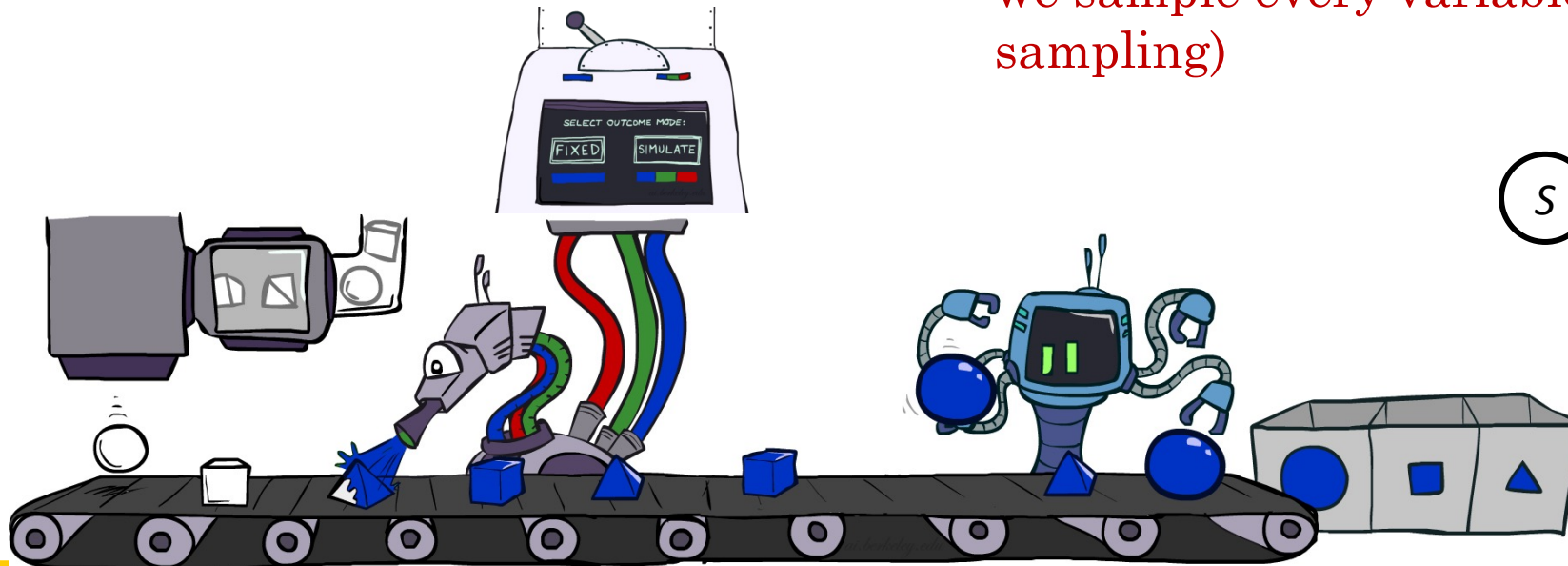
- Input: evidence instantiation
- $w = 1.0$
- for  $i = 1, 2, \dots, n$ 
  - if  $X_i$  is an evidence variable
    - $X_i = \text{observation } x_i \text{ for } X_i$
    - Set  $w = w * P(x_i \mid \text{Parents}(X_i))$
  - else
    - Sample  $x_i$  from  $P(X_i \mid \text{Parents}(X_i))$
- return  $(x_1, x_2, \dots, x_n), w$





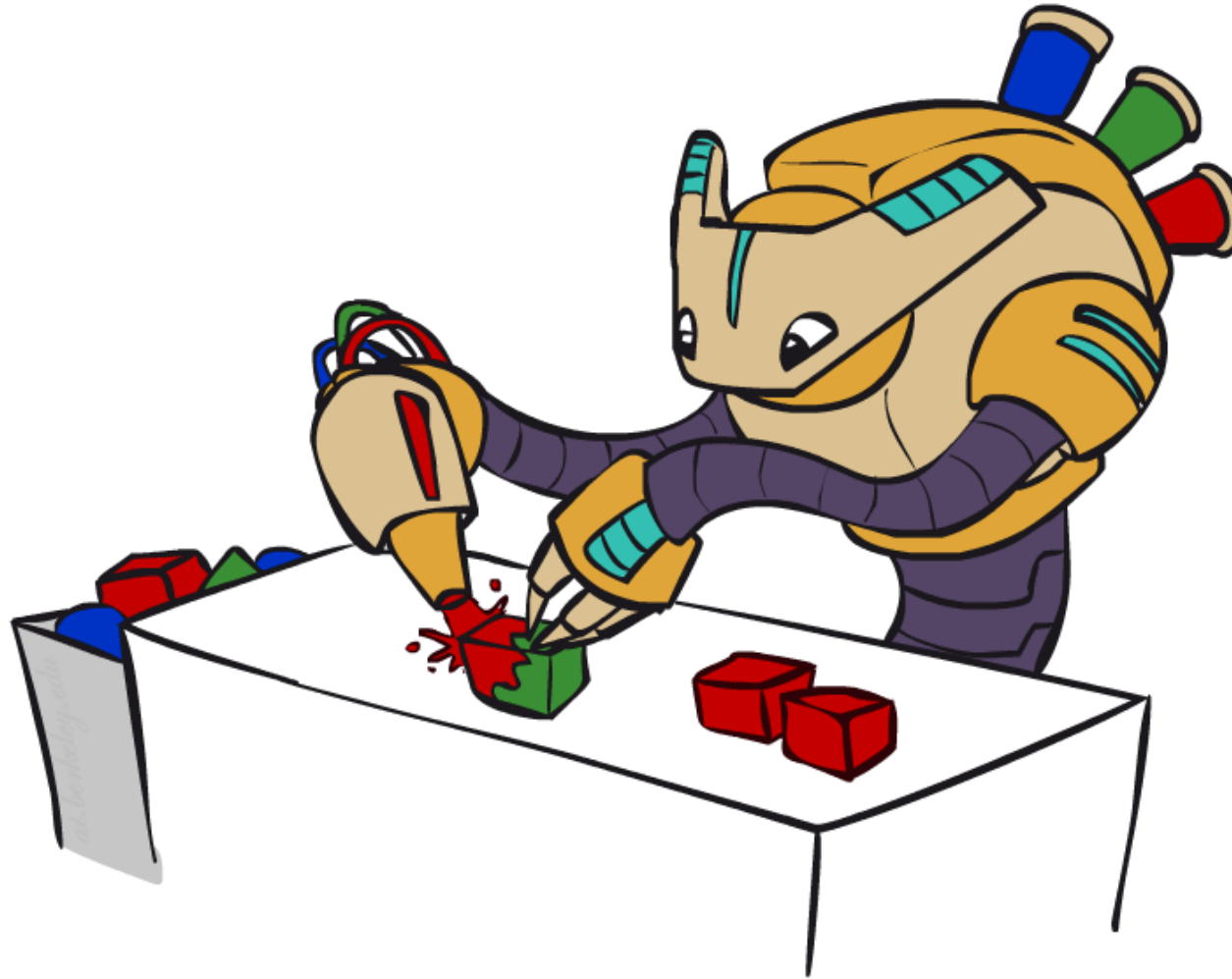
# Likelihood Weighting

- Likelihood weighting is good
  - We have taken evidence into account as we generate the sample
  - E.g. here,  $W$ 's value will get picked based on the evidence values of  $S$ ,  $R$
  - More of our samples will reflect the state of the world suggested by the evidence
- Likelihood weighting doesn't solve all our problems
  - Evidence influences the choice of downstream variables, but not upstream ones ( $C$  isn't more likely to get a value matching the evidence)
- We would like to consider evidence when we sample every variable (leads to Gibbs sampling)



# Gibbs Sampling

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# Gibbs Sampling

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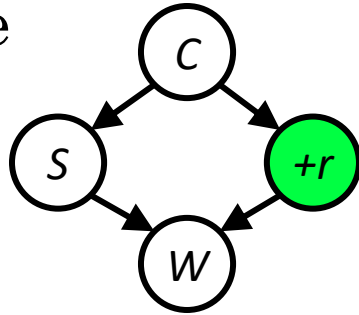
- *Procedure*: keep track of a full instantiation  $x_1, x_2, \dots, x_n$ . Start with an arbitrary instantiation consistent with the evidence. Sample one variable at a time, conditioned on all the rest, but keep evidence fixed. Keep repeating this for a long time.
- *Property*: in the limit of repeating this infinitely many times the resulting samples come from the correct distribution (i.e. conditioned on evidence).
- *Rationale*: both upstream and downstream variables condition on evidence.
- In contrast: likelihood weighting only conditions on upstream evidence, and hence weights obtained in likelihood weighting can sometimes be very small. Sum of weights over all samples is indicative of how many “effective” samples were obtained, so we want high weight.



# Gibbs Sampling Example: $P(S \mid +r)$

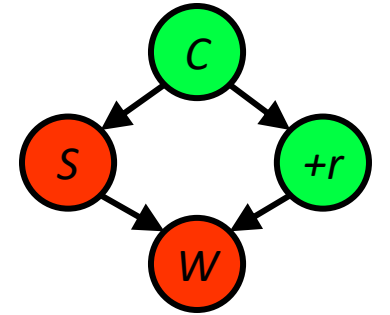
- Step 1: Fix evidence

- $R = +r$



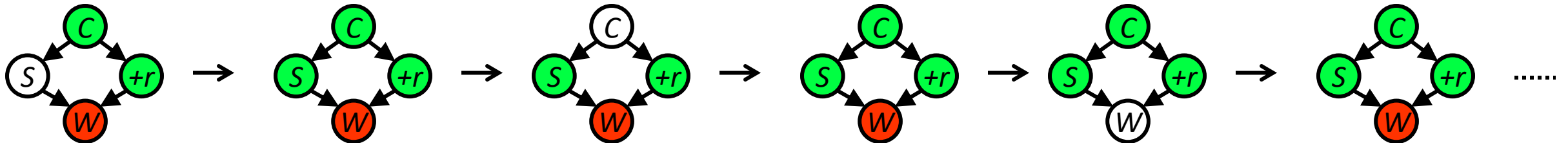
- Step 2: Initialize other variables

- Randomly



- Steps 3: Repeat

- Choose a non-evidence variable X
  - Resample X from  $P(X \mid \text{all other variables})$



Sample from  $P(S \mid +c, -w, +r)$

Sample from  $P(C \mid +s, -w, +r)$

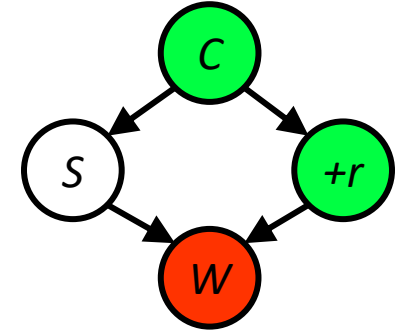
Sample from  $P(W \mid +s, +c, +r)$



# Efficient Resampling of One Variable

- Sample from  $P(S \mid +c, +r, -w)$

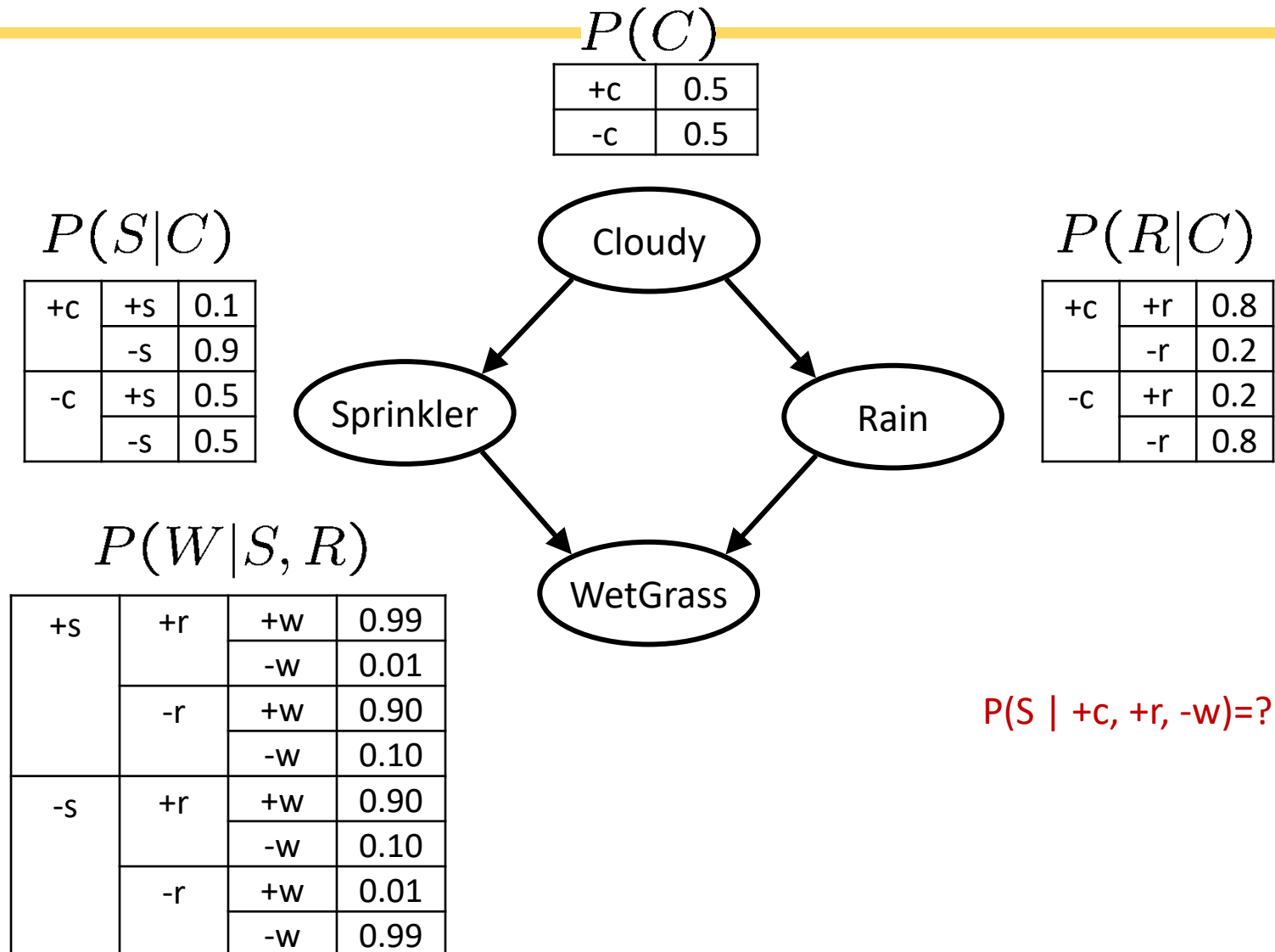
$$\begin{aligned} P(S \mid +c, +r, -w) &= \frac{P(S, +c, +r, -w)}{P(+c, +r, -w)} \\ &= \frac{P(S, +c, +r, -w)}{\sum_s P(s, +c, +r, -w)} \\ &= \frac{P(+c)P(S \mid +c)P(+r \mid +c)P(-w \mid S, +r)}{\sum_s P(+c)P(s \mid +c)P(+r \mid +c)P(-w \mid s, +r)} \\ &= \frac{P(+c)P(S \mid +c)P(+r \mid +c)P(-w \mid S, +r)}{P(+c)P(+r \mid +c) \sum_s P(s \mid +c)P(-w \mid s, +r)} \\ &= \frac{P(S \mid +c)P(-w \mid S, +r)}{\sum_s P(s \mid +c)P(-w \mid s, +r)} \end{aligned}$$



- Many things cancel out – only CPTs with S remain!
- More generally: only CPTs that have resampled variable need to be considered, and joined together



# Example

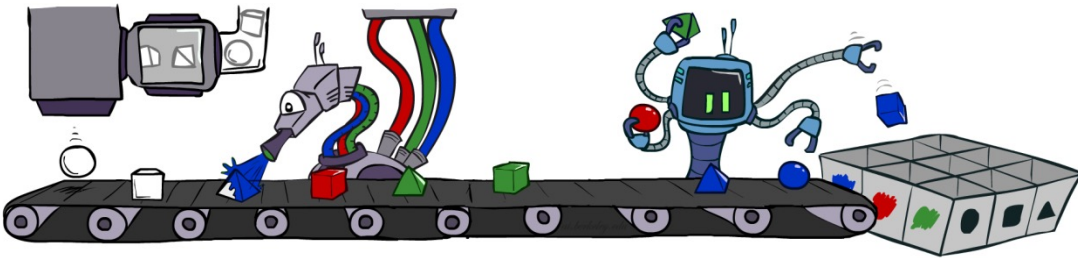


$P(S \mid +c, +r, -w) = ?$

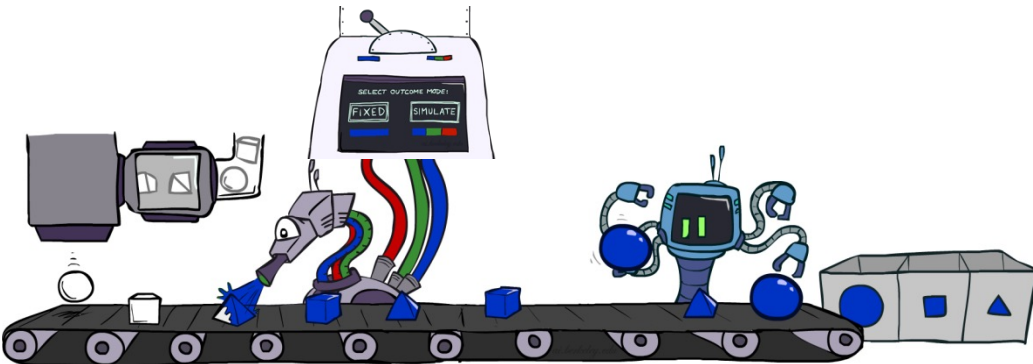


# Bayes' Net Sampling Summary

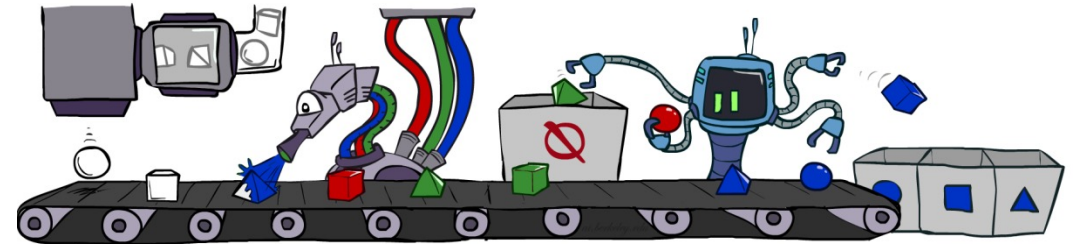
- Prior Sampling  $P(Q)$



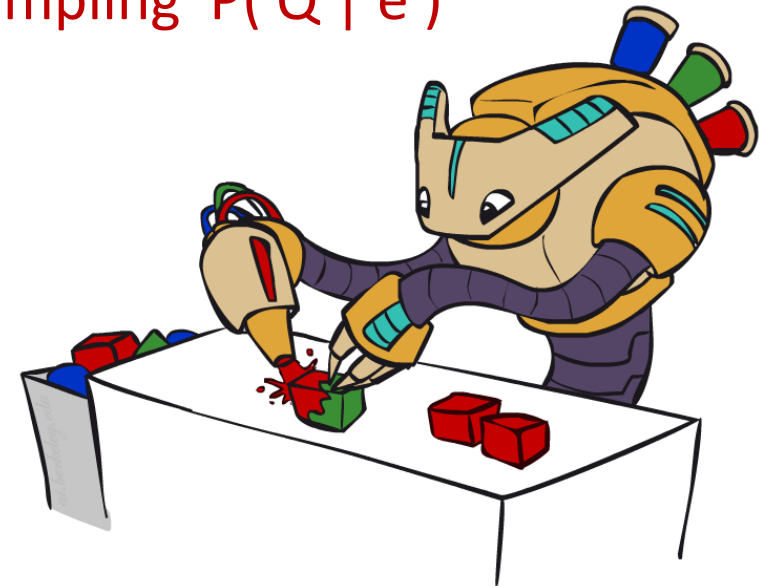
- Likelihood Weighting  $P(Q | e)$



- Rejection Sampling  $P(Q | e)$



- Gibbs Sampling  $P(Q | e)$



# Further Reading on Gibbs Sampling\*

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- Gibbs sampling produces sample from the query distribution  $P(Q | e)$  in limit of re-sampling infinitely often
- Gibbs sampling is a special case of more general methods called Markov chain Monte Carlo (MCMC) methods
  - Metropolis-Hastings is one of the more famous MCMC methods (in fact, Gibbs sampling is a special case of Metropolis-Hastings)
- You may read about Monte Carlo methods – they're just sampling

