
CS 471/571 (Fall 2023): Introduction to Artificial Intelligence

Lecture 18: Bayes Nets – Part 2

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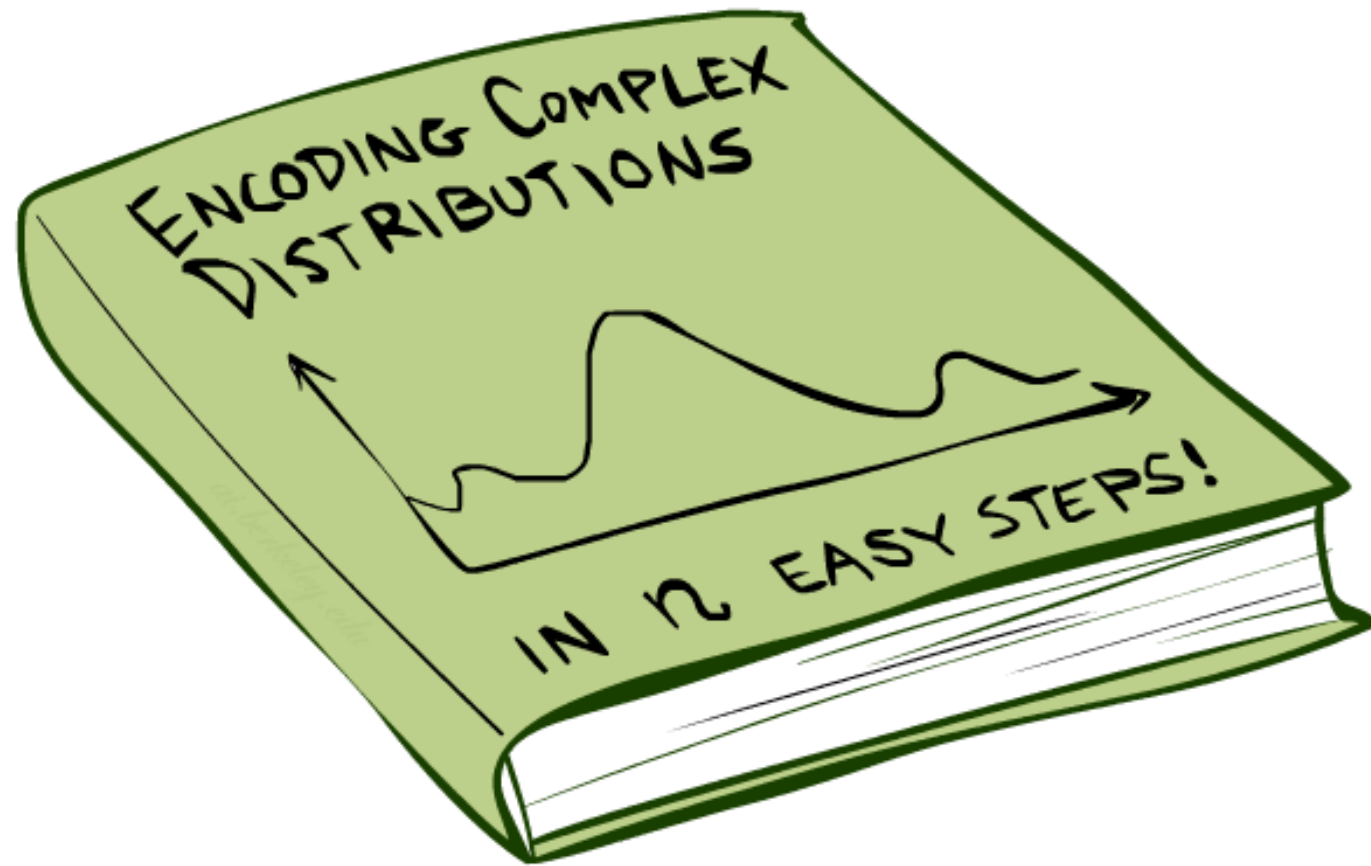
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Reminder:

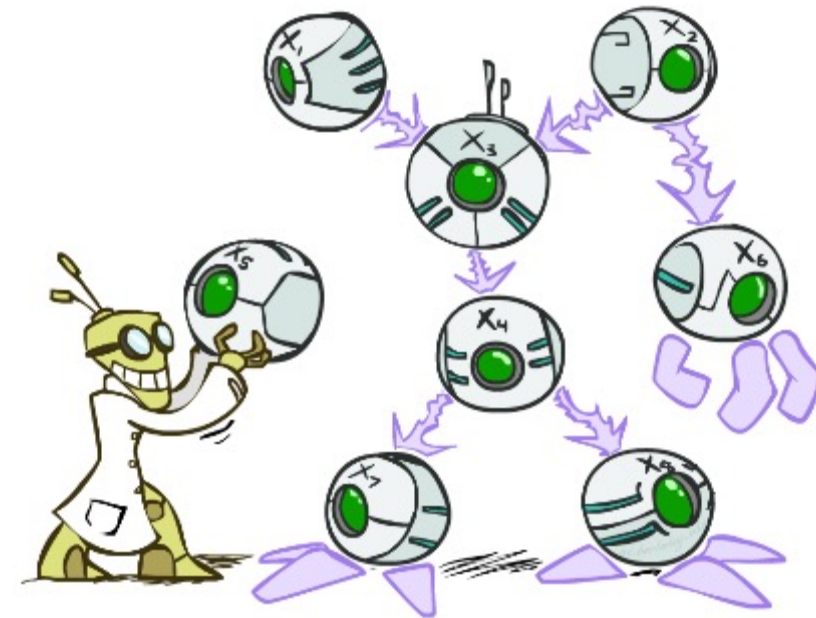
- Written assignment 3:
 - Deadline: Nov 08, 2023
- Programming project 3:
 - Deadline: Nov 20, 2023

Bayes' Nets: Big Picture



Bayes' Nets: Big Picture

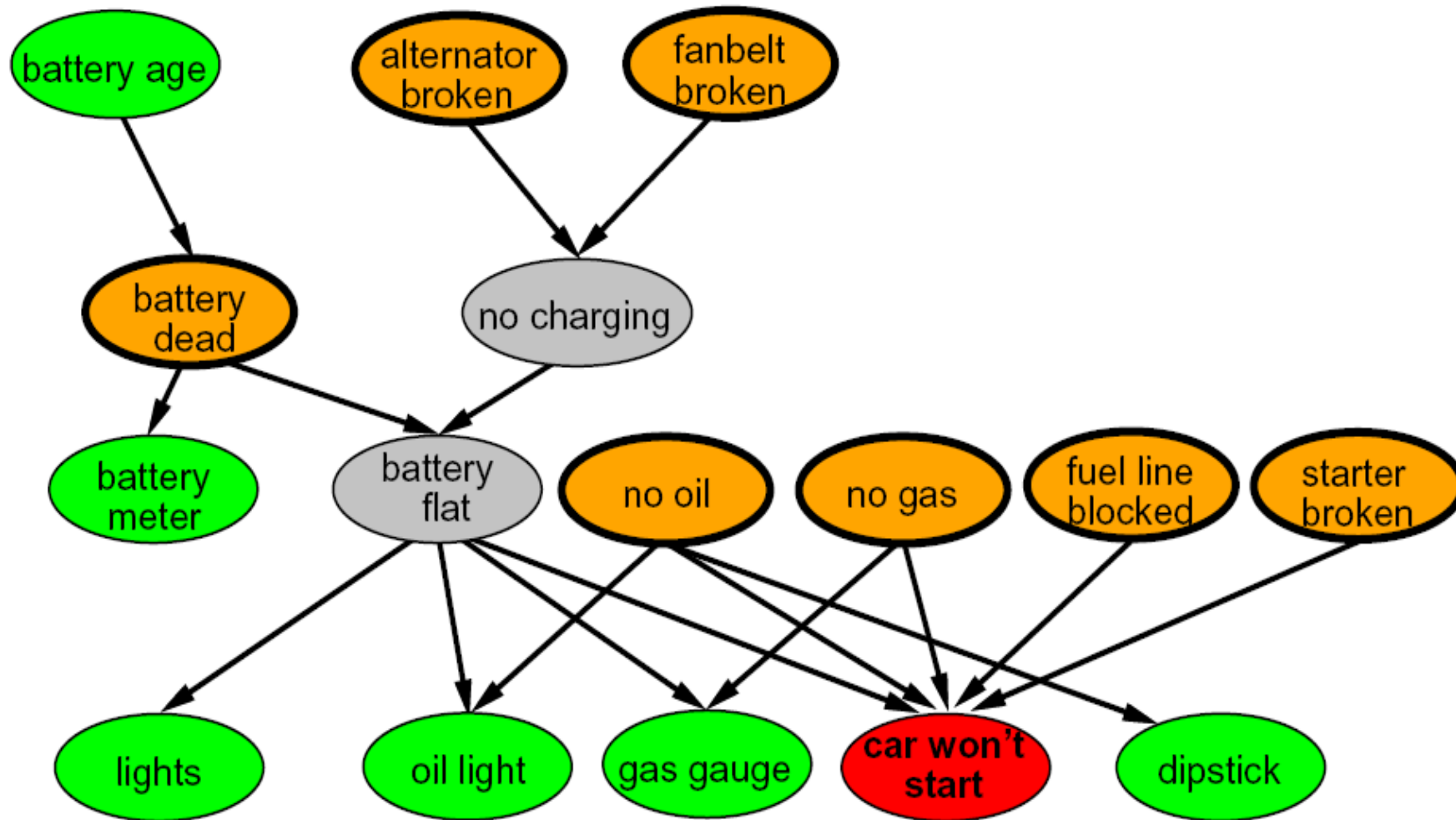
- Two problems with using full joint distribution tables as our probabilistic models:
 - Unless there are only a few variables, the joint is WAY too big to represent explicitly
 - Hard to learn (estimate) anything empirically about more than a few variables at a time
- Bayes' nets:** a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
 - More properly called **graphical models**
 - We describe how variables locally interact
 - Local interactions chain together to give global, indirect interactions
 - For about 10 min, we'll be vague about how these interactions are specified



Example Bayes' Net: Insurance

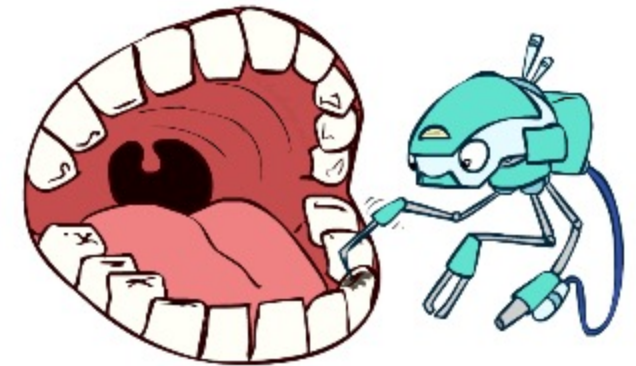
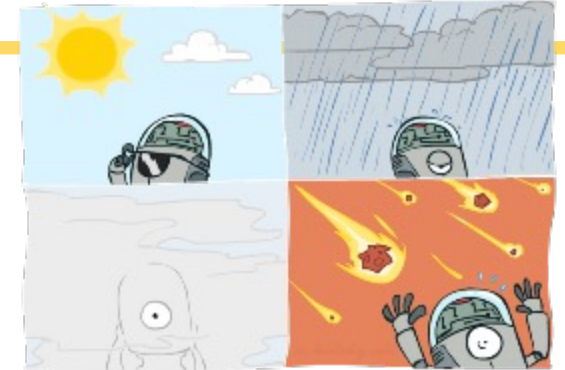
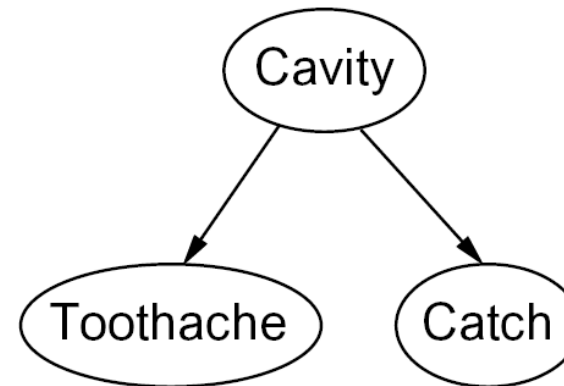


Example Bayes' Net: Car



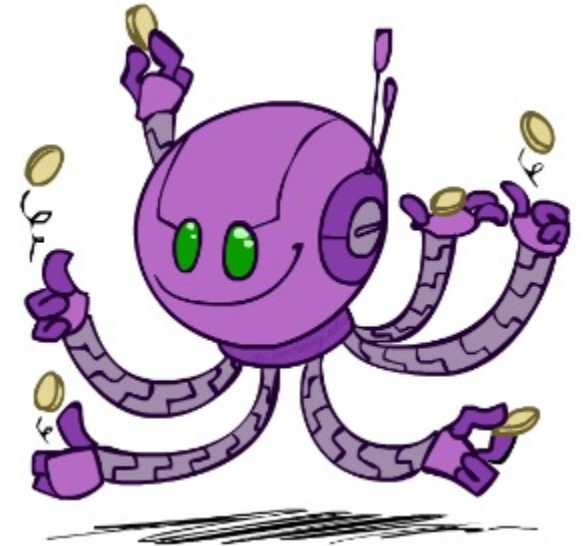
Graphical Model Notation

- Nodes: variables (with domains)
 - Can be assigned (observed) or unassigned (unobserved)
- Arcs: interactions
 - Similar to CSP constraints
 - Indicate “direct influence” between variables
 - Formally: encode conditional independence (more later)
- For now: imagine that arrows mean direct causation (in general, they don't!)



Example: Coin Flips

- N independent coin flips

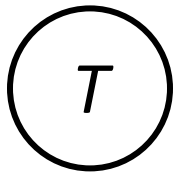
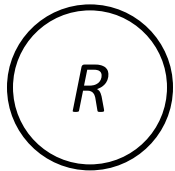


- No interactions between variables: **absolute independence**



Example: Traffic

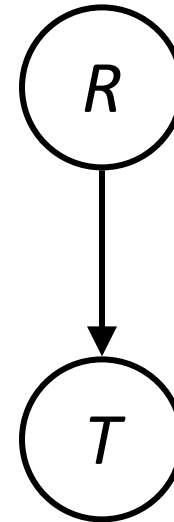
- Variables:
 - R: It rains
 - T: There is traffic
- Model 1: independence



- Why is an agent using model 2 better?



- Model 2: rain causes traffic



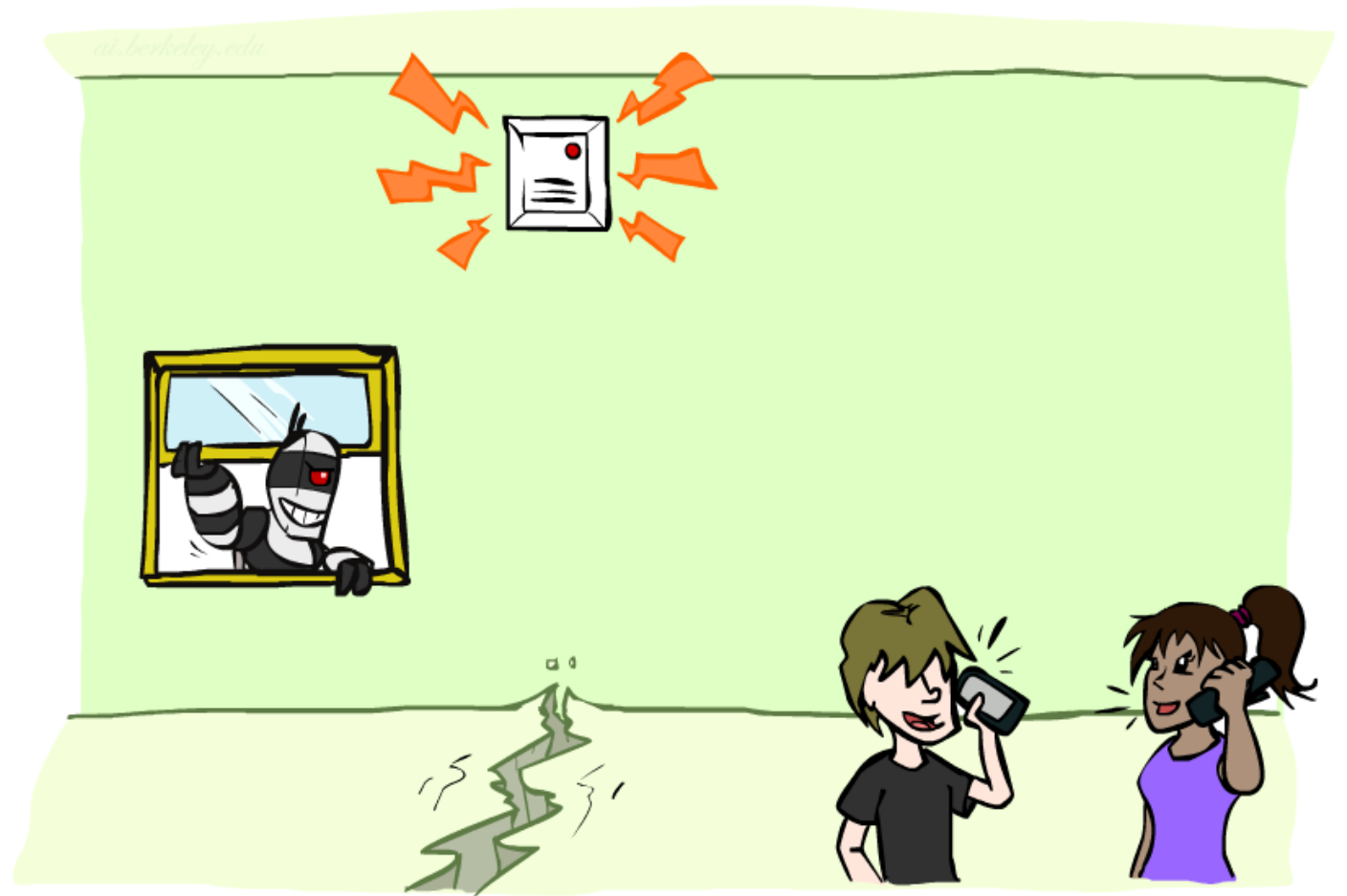
Example: Traffic II

- Let's build a causal graphical model!
- Variables
 - T: Traffic
 - R: It rains
 - L: Low pressure
 - D: Roof drips
 - B: Ballgame
 - C: Cavity



Example: Alarm Network

- Variables
 - B: Burglary
 - A: Alarm goes off
 - M: Mary calls
 - J: John calls
 - E: Earthquake!



Bayes' Net Semantics



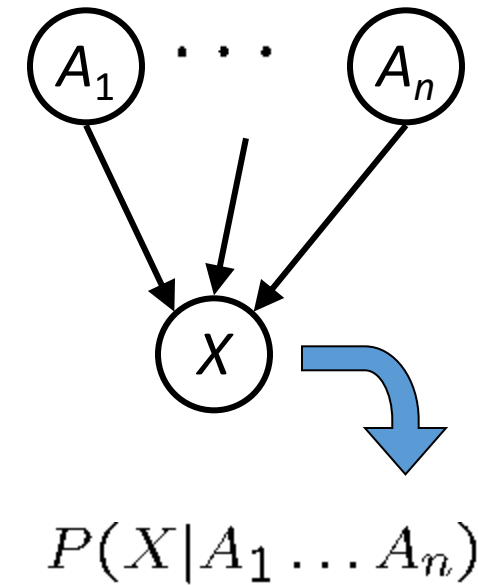


Bayes' Net Semantics

- A set of nodes, one per variable X
- A directed, acyclic graph
- A conditional distribution for each node
 - A collection of distributions over X , one for each combination of parents' values

$$P(X|a_1 \dots a_n)$$

- CPT: conditional probability table
- Description of a noisy “causal” process



A Bayes net = Topology (graph) + Local Conditional Probabilities



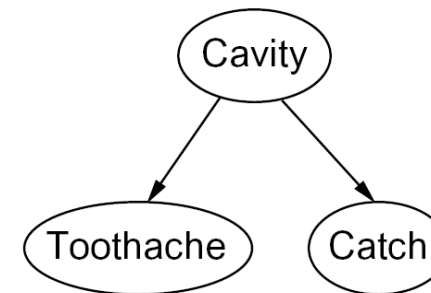
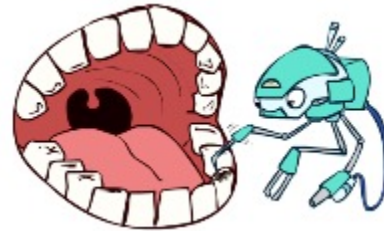


Probabilities in BNs

- Bayes' nets **implicitly** encode joint distributions
 - As a product of local conditional distributions
 - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

- Example:



$$P(+cavity, +catch, -toothache)$$





Probabilities in BNs

- Why are we guaranteed that setting

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

results in a proper joint distribution?

- Chain rule (valid for all distributions): $P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | x_1 \dots x_{i-1})$

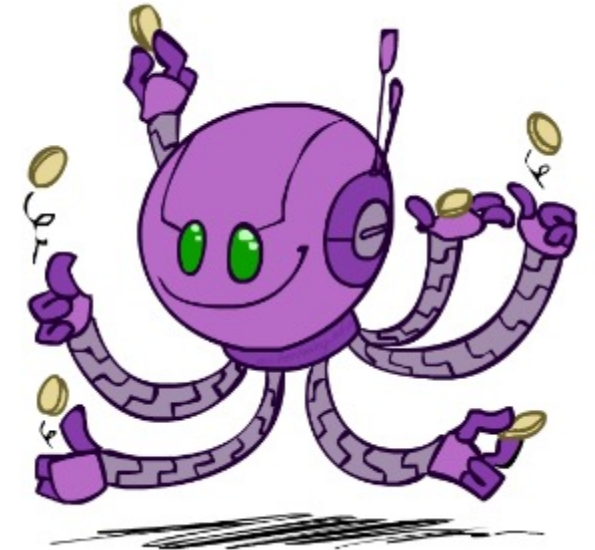
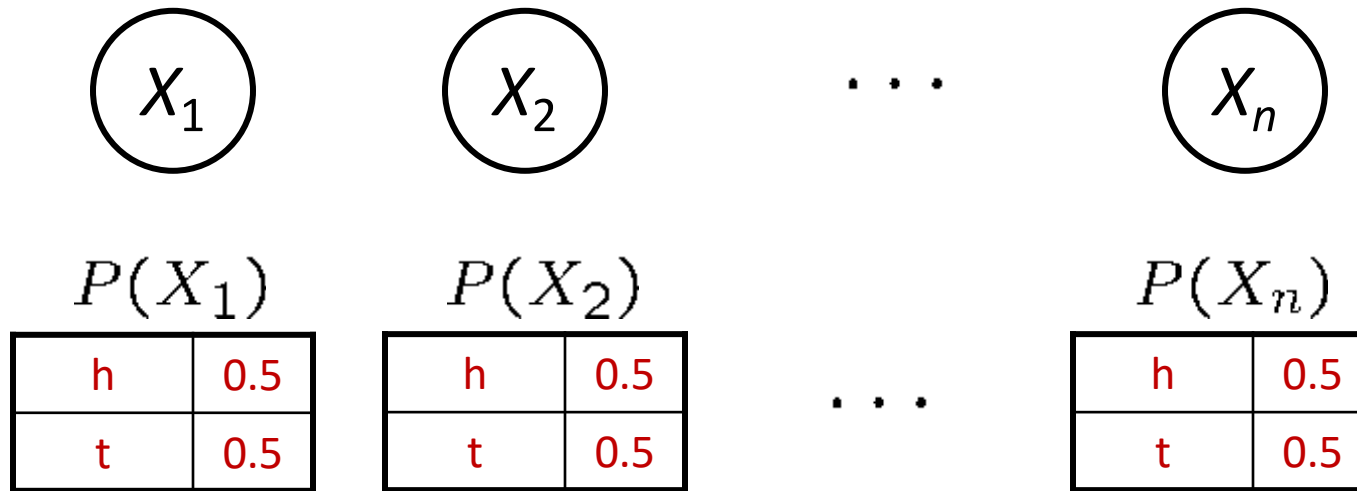
- Assume conditional independences: $P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$

→ Consequence:
$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

- Not every BN can represent every joint distribution
 - The topology enforces certain conditional independencies



Example: Coin Flips

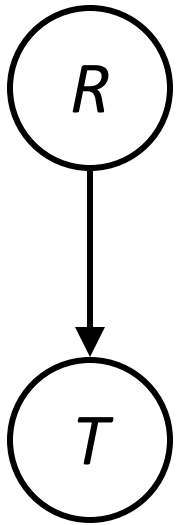


$$P(h, h, t, h) =$$

Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.



Example: Traffic



$P(R)$	
$+r$	$1/4$
$-r$	$3/4$

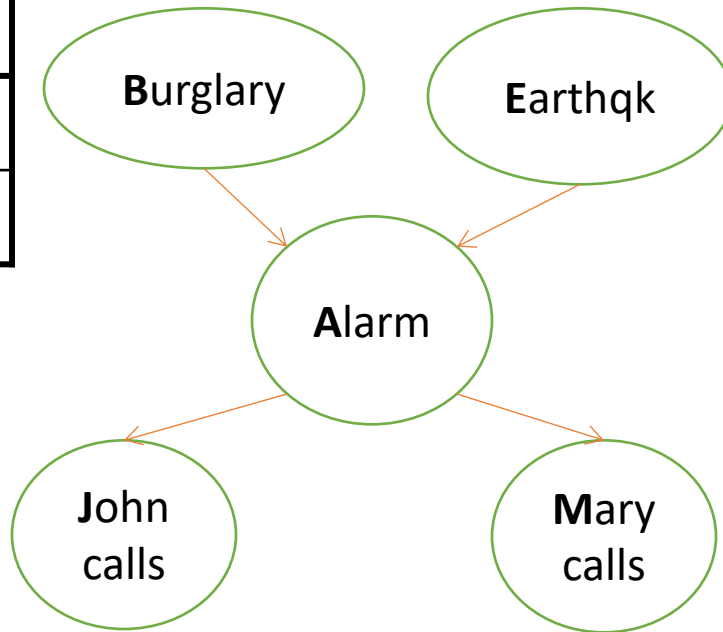
$$P(+r, -t) =$$

$P(T R)$					
$+r$	<table><tr><td>$+t$</td><td>$3/4$</td></tr><tr><td>$-t$</td><td>$1/4$</td></tr></table>	$+t$	$3/4$	$-t$	$1/4$
$+t$	$3/4$				
$-t$	$1/4$				
$-r$	<table><tr><td>$+t$</td><td>$1/2$</td></tr><tr><td>$-t$</td><td>$1/2$</td></tr></table>	$+t$	$1/2$	$-t$	$1/2$
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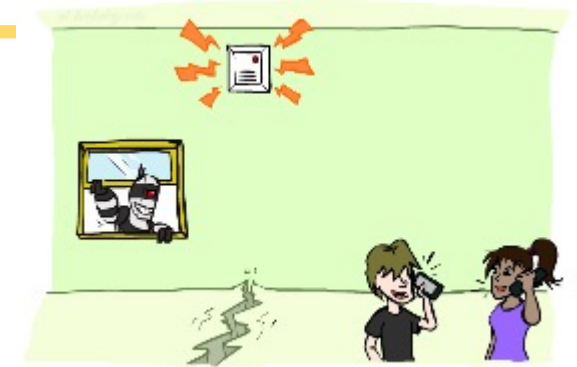


Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999



E	P(E)
+e	0.002
-e	0.998



A	J	P(J A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

B	E	A	P(A B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999



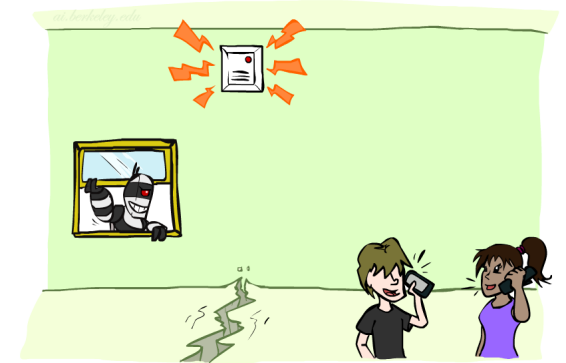
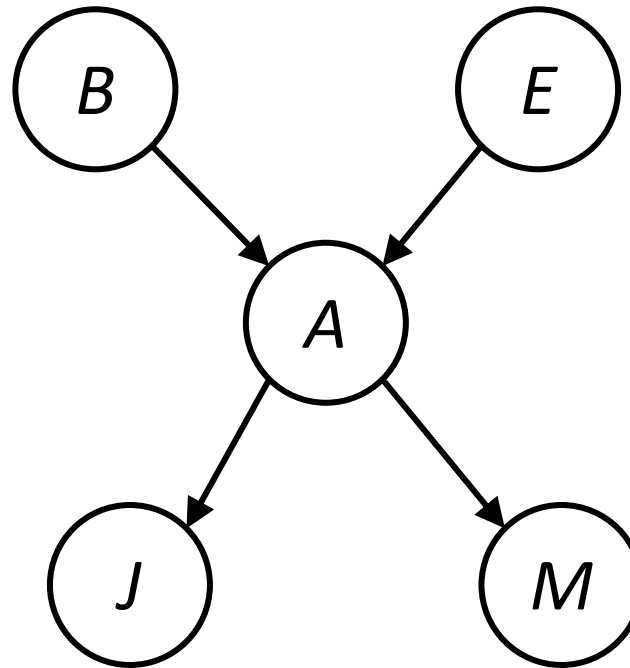
Example: Alarm Network

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A	J	P(J A)
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A	M	P(M A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99



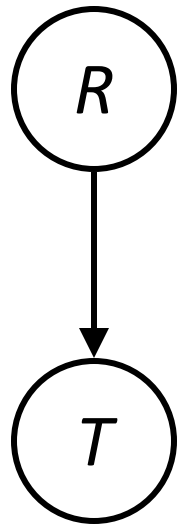
$$P(+b, -e, +a, -j, +m) =$$

B	E	A	P(A B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999



Example: Traffic

- Causal direction



$P(R)$

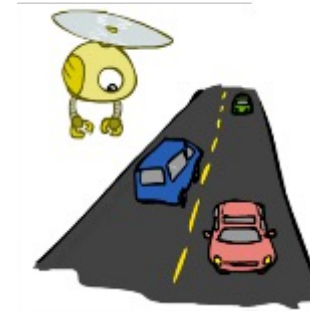
$+r$	$1/4$
$-r$	$3/4$

$P(T|R)$

$+r$	$+t$	$3/4$
	$-t$	$1/4$
$-r$	$+t$	$1/2$
	$-t$	$1/2$

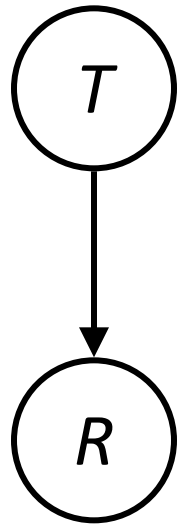
$P(T, R)$

$+r$	$+t$	$3/16$
$+r$	$-t$	$1/16$
$-r$	$+t$	$6/16$
$-r$	$-t$	$6/16$



Example: Reverse Traffic

- Reverse causality?



$P(T)$

+t	9/16
-t	7/16

$P(R|T)$

+t	+r	1/3
	-r	2/3
-t	+r	1/7
	-r	6/7



$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16



Size of a Bayes' Net

- How big is a joint distribution over N Boolean variables?

$$2^N$$

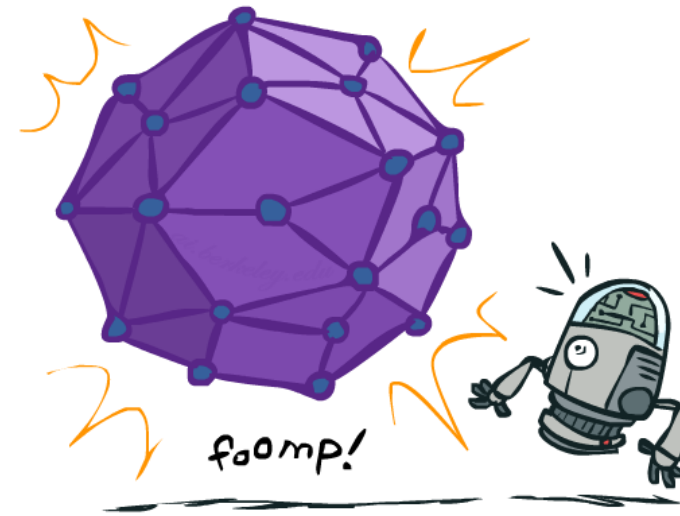
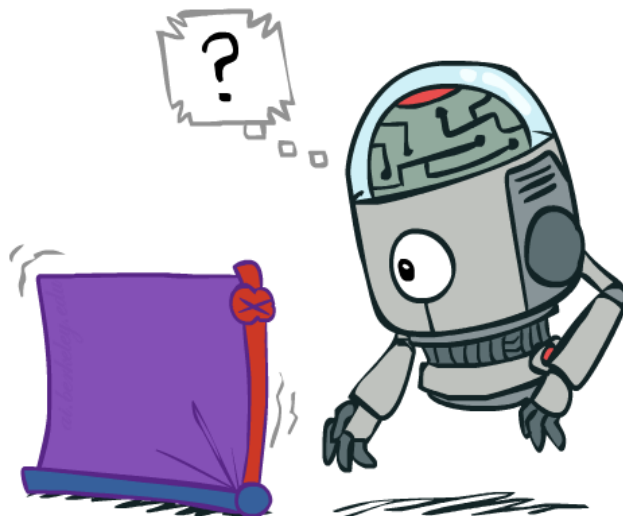
- How big is an N -node net if nodes have up to k parents?

$$O(N * 2^{k+1})$$

- Both give you the power to calculate

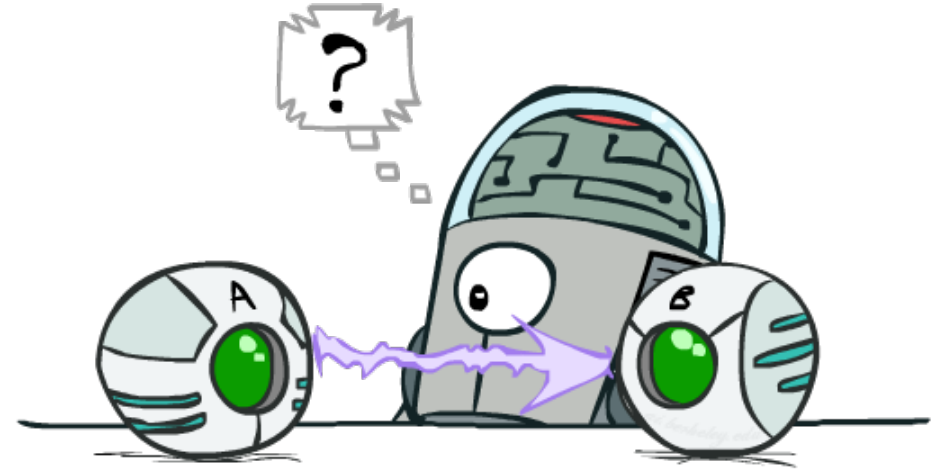
$$P(X_1, X_2, \dots, X_n)$$

- BNs: Huge space savings!
- Also easier to elicit local CPTs
- Also faster to answer queries (coming)



Causality?

- When Bayes' nets reflect the true causal patterns:
 - Often simpler (nodes have fewer parents)
 - Often easier to think about
 - Often easier to elicit from experts
- BNs need not actually be causal
 - Sometimes no causal net exists over the domain (especially if variables are missing)
 - E.g. consider the variables *Traffic* and *Drips*
 - End up with arrows that reflect correlation, not causation
- What do the arrows really mean?
 - Topology may happen to encode causal structure
 - **Topology really encodes conditional independence**



$$P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$$



Bayes' Nets

- So far: how a Bayes' net encodes a joint distribution
- Next: how to answer queries about that distribution
 - Today:
 - First assembled BNs using an intuitive notion of conditional independence as causality
 - Then saw that key property is conditional independence
 - Main goal: answer queries about conditional independence and influence
- After that: how to answer numerical queries (inference)

