CS 471/571(Fall 2023): Introduction to Artificial Intelligence

Lecture 17: Bayes Nets

Thanh H. Nguyen

Source: http://ai.berkeley.edu/home.html

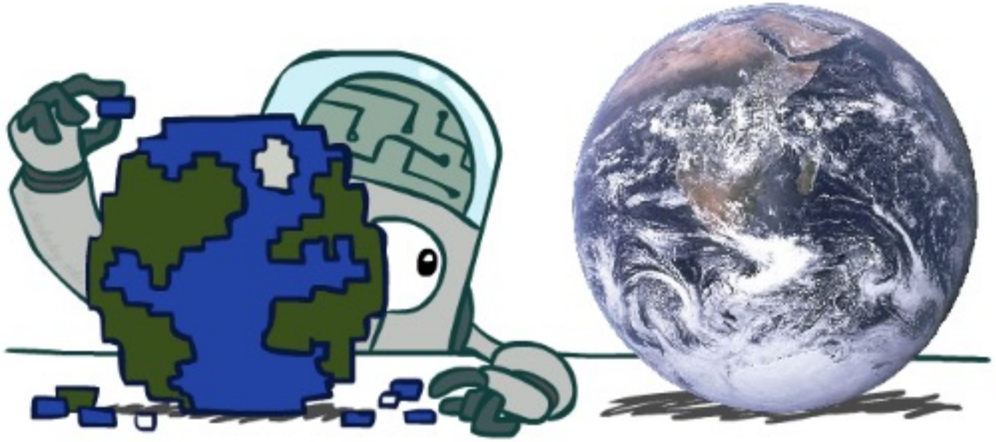
Reminder:

- Written assignment 3:
  - Deadline: Nov 08, 2023
- Programming project 3:
  - Will be posted tomorrow
  - Deadline: Nov 20, 2023

Probabilistic Models

- Models describe how (a portion of) the world works

- Models are always simplifications
  - May not account for every variable
  - May not account for all interactions between variables
  - "All models are wrong; but some are useful."
    – George E. P. Box

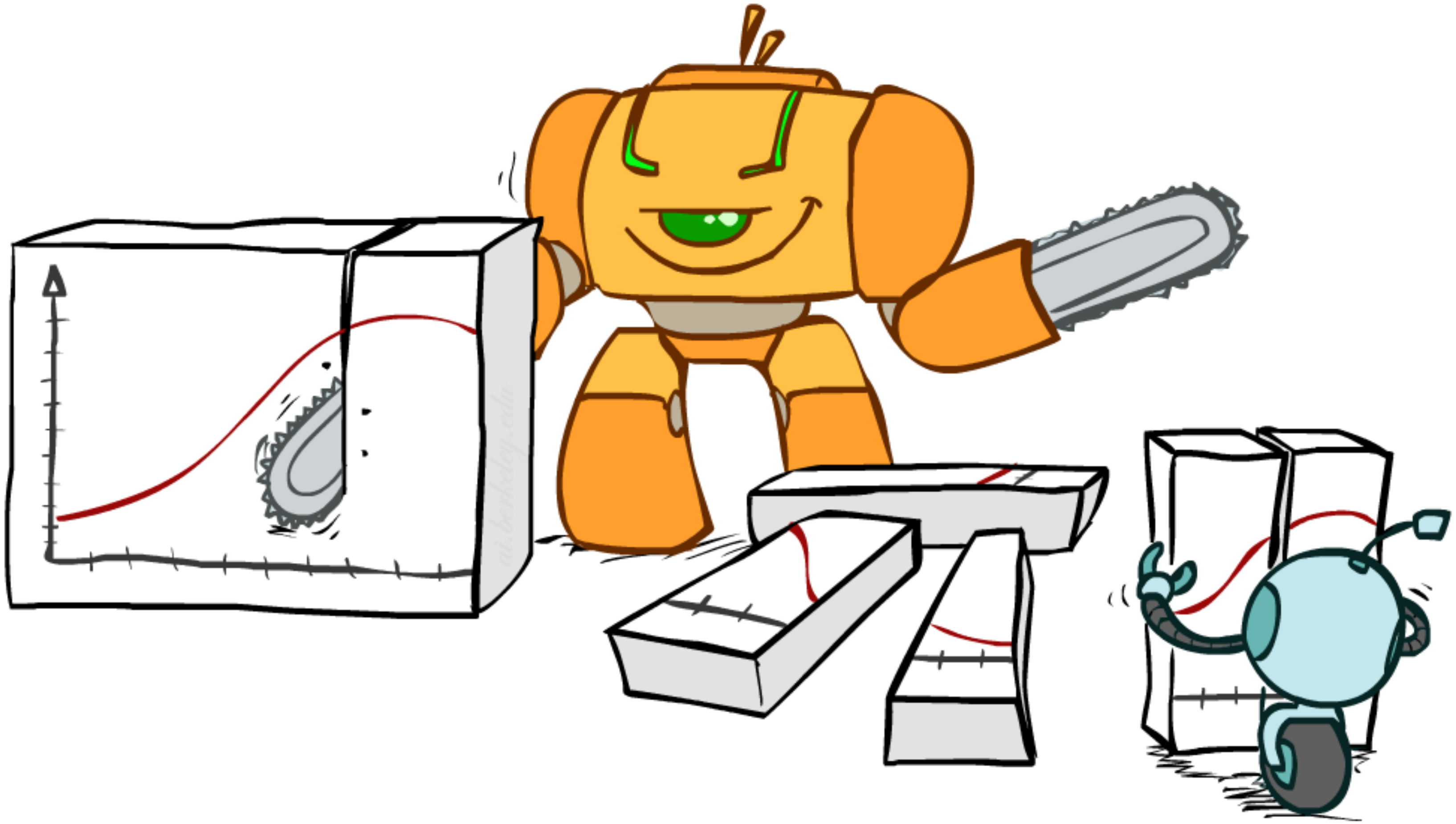


- What do we do with probabilistic models?
  - We (or our agents) need to reason about unknown variables, given evidence
  - Example: explanation (diagnostic reasoning)
  - Example: prediction (causal reasoning)

Probability Recap

- Conditional probability $$P(x|y) = \frac{P(x,y)}{P(y)}$$
- Product rule $$P(x,y) = P(x|y)P(y)$$
- Chain rule $$\begin{aligned} P(X_1, X_2, \ldots X_n) &= P(X_1)P(X_2|X_1)P(X_3|X_1,X_2)\ldots \\ &= \prod_{i=1}^{n} P(X_i|X_1,\ldots,X_{i-1}) \end{aligned}$$

Bayes Rule



Bayes Rule

- Two ways to factor a joint distribution over two variables:

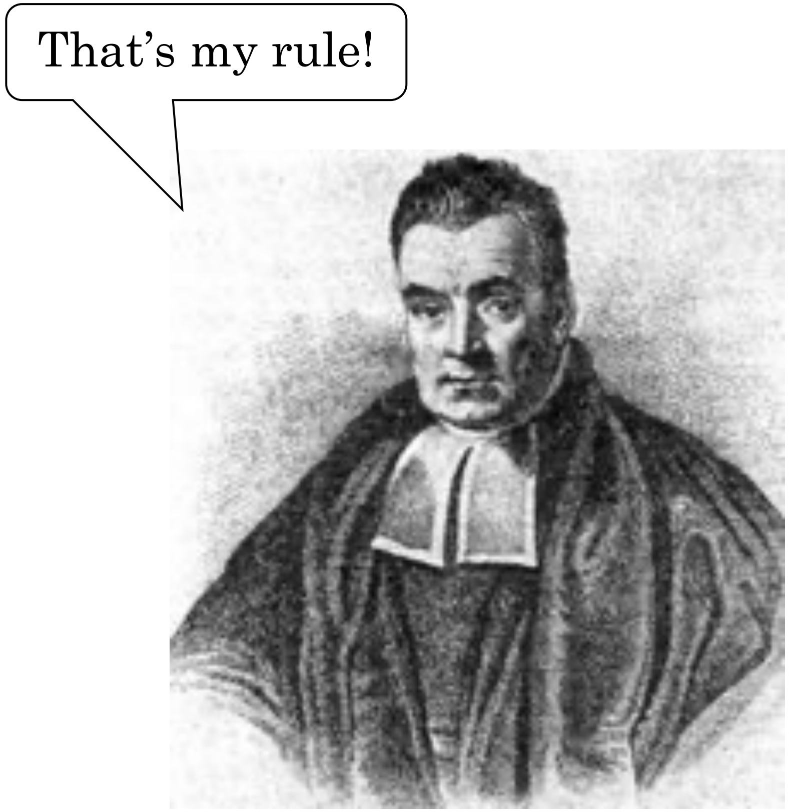
$$P(x, y) = P(x|y)P(y) = P(y|x)P(x)$$

- Dividing, we get:

$$P(x|y) = \frac{P(y|x)}{P(y)}P(x)$$

- Why is this at all helpful?
  - Lets us build one conditional from its reverse
  - Often one conditional is tricky but the other one is simple

- In the running for most important AI equation!



Quiz

- Given:

$P(W)$

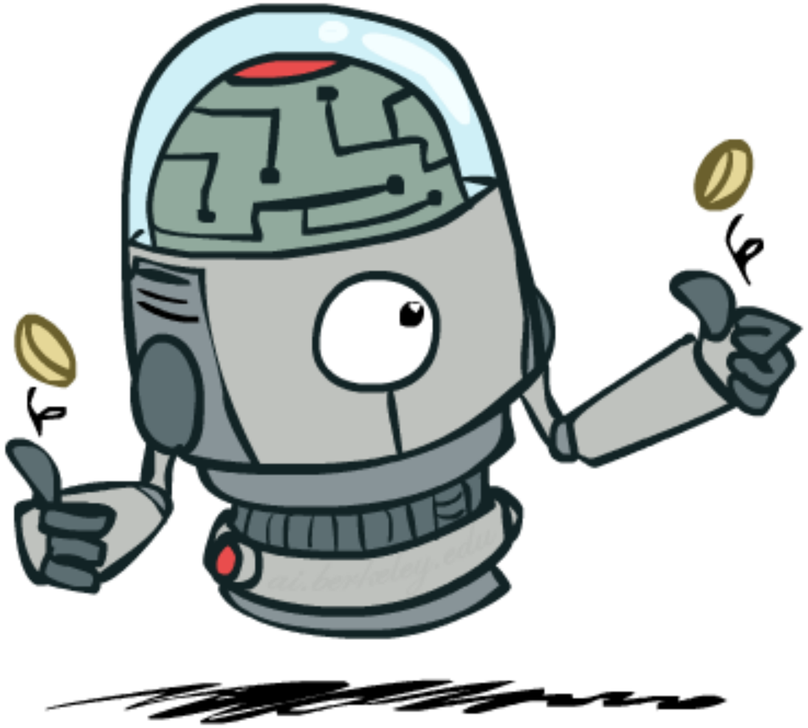
R	P
sun	0.8
rain	0.2

$P(D|W)$

D	W	P
wet	sun	0.1
dry	sun	0.9
wet	rain	0.7
dry	rain	0.3

- What is P(W | dry) ?

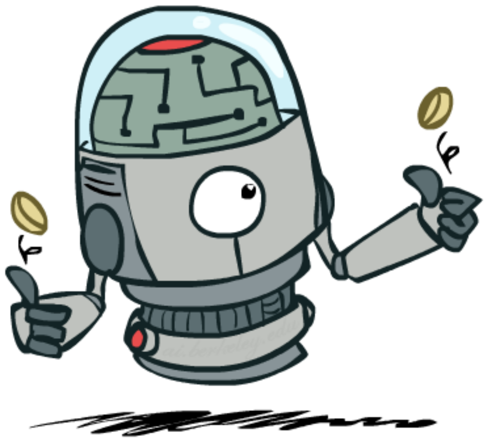
Independence



Independence

- Two variables are *independent* if:

$$\forall x, y : P(x, y) = P(x)P(y)$$



  - This says that their joint distribution *factors* into a product two simpler distributions
  - Another form: $\forall x, y : P(x|y) = P(x)$
  - We write: $X \perp\!\!\!\perp Y$
- Independence is a simplifying *modeling assumption*
  - What could we assume for {Weather, Traffic, Cavity, Toothache}?

Example: Independence?

$P(T)$

T	P
hot	0.5
cold	0.5

$P_1(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$P_2(T, W)$

T	W	P
hot	sun	0.3
hot	rain	0.2
cold	sun	0.3
cold	rain	0.2

$P(W)$

W	P
sun	0.6
rain	0.4

Example: Independence

- N fair, independent coin flips:

$P(X_1)$

H	0.5
T	0.5

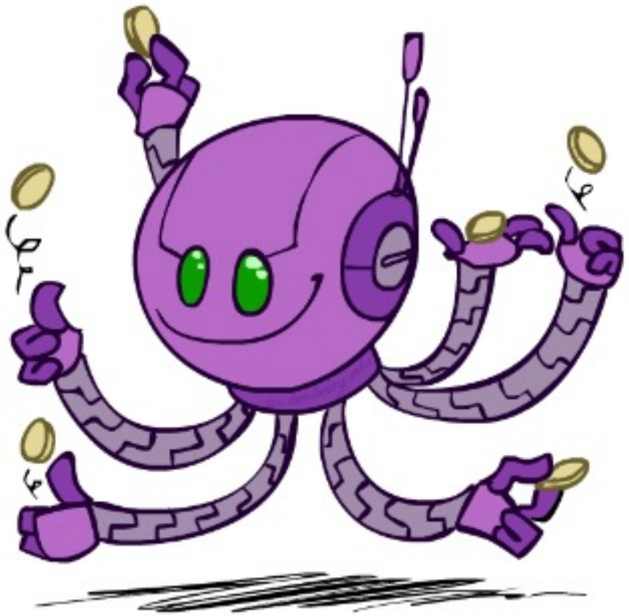
$P(X_2)$

H	0.5
T	0.5

$\ldots$

$P(X_n)$

H	0.5
T	0.5



$P(X_1, X_2, \ldots X_n)$

2^n

Conditional Independence

- Unconditional (absolute) independence very rare
- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments.
- X is conditionally independent of Y given Z

$$X \perp\!\!\!\perp Y | Z$$

if and only if:

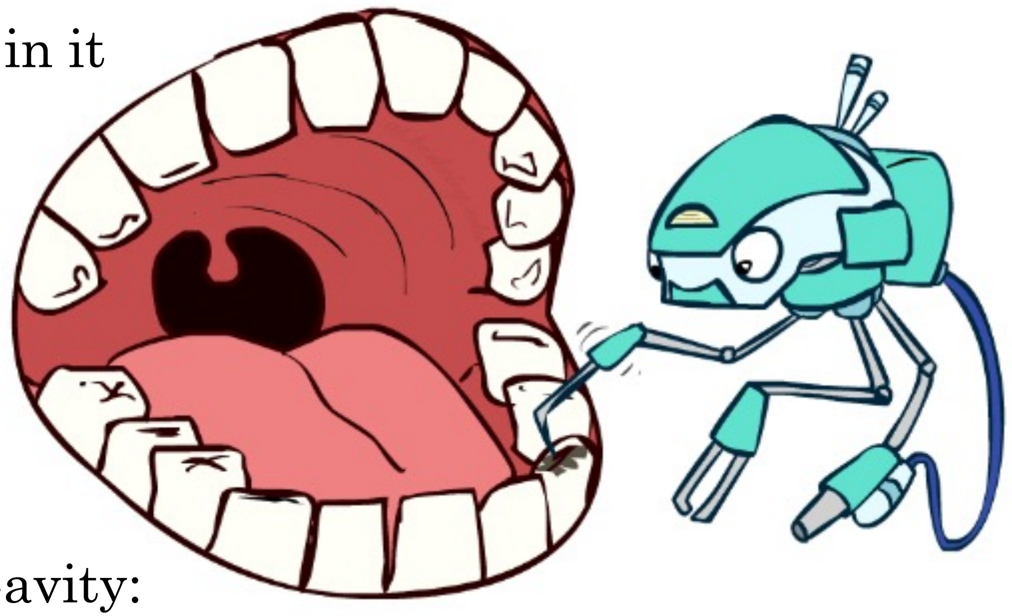
$$\forall x, y, z : P(x, y|z) = P(x|z)P(y|z)$$

or, equivalently, if and only if

$$\forall x, y, z : P(x|z, y) = P(x|z)$$

Conditional Independence

- P(Toothache, Cavity, Catch)
- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
  - $P(+catch \mid +toothache, +cavity) = P(+catch \mid +cavity)$
- The same independence holds if I don't have a cavity:
  - $P(+catch \mid +toothache, -cavity) = P(+catch \mid -cavity)$
- Catch is *conditionally independent* of Toothache given Cavity:
  - $P(Catch \mid Toothache, Cavity) = P(Catch \mid Cavity)$



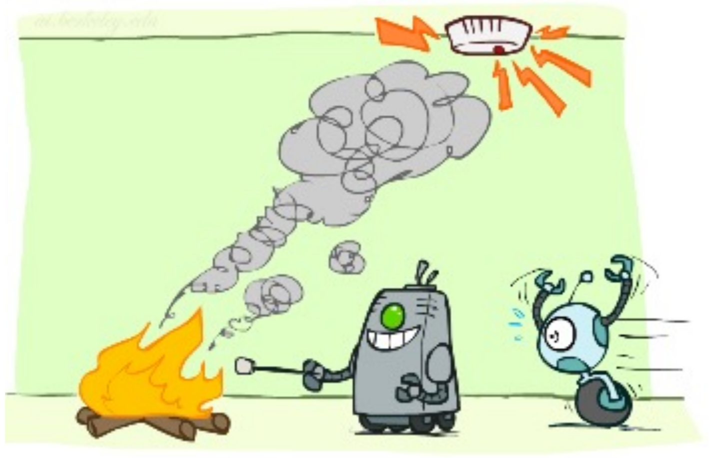
- Equivalent statements:
  - $P(Toothache \mid Catch, Cavity) = P(Toothache \mid Cavity)$
  - $P(Toothache, Catch \mid Cavity) = P(Toothache \mid Cavity)\, P(Catch \mid Cavity)$
  - One can be derived from the other easily

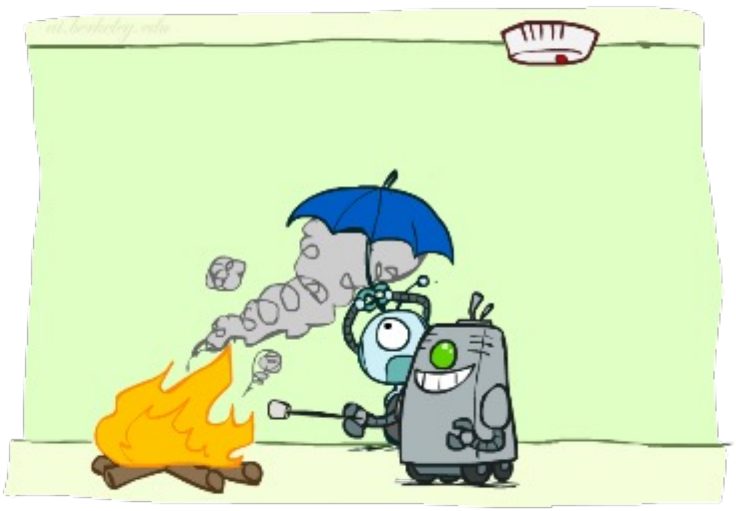
Conditional Independence

- What about this domain:
  - Traffic
  - Umbrella
  - Raining

Conditional Independence

- What about this domain:
  - Fire
  - Smoke
  - Alarm





Conditional Independence and the Chain Rule

- Chain rule: $P(X_1, X_2, \ldots X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2)\ldots$

- Trivial decomposition:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain}, \text{Traffic})$$

- With assumption of conditional independence:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain})$$

- Bayes'nets / graphical models help us express conditional independence assumptions

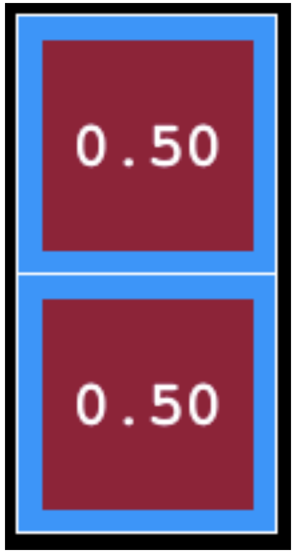
Ghostbusters Chain Rule

- Each sensor depends only on where the ghost is

- That means, the two sensors are conditionally independent, given the ghost position

- T: Top square is red
  B: Bottom square is red
  G: Ghost is in the top

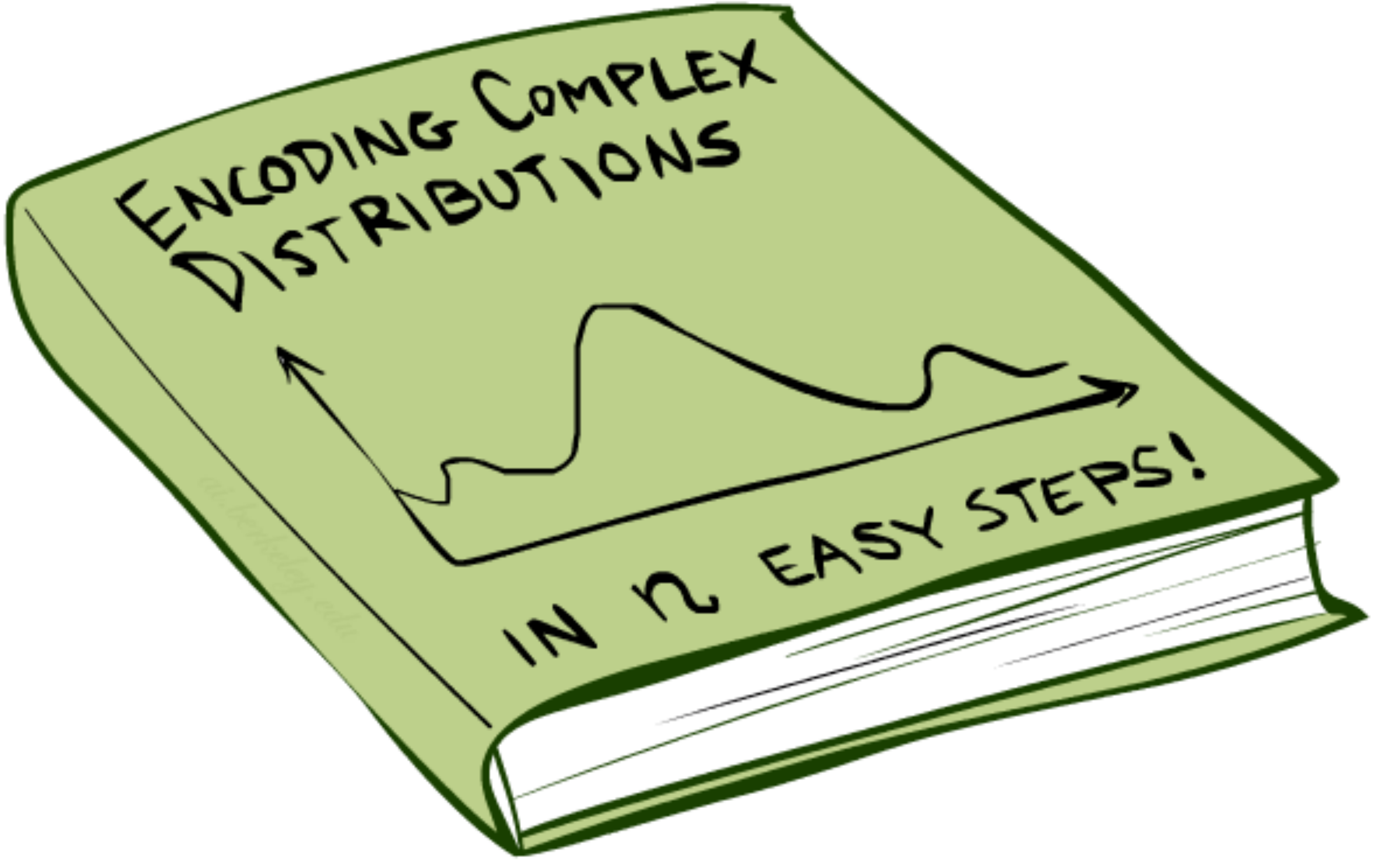
- Givens:
  $P(+g) = 0.5$
  $P(-g) = 0.5$
  $P(+t \mid +g) = 0.8$
  $P(+t \mid -g) = 0.4$
  $P(+b \mid +g) = 0.4$
  $P(+b \mid -g) = 0.8$



$P(T,B,G) = P(G)\, P(T|G)\, P(B|G)$

T	B	G	P(T,B,G)
+t	+b	+g	0.16
+t	+b	-g	0.16
+t	-b	+g	0.24
+t	-b	-g	0.04
-t	+b	+g	0.04
-t	+b	-g	0.24
-t	-b	+g	0.06
-t	-b	-g	0.06

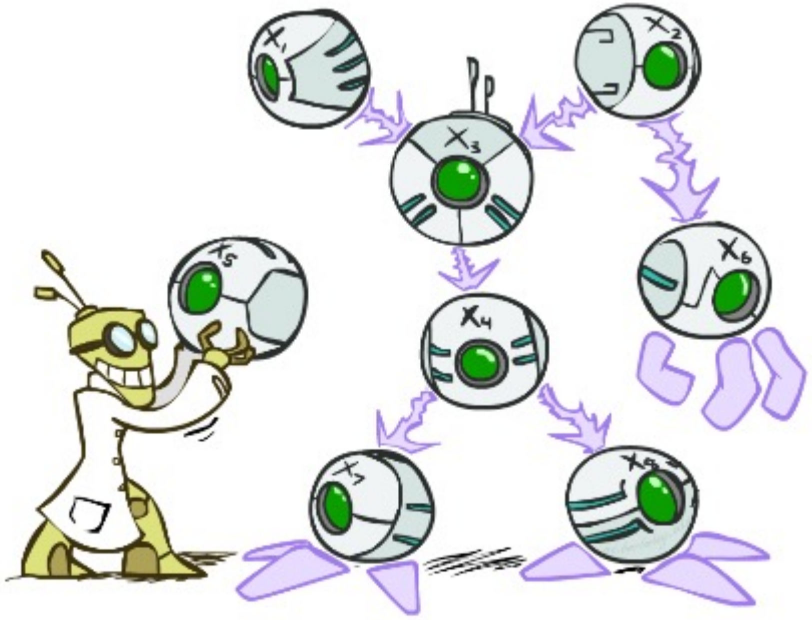
Bayes' Nets: Big Picture



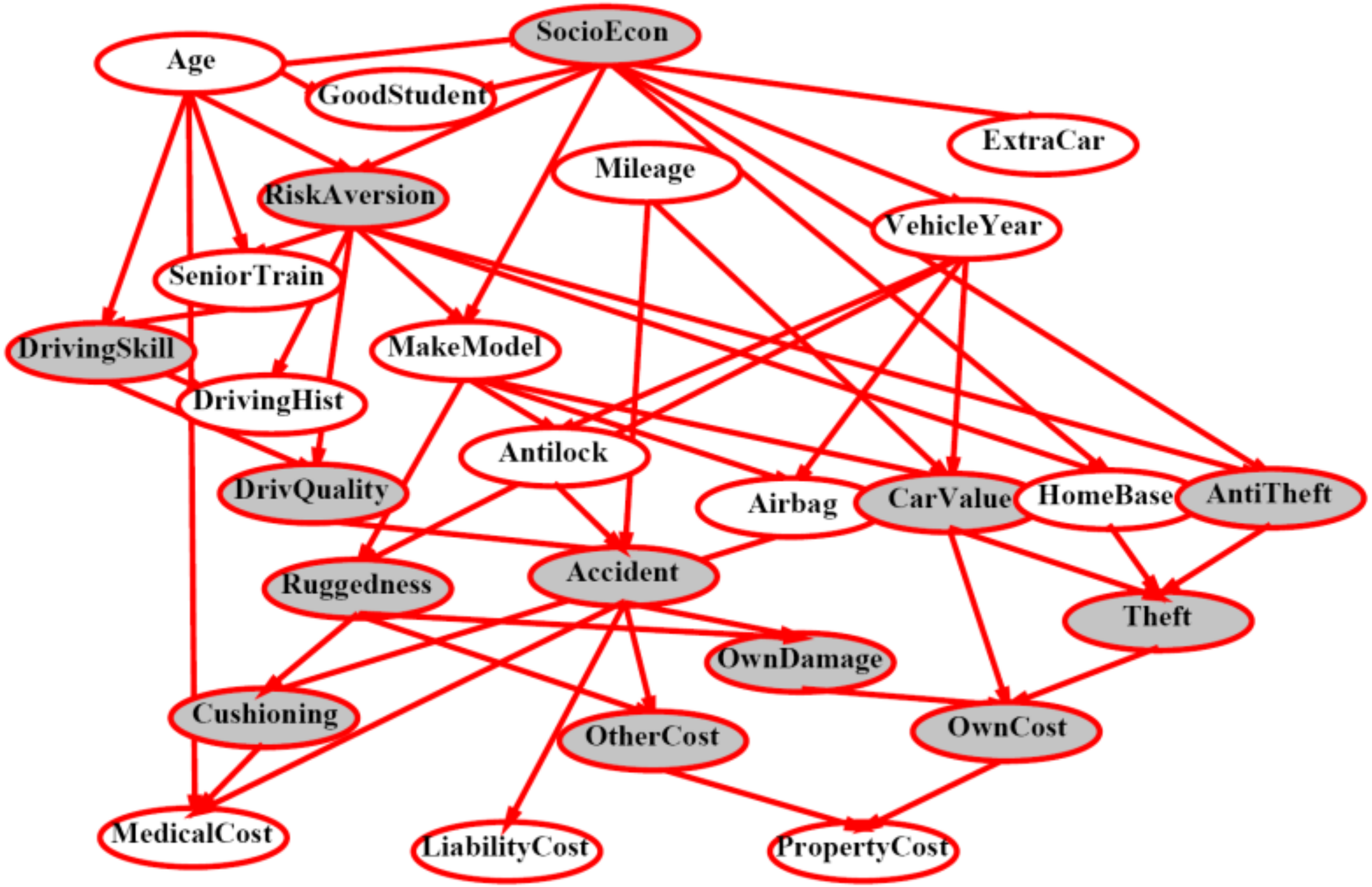
Bayes' Nets: Big Picture

- Two problems with using full joint distribution tables as our probabilistic models:
  - Unless there are only a few variables, the joint is WAY too big to represent explicitly
  - Hard to learn (estimate) anything empirically about more than a few variables at a time

- Bayes' nets: a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
  - More properly called graphical models
  - We describe how variables locally interact
  - Local interactions chain together to give global, indirect interactions
  - For about 10 min, we'll be vague about how these interactions are specified



Example Bayes' Net: Insurance



Example Bayes' Net: Car