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Written Homework 4

Q1) Bayes Nets: Independence

1. X6 ⊥⊥ X1 | X2, X4: **True.** In this case, X2→X4←X1→X6. There is no active path between X6 and X1 given X2 and X4 because X2→X4←X1 forms a V-structure, and conditioning on the middle node (X4) blocks the path.

2. X6 ⊥⊥ X9 | X4: **True.** In this case, X4→X6→X10←X5→X9. The path between X6 and X9 is blocked by the evidence X4, and there are no other active paths.

3. X3 ⊥⊥ X9 | X8: **False.** In this case, X3→X7→X8→X9. The path between X3 and X9 is not blocked by the evidence X8, so they are not independent.

4. X1 ⊥⊥ X2 | X6: **False.** In this case, X1→X4←X2→X5→X10←X6. The path between X1 and X2 is not blocked by the evidence X6, so they are not independent.

5. X4 ⊥⊥ X8 | X3, X7: **True.** In this case, X3→X7→X4→X6→X10←X5→X8→X9. The path between X4 and X8 is blocked by the evidence X3 and X7, so they are independent.

Q2 Bayes Nets: Inference

given

table 1:
A | P(A)
0 | .200
1 | .800

table 2:
B | A | P(B|A)
0 | 0 | .400
1 | 0 | .600
0 | 1 | .200
1 | 1 | .800

table 3:
C | B | P(C|B)
0 | 0 | .600
1 | 0 | .400
0 | 1 | .600
1 | 1 | .400
table 4:
D | B | P(D|B)

0 | 0 | .800
1 | 0 | .200
0 | 1 | .600
1 | 1 | .400

Table 5:
E | C | D | P(E|C,D)
0 | 0 | 0 | .200
1 | 0 | 0 | .800
0 | 1 | 0 | .600
1 | 1 | 0 | .400
0 | 0 | 1 | .800
1 | 0 | 1 | .200
0 | 1 | 1 | .800
1 | 1 | 1 | .200

1.1) the conditional probability P(B=1|E=1), P(B=1|E=1) using the variable elimination method:

First, we remove all data that doesn’t corresponded with our evidence b=1 e=1.

<u>table 1:</u>
<u>A | P(A)</u>
<u>0 | .200</u>
<u>1 | .800</u>

<u>table 2:</u>
<u>B | A | P(B|A)</u>
0 | 0 | .400
1 | 0 | .600
0 | 1 | .200
1 | 1 | .800

table 3:
C | B | P(C|B)
0 | 0 | .600
1 | 0 | .400
0 | 1 | .600
1 | 1 | .400

table 4:
D | B | P(D|B)
0 | 0 | .800
1 | 0 | .200
0 | 1 | .600
1 | 1 | .400

Table 5:

E	C	D	P(E|C,D)
1	0	0	.800
1	1	0	.400
1	0	1	.200
1	1	1	.200

A elimination.
P(A) , P(B=1|A) => P(B)

A	P(A)
0	.200
1	.800

table 2:

B	A	P(B|A)
0	0	.400
1	0	.600
0	1	.200
1	1	.800

B	A	P(B, A)
1	0	(.6 *.2) = .12
1	1	(.8 * .8) = .64
0	0	(.2 * .4) = .08
0	1	(.2 *.8) = .16

Sum out A

B	P(B)
1	.12+ .64 = .76
0	.08 +.16 = .24

C elimination.
P(B) , P(C|B=1) , P(E=1| C,D) => P(C, B=1, E=1, D)

B	C	D	E	P(B=1, C, D, E=1)
1	0	0	1	(.76 *.6 * .8) = .3648
1	0	1	1	(.76 * .6 * .2) = .0912
1	1	0	1	(.76 * .4 * .4) = .1216
1	1	1	1	(.76 * .4 * .2) = .0608
0	0	0	1	(.24 * .6 * .8) =. 1152
0	0	1	1	(.24 * .6 * .2) = .0288
0	1	0	1	(.24 * .4 * .4) = .0384
0	1	1	1	(.24 * .4 * .2) = .0192

Sum out C

B | D | E | P(B=1, D, E=1)
1 | 0 | 1 | (.3648 +.1216) = .4864
1 | 1 | 1 | (.0912+.0608) = .152
0 | 0 | 1 | (.1152 +.0384 = .1536
0 | 1 | 1 | (.0192 + .0288) = .048

D elimination
P(D|B) , P(B=1, D, E=1) => P(B=1, D, E=1)

B | D | E | P(B=1, D, E=1)
1 | 0 | 1 | (.4864 * .6) = .29184
1 | 1 | 1 | (.152 * .4) = .0608
0 | 0 | 1 | (.1536 * .8) = .12288
0 | 1 | 1 | (.048 * .2) = .0096

Sum out D
B | E | P(B=1, E=1)
1 | 1 | (.29184 + .0608) = .35264
0 | 1 | (.12288 + .0096) = .13248

Normalize

B | E | P(B=1|E=1)
1 | 1 | .35264/ (.35264 +.13248) = .726913
0 | 1 | .13248/ (.35264 +.13248) = .273087

P(B=1|E=1) = .726913

2) Compute P (A = 1 | C = 0, E = 0). Similarly, in this question, we first keep table entries that are consistent with the evidence C = 0, E = 0. We then perform variable elimination on hidden variables B, D

First, we remove all data that doesn’t corresponded with our evidence C=0 E=0.

table 1:

A	P(A)
0	.200
1	.800

table 2:

B	A	P(B|A)
0	0	.400
1	0	.600
0	1	.200
1	1	.800

table 3:

C	B	P(C|B)
0	0	.600
0	1	.600

table 4:

D	B	P(D|B)
0	0	.800
1	0	.200
0	1	.600
1	1	.400

Table 5:

E	C	D	P(E|C,D)
0	0	0	.200
0	0	1	.800

Eliminate B

P(B|A) , P(C|B) , P(D|B) => P(A,B,C,D)

A	B	C	D	P(A,B,C)
1	0	0	0	((.2*.6 *.8)= .096
1	1	0	0	(.8 * .6 * .6) = .288
0	0	0	0	(.4 * .6 * .8) = .192
0	1	0	0	(.6*.6*.6) = .216
1	0	0	1	(.2*.6*.2)= .024
1	1	0	1	(.8 * .6 * .4) = .192
0	0	0	1	(.4 * .6 * .2) = .048
0	1	0	1	(.2 * .6 * .4) (.6*.6*.4) = .144

Sum out B

A	C	D	P(A,C,D)
0	0	1	(.096+ .048) =.192
1	0	1	(.192 + .024) = .216
0	0	0	(.192 + .216) = .408
1	0	0	(.288 +.096) = .384

P(E=0|C=0 ,D), P(A,C,D) => P(A,C=0,D, E=0)

Table 5:

E	C	D	$P(E \mid C,D)$
0	0	0	.200
0	0	1	.800

A	D	C	E	P(A,D,C, E)
0	1	0	0	(.192 * .800) = .1536
1	1	0	0	(.216 * .800) = .1728
0	0	0	0	(.408 * .200) = .0816
1	0	0	0	(.384 * .200) = .0688

Sum out D

A	C	E	P(A,C, E)
0	0	0	(.1536+.0816) = .2352
1	0	0	(.1728+.0688) = .2496

Normalize

A	C	E	$(P(A \mid C = 0, E = 0)$
0	0	0	.2352 / (.2352 +.2496) = .190661
1	0	0	.2496 / (.2352 +.2496) = .809339

P (A = 1 | C = 0, E = 0) = .809339

Q3 Bayes Net Sampling

Q3.1

Let's use these samples to generate values for A, B, C, D, and E.

1. Sample A:
   - Using the first sample (0.320) and the probability distribution from Table 1:
     - $P(A=0)=0.200$
     - $P(A=1)=0.800$
   - Since $0.320 > 0.200$, set $A=1$
2. Sample B:
   - Using the second sample (0.037) and the conditional probability distribution from Table 2 given $A=1$:
     - $P(B=0|A=1)=0.200$
     - $P(B=1|A=1)=0.800$
   - Since $0.037 < 0.200$, set $B=0$.
3. Sample C:
   - Using the third sample (0.303) and the conditional probability distribution from Table 3 given $B=0$:
     - $P(C=0|B=0)=0.600$
     - $P(C=1|B=0)=0.400$
   - Since $0.303 < 0.600$, set $C=0$.
4. Sample D:
   - Using the fourth sample (0.318) and the conditional probability distribution from Table 4 given $B=0$:
     - $P(D=0|B=0)=0.800$
     - $P(D=1|B=0)=0.200$
   - Since $0.318 < 0.800$, set $D=0$.
5. Sample E:
   - Using the fifth sample (0.032) and the conditional probability distribution from Table 5 given $C=0,D=0$:
     - $P(E=0|C=0,D=0)=0.200$
     - $P(E=1|C=0,D=0)=0.800$
   - Since $0.032 < 0.200$, set $E=0$.

The resulting assignment is: A=1, B=0, C=0, D=0, E=0

Now, let's check the evidence $B=1,E=1$:

- $B=0$ (does not match evidence, reject the sample).

Since the sample was rejected at the second variable (B), we mark the assignment for C,D,E as "none" as we don't need to consider their values.

The rejected variable is B.

Q3.2

Let's go step by step:

1. **Sample A:** Use the first value from the table (0.249) to determine the value of A.
    1. $P(A=0)=0.200$ and $P(A=1)=0.800$.
    2. Since $0.249 > 0.200$, set $A=1$.
2. **Sample B:** to determine the value of B given A = 1. Given B=1 thus no sample
    1. set B=1
    2. weight $P(B=1|A=1) = .8$
3. **Sample C:** Use the third value from the table (0.299) to determine the value of C given B = 0.
    1. $P(C=0|B=0)=0.600$ and $P(C=1|B=0)=0.400$.
    2. Since $0.299 < 0.600$, set $C=0$.
4. **Sample D:** Use the fourth value from the table (0.773) to determine the value of D given B = 0.
    1. $P(D=0|B=0)=0.800$ and $P(D=1|B=0)=0.200$.
    2. Since $0.773 < 0.800$, set $D=0$.
5. **Sample E:**) to determine the value of E given C = 0 and D = 0. Given E=1 thus no sample
    1. set E=1.
    2. weight $P(E=1|C=0\ D=0) = .8$

Now we have a sample: A=1,B=1,C=0,D=0,E=1

Next, calculate the weight:

$\text{Weight}=1\times0.800\times0.800=0.640$

So, the sample A=1,B=1,C=0,D=0,E=1 has a weight of 0. 640.

Q3.3

3.3.1
Gibbs Sampling Step 1: Update B

Current Sample:

- A=1
- B=0
- C=1
- D=1
- E=1

Step 2: Sample B given the values.
P(B=0) = P(B|A) * P(C|B) *P(D|B)
.2*.4*.2* =.016
P(B=1) = P(B|A) * P(C|B) *P(D|B)
.8*.4*.4=.128

Normalize
P(B=0) =.016/(.016 +.128) = .1111
P(B=1) = .128/(.016 +.128) = .8889
.320 > .1111 = B =1

Step 3: Keep C unchanged

Step 4: Keep D unchanged

Step 5: Keep E unchanged as it's evidence.

The new sample after updating BB would be (A=1,B=1,C=1,D=1,E=1)

3.3.2
Gibbs Sampling Step 2: Update D

Current Sample:

- A=1
- B=1
- C=1
- D=1
- E=1

Step 1: Keep A unchanged

Step 2: Keep B unchanged

Step 3: Keep C unchanged

Step 4: Sample D given the values
P(D=0) = P(D|B) * P(E|C D)
.6*.4 = .24
P(D=1) = P(D|B) * P(E|C D)
.4*.2 = .08

Normalize to get probabilities and sample D.
P(B=0) =.24/(.08 +.24) = .75
P(B=1) = .08/(.08 +.24) = .25

.037 < .25 = D =0

Step 5: Keep E unchanged as it's evidence.

The new sample after updating D would be (A=1,B=1,C=,D=0,E=1)