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# CS 471/571 (Fall 2023): Introduction to Artificial Intelligence

## Lecture 12: Reinforcement Learning (Part 1)

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Source: <http://ai.berkeley.edu/home.html>



# Reminder and Announcement

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- Project 2
  - Deadline: November 03rd, 2023

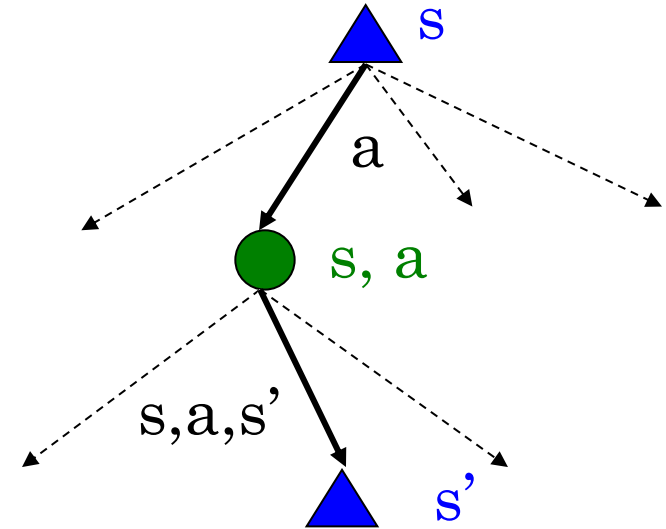
# Recap: MDPs

- Markov decision processes:

- States  $S$
- Actions  $A$
- Transitions  $P(s' | s, a)$  (or  $T(s, a, s')$ )
- Rewards  $R(s, a, s')$  (and discount  $\gamma$ )
- Start state  $s_0$

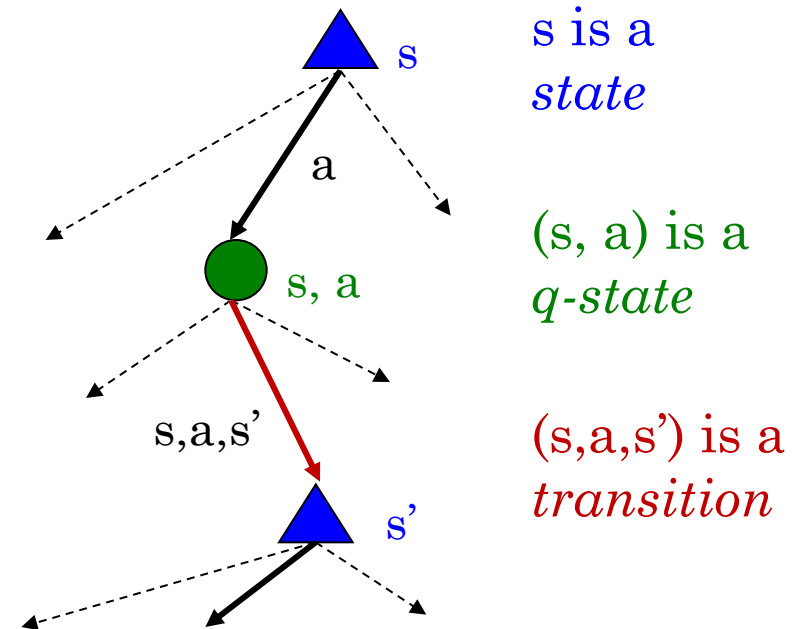
- Quantities:

- Policy = map of states to actions
- Utility = sum of discounted rewards
- Values = expected future utility from a state (max node)
- Q-Values = expected future utility from a q-state (chance node)



# Optimal Quantities

- The value (utility) of a state  $s$ :  
 $V^*(s)$  = expected utility starting in  $s$  and acting optimally
- The value (utility) of a q-state  $(s,a)$ :  
 $Q^*(s,a)$  = expected utility starting out having taken action  $a$  from state  $s$  and (thereafter) acting optimally
- The optimal policy:  
 $\pi^*(s)$  = optimal action from state  $s$



# Policy Iteration

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- Alternative approach for optimal values:
  - **Step 1: Policy evaluation:** calculate utilities for some fixed policy (not optimal utilities!) until convergence
  - **Step 2: Policy improvement:** update policy using one-step look-ahead with resulting converged (but not optimal!) utilities as future values
  - Repeat steps until policy converges
- This is **policy iteration**
  - It's still optimal!
  - Can converge (much) faster under some conditions



# Policy Iteration

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- Evaluation: For fixed current policy  $\pi$ , find values with policy evaluation:
  - Iterate until values converge:

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') [R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')]$$

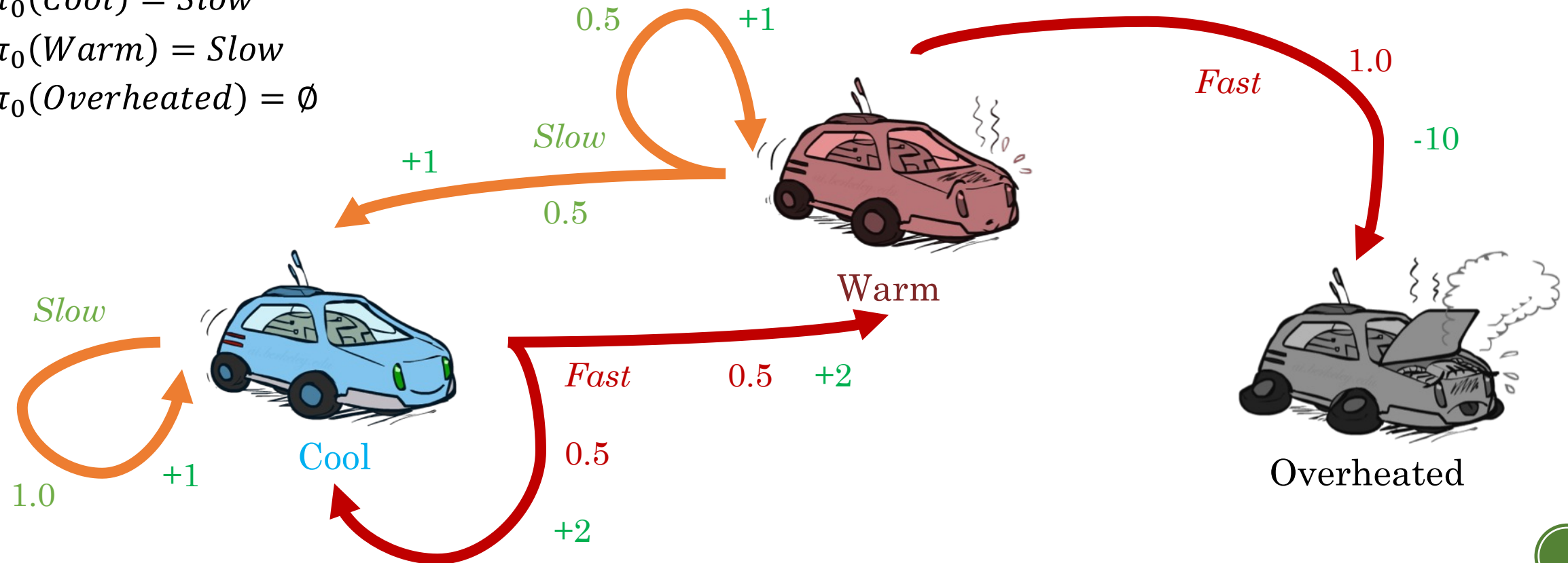
- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:

$$\pi_{i+1}(s) = \arg \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^{\pi_i}(s')]$$



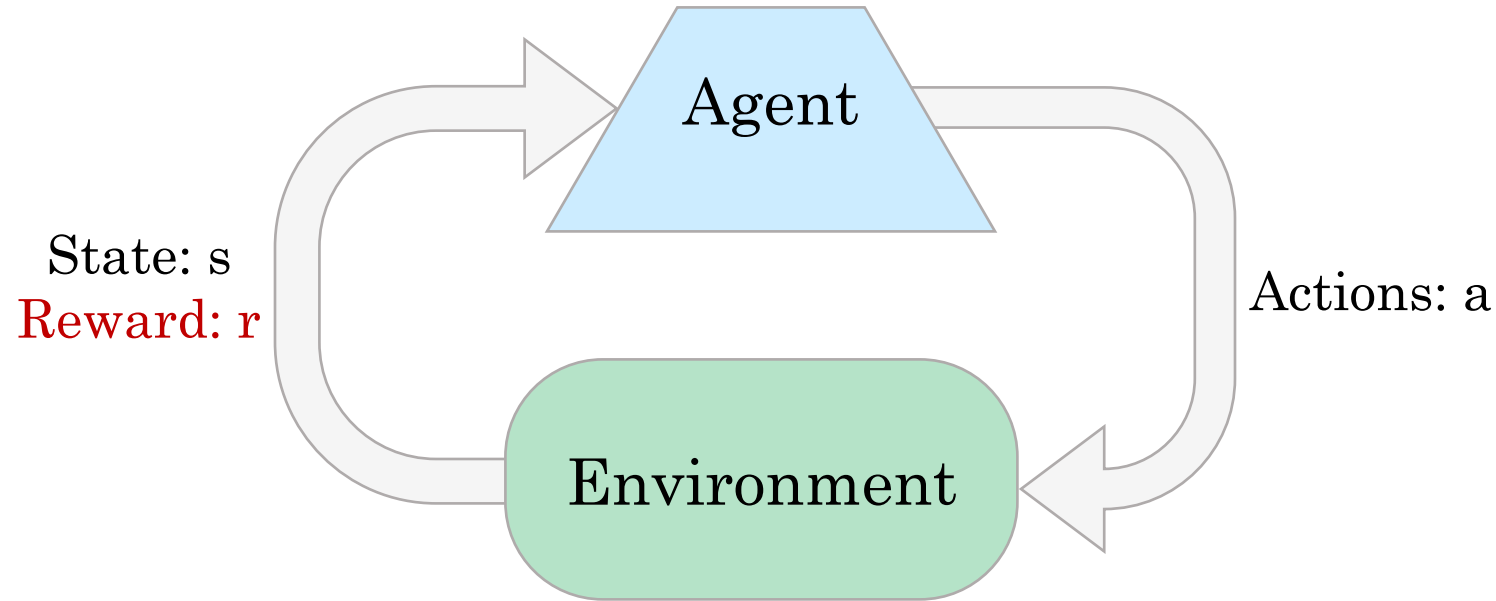
# Example: Racing

- Discount:  $\gamma = 0.1$
- Initial policy
  - $\pi_0(\text{Cool}) = \text{Slow}$
  - $\pi_0(\text{Warm}) = \text{Slow}$
  - $\pi_0(\text{Overheated}) = \emptyset$



# Reinforcement Learning

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- Basic idea:
  - Receive feedback in the form of **rewards**
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to **maximize expected rewards**
  - All learning is based on observed samples of outcomes!





# Example: Learning to Walk

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Initial



A Learning Trial



After Learning  
[1K Trials]



# Example: Learning to Walk

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[Kohl and Stone, ICRA 2004]

Initial



# Example: Learning to Walk

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[Kohl and Stone, ICRA 2004]

Training



# Example: Learning to Walk

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[Kohl and Stone, ICRA 2004]

Finished



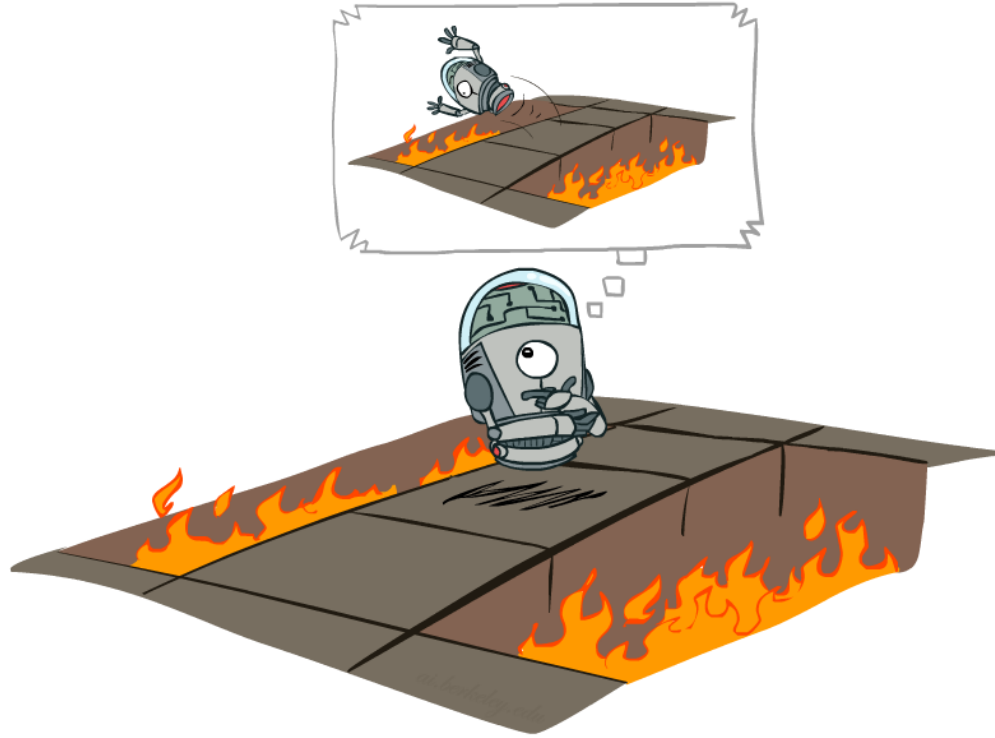
# Reinforcement Learning

- Still assume a Markov decision process (MDP):
  - A set of states  $s \in S$
  - A set of actions (per state)  $A$
  - A model  $T(s,a,s')$
  - A reward function  $R(s,a,s')$
- Still looking for a policy  $\pi(s)$
- New twist: don't know  $T$  or  $R$ 
  - I.e. we don't know which states are good or what the actions do
  - Must actually try out actions and states to learn



# Offline (MDPs) vs. Online (RL)

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Offline  
Solution



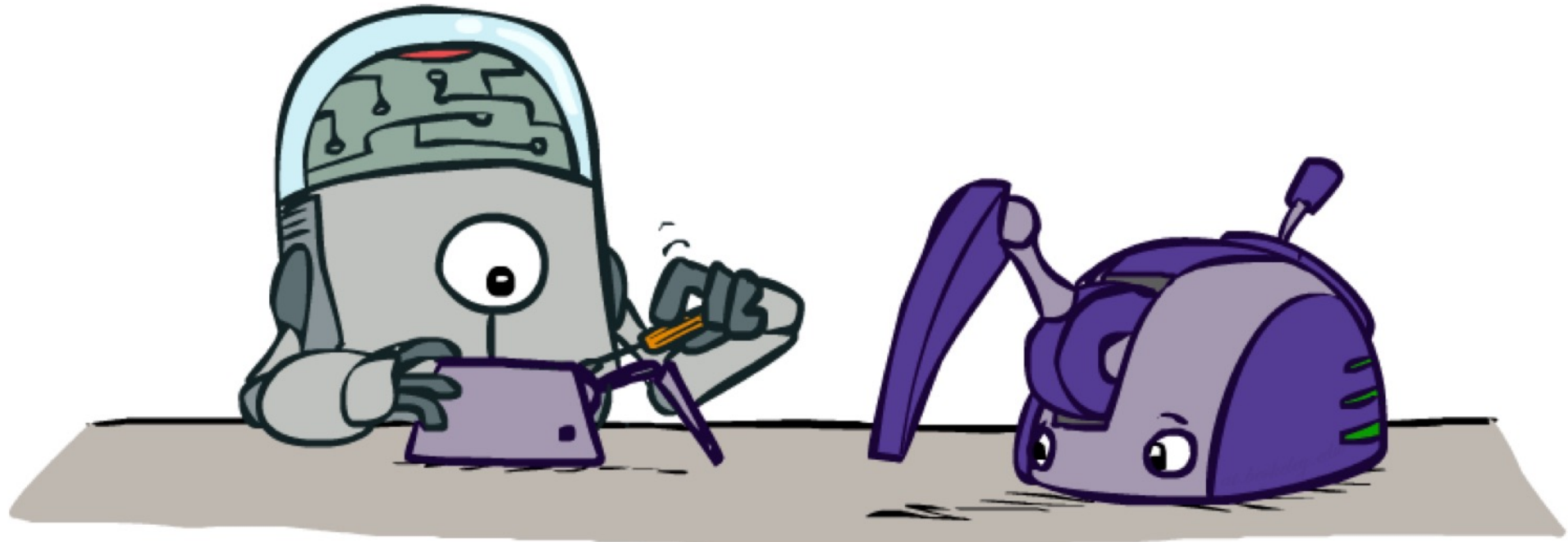
Online  
Learning





# Model-Based Learning

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# Example: Expected Age

Goal: Compute expected age of UO students

Known  $P(A)$

$$E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \dots$$

Without  $P(A)$ , instead collect samples  $[a_1, a_2, \dots, a_N]$

Unknown  $P(A)$ : “Model Based”

Why does this work?  
Because eventually you learn the right model.

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown  $P(A)$ : “Model Free”

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.



# Model-Based Learning

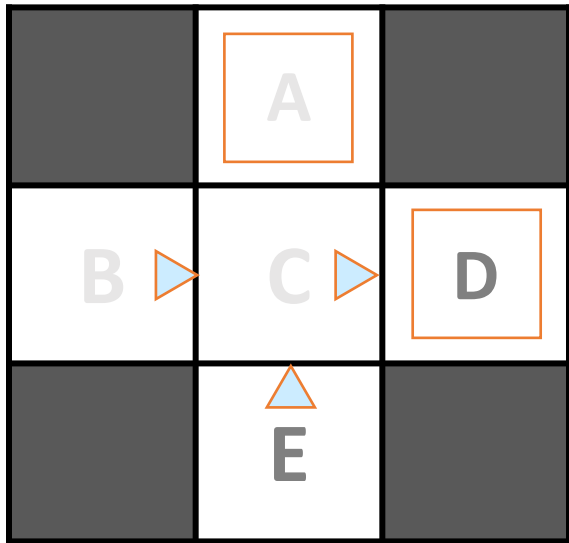
- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
  - Count outcomes  $s'$  for each  $s, a$
  - Normalize to give an estimate of  $\hat{T}(s, a, s')$
  - Discover each  $\hat{R}(s, a, s')$  when we experience  $(s, a, s')$
- Step 2: Solve the learned MDP
  - For example, use value iteration, as before



# Example: Model-Based Learning

Input Policy

$\pi$



Assume:  $\gamma = 1$

Observed Episodes  
(Training)

Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 2

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 3

E, north, C, -1  
C, east, D, -1  
D, exit, x, +10

Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

T(B, east, C) = 1.00  
T(C, east, D) = 0.75  
T(C, east, A) = 0.25  
...

$\hat{R}(s, a, s')$

R(B, east, C) = -1  
R(C, east, D) = -1  
R(D, exit, x) = +10  
...