

CSCI 3155: Lab Assignment 3

Learning Goals. The primary learning goals of this lab are to understand the following:

- how to read a formal specification of a language semantics;
- how dynamic scoping arises;
- the distinction between a big-step and a small-step operational semantics;
- evaluation order; and
- substitution and program transformation;

PL Ideas Semantics: evaluation order. Operational semantics. Substitution and program transformation.

FP Skills Iteration. Introduction to higher-order functions.

Concretely, we will extend JAVASCRIPTY with recursive functions and implement two interpreters. The first will be a big-step interpreter that is an extension of Lab 2 but implements dynamic scoping “by accident.” The second will be a small-step interpreter that exposes evaluation order by *iterating* a single-step transition relation and implements static scoping by substitution.

General Guidelines. During recitation find a partner for this lab assignment (should be different for every lab assignment). You will work on this assignment closely with your partner. However, note that **each student is individually responsible** for completing the assignment so that you can do well in your interview.

You are welcome to talk about these questions beyond your teams. However, we ask that you code in pairs. See the collaboration policy for details, including the following:

*Bottom line, feel free to use resources that are available to you as long as the use is **reasonable** and you **cite** them in your submission. However, copying answers directly or indirectly from solution manuals, web pages, or your peers is certainly unreasonable.*

Also, recall the evaluation guideline from the course syllabus.

Both your ideas and also the clarity with which they are expressed matter—both in your English prose and your code!

We will consider the following criteria in our grading:

- How well does your submission answer the questions? *For example, a common mistake is to give an example when a question asks for an explanation. An example may be useful in your explanation, but it should not take the place of the explanation.*
- How clear is your submission? *If we cannot understand what you are trying to say, then we cannot give you points for it. Try reading your answer aloud to yourself or a friend; this technique is often a great way to identify holes in your reasoning. For code, not every program that “works” deserves full credit. We must be able to read and understand your intent. Make sure you state any pre-conditions or invariants for your functions (either in comments, as assertions, or as require clauses as appropriate).*

Try to make your code as concise and clear as possible. Challenge yourself to find the most crisp, concise way of expressing the intended computation. This may mean using ways of expression computation currently unfamiliar to you.

Finally, make sure that your file compiles and runs via `sbt test`. A program that does not compile will *not* be graded—no interview will be conducted.

Submission Instructions. We are using the following tools:

- Github for assignment distribution and submission;
- INGINious for auto-testing submission;
- Moodle for archival and as a backup for assignment submission.

1. You will be editing and submitting the the following files to Github:

- `src/main/scala/jsy/student/Lab3.scala` with your solution to the coding exercises;
- `src/test/scala/jsy/student/Lab3Spec.scala` with your additional tests; and
- `lab3-writeup.pdf` or `lab3-writeup.md` for a pdf or a Markdown document that should be pushed to the root directory of your repository with your response to the written questions (scanned, clearly legible handwritten write-ups are acceptable).
You will not get credit for write-ups in any other file format.
- `lab3-yourteamname.jsy` with a challenging test case for your JAVASCRIPTY interpreter.

You are also likely to edit `src/main/scala/jsy/student/Lab3Worksheet.sc` for any scratch work.

You need to have a Github identity and should have your full name in your Github profile so that we can associate you with your submissions if there are any confusions.

Following good git practice, please make commits in small bits corresponding to completing small conceptual parts and push often so that your progress is evident. We expect that you have some familiarity with git from prior courses or experience. If not, please discuss with your classmates and the course staff (e.g., via Piazza).

2. At any point, you may submit your `Lab3.scala` file to INGINious for auto-testing. You need to submit to INGINious for the auto-testing part of your score, as well as to continue to the interview.
3. Sign-up for an interview slot for an evaluator. To fairly accommodate everyone, the interview times are strict and **will not be rescheduled**. Missing an interview slot means missing the interview evaluation component of your lab score. Please take advantage of your interview time to maximize the feedback that you are able receive. Arrive at your interview ready to show your team's implementation and your written responses. Implementations that do not compile and run will not be evaluated.
4. As a backup for any submission issues and archiving, please help us by also uploading to the moodle exactly the files named above, that is,
 - `Lab3.scala`
 - `Lab3Spec.scala`
 - `lab3-writeup.pdf` or `lab3-writeup.md`
 - `lab3-yourteamname.jsy`

Getting Started. First, form a team of two and pick a team name. For our bookkeeping, please prefix your team name with `lab3-` (e.g., `lab3-anatomists`).

You must work in teams of two, and you will form teams in lab section. If you miss lab section on the day teams are formed, you need to find a partner on your own. If you really, really cannot find a partner, then please contact the course staff (via Piazza).

Then, log into moodle and follow the Github Classroom link for setting up your Lab 3 repository with your team name. The first person will create the team, and the second person will select the team name from the existing team names.

If you would like to look at the code before getting your own copy for submission, you may go to <https://github.com/csci3155/pppl-lab3>.

Checkpoint. The checkpoint is to encourage you to start the coding portion of the assignment early and it requires you to submit your partial solution on INGINious a week before the assignment is due. You do not need to complete all coding a week early but we want you to start working on it. This means that submitting the empty template that fails all tests is **not sufficient**. Failing to submit to the checkpoint will prevent you from proceeding to the interview. However, as long as you pass the checkpoint, this early score from the checkpoint will not affect your grade for the assignment or your overall grade for the course.

1. **Feedback.** Complete the survey on the linked from the moodle after completing this assignment. Any non-empty answer will receive full credit.
2. **JavaScripty Interpreter: Tag Testing, Recursive Functions, and Dynamic Scoping.**

We now have the formal tools to specify exactly how a JAVASCRIPTY program should behave. Unless otherwise specified, we will continue to try to match JavaScript semantics as implemented by Node.js/Google's V8 JavaScript Engine. Thus, it is still useful to write little test

expressions	$e ::= x \mid n \mid b \mid str \mid \text{undefined} \mid uop\ e_1 \mid e_1\ bop\ e_2$ $\mid e_1\ ?\ e_2\ :\ e_3 \mid \text{const } x = e_1;\ e_2 \mid \text{console.log}(e_1)$ $\mid p(x) \Rightarrow e_1 \mid e_1(e_2) \mid \text{typeerror}$
values	$v ::= n \mid b \mid \text{undefined} \mid str \mid p(x) \Rightarrow e_1 \mid \text{typeerror}$
unary operators	$uop ::= - \mid !$
binary operators	$bop ::= , \mid + \mid - \mid * \mid / \mid === \mid !== \mid < \mid <= \mid > \mid >= \mid \&\& \mid $
variables	x
numbers (doubles)	n
booleans	$b ::= \text{true} \mid \text{false}$
strings	str
function names	$p ::= x \mid \varepsilon$
value environments	$E ::= \cdot \mid E[x \mapsto v]$

Figure 1: Abstract Syntax of JAVASCRIPTY

statements	$s ::= \text{const } x = e \mid e \mid \{ s_1 \} \mid ; \mid s_1\ s_2$
expressions	$e ::= \dots \mid (e_1)$ $\mid \text{const } x = e_1;\ e_2$ $\mid p(x) \Rightarrow e_1 \mid x \Rightarrow e_1 \mid x \Rightarrow blk \mid (x) \Rightarrow e_1 \mid (x) \Rightarrow blk$ $\mid \text{function } p(x)\ blk$
function body blocks	$blk ::= \{ s\ \text{return } e_1 \}$

Figure 2: Concrete Syntax of JAVASCRIPTY

JavaScript programs and run it through Node.js to see how the test should behave. Finding bugs in the JAVASCRIPTY specification with respect to JavaScript is certainly deserving of extra credit.

In this lab, we extend JAVASCRIPTY with recursive functions. This language is very similar to the LETREC language in Section 3.4 of Friedman and Wand.

For this question, we try to implement functions as an extension of Lab 2 in the most straightforward way. What we will discover is that we have made a historical mistake and have ended up with a form of dynamic scoping.

The syntax of JAVASCRIPTY for this lab is given in Figure 1. Note that the grammar specifies the abstract syntax using notation borrowed from the concrete syntax. The new constructs are **highlighted**. We have function expressions $p(x) \Rightarrow e_1$ and function calls $e_1(e_2)$.

In a function expression, the function name p can either be an identifier or empty. When the identifier for the function name is present, it can be used for recursion. For simplicity, all functions are one argument functions. Since functions are first-class values, we can get multi-argument functions via currying.

We have also added a “marker” `typeerror` to the expression language. This marker is not part of the source language but is used in our definition of the evaluation judgment form. We discuss this in more detail further below.

As before, the concrete syntax accepted by the parser is slightly less flexible than the abstract syntax in order to match the syntactic structure of JavaScript. For function expres-

```

case class Function(p: Option[String], x: String, e1: Expr) extends Expr
  Function(p, x, e1)    p(x) => e1
case class Call(e1: Expr, e2: Expr) extends Expr
  Call(e1, e2)    e1(e2)

```

Figure 3: Representing in Scala the abstract syntax of JAVASCRIPTY. After each **case class** or **case object**, we show the correspondence between the representation and the concrete syntax.

sions, only the anonymous version is allowed using the arrow syntax (\Rightarrow), which may optionally have parentheses around the parameter and whose body may be an expression e or a block blk . A function body block is surrounded by curly braces (i.e., $\{ \}$) and consists of a statement s for **const** bindings followed by a **return** with an expression e_1 .

The **function** keyword syntax may have a function name but whose body must be a block.

An abstract syntax tree representation is provided for you in `ast.scala`. We also provide a parser and main driver for testing. The correspondence between the concrete syntax and the abstract syntax representation is shown in Figure 3.

A big-step operational semantics of JAVASCRIPTY is given in Figure 4. This figure may be one of the first times that you are reading a formal semantics of a programming language. It may seem daunting at first, but it will become easier with practice. This lab is such an opportunity to practice.

A formal semantics enables us to describe the semantics of a programming language clearly and concisely. The initial barrier is getting used to the meta-language of judgment forms and inference rules. However, once you cross that barrier, you will see that we are telling you exactly how to implement the interpreter—it will almost feel like cheating!

In Figure 4, we define the judgment form $E \vdash e \Downarrow v$, which says informally, “In value environment E , expression e evaluates to value v .” This relation has three parameters: E , e , and v . You can think of the other parts of the judgment as just punctuation. This judgment form corresponds directly to the `eval` function that we are asked to implement (not a coincidence). It similarly has three parts:

```
def eval(env: Env, e: Expr): Expr
```

It takes as input a value environment env (E) and an expression e (e) returns a value v .

It is very informative to compare your Scala code from Lab 2 with the inference rules that define $E \vdash e \Downarrow v$. One thing you should observe is that all of the rules are implemented, except for `EVALCALL`, `EVALCALLREC`, and part of `EVALEQUALITY1` and `EVALEQUALITY2`. In essence, implementing those rules is your task for this question.

In Lab 2, all expressions could be evaluated to something (because of conversions). With functions, we encounter one of the very few run-time errors in JavaScript: trying to call something that is not a function. In JavaScript and in JAVASCRIPTY, calling a non-function raises a run-time error. In the formal semantics, we model this with evaluating to the “marker” `typeerror`.

$E \vdash e \Downarrow v$

$\frac{\text{EVALVAR}}{E \vdash x \Downarrow E(x)}$	$\frac{\text{EVALVAL}}{E \vdash v \Downarrow v}$	$\frac{\text{EVALNEG}}{E \vdash -e_1 \Downarrow n'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ n' = -\text{toNumber}(v_1) \end{array}$	$\frac{\text{EVALNOT}}{E \vdash !e_1 \Downarrow b'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ b' = \neg \text{toBoolean}(v_1) \end{array}$
$\frac{\text{EVALSEQ}}{E \vdash e_1, e_2 \Downarrow v_2} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E \vdash e_2 \Downarrow v_2 \end{array}$	$\frac{\text{EVALPLUSNUMBER}}{E \vdash e_1 + e_2 \Downarrow n'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E \vdash e_2 \Downarrow v_2 \\ n' = \text{toNumber}(v_1) + \text{toNumber}(v_2) \\ v_1 \neq \text{str}_1 \quad v_2 \neq \text{str}_2 \end{array}$		
$\frac{\text{EVALPLUSSTRING}_1}{E \vdash e_1 + e_2 \Downarrow \text{str}'} \quad \begin{array}{l} E \vdash e_1 \Downarrow \text{str}_1 \\ E \vdash e_2 \Downarrow v_2 \\ \text{str}' = \text{str}_1 + \text{toString}(v_2) \end{array}$		$\frac{\text{EVALPLUSSTRING}_2}{E \vdash e_1 + e_2 \Downarrow \text{str}'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E \vdash e_2 \Downarrow \text{str}_2 \\ \text{str}' = \text{toString}(v_1) + \text{str}_2 \end{array}$	
$\frac{\text{EVALARITH}}{E \vdash e_1 \text{ bop } e_2 \Downarrow n'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E \vdash e_2 \Downarrow v_2 \\ n' = \text{toNumber}(v_1) \text{ bop } \text{toNumber}(v_2) \\ \text{bop} \in \{-, *, /\} \end{array}$			
$\frac{\text{EVALINEQUALITYNUMBER}_1}{E \vdash e_1 \text{ bop } e_2 \Downarrow b'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E \vdash e_2 \Downarrow v_2 \\ v_1 \neq \text{str}_1 \\ b' = \text{toNumber}(v_1) \text{ bop } \text{toNumber}(v_2) \\ \text{bop} \in \{<, <=, >, >=\} \end{array}$		$\frac{\text{EVALINEQUALITYNUMBER}_2}{E \vdash e_1 \text{ bop } e_2 \Downarrow b'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E \vdash e_2 \Downarrow v_2 \\ v_2 \neq \text{str}_2 \\ b' = \text{toNumber}(v_1) \text{ bop } \text{toNumber}(v_2) \\ \text{bop} \in \{<, <=, >, >=\} \end{array}$	
$\frac{\text{EVALINEQUALITYSTRING}}{E \vdash e_1 \text{ bop } e_2 \Downarrow b'} \quad \begin{array}{l} E \vdash e_1 \Downarrow \text{str}_1 \\ E \vdash e_2 \Downarrow \text{str}_2 \\ b' = \text{str}_1 \text{ bop } \text{str}_2 \\ \text{bop} \in \{<, <=, >, >=\} \end{array}$			
$\frac{\text{EVAEQUALITY}_2}{E \vdash e_1 \text{ bop } e_2 \Downarrow b'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E \vdash e_2 \Downarrow v_2 \\ v_2 \neq p_1(x_2) \Rightarrow e_2 \\ b' = (v_1 \text{ bop } v_2) \\ \text{bop} \in \{==, !=\} \end{array}$		$\frac{\text{EVALANDTRUE}}{E \vdash e_1 \&\& e_2 \Downarrow v_2} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ \text{true} = \text{toBoolean}(v_1) \\ E \vdash e_2 \Downarrow v_2 \end{array}$	
$\frac{\text{EVALANDFALSE}}{E \vdash e_1 \&\& e_2 \Downarrow v_1} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ \text{false} = \text{toBoolean}(v_1) \end{array}$	$\frac{\text{EVALORTTRUE}}{E \vdash e_1 e_2 \Downarrow v_1} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ \text{true} = \text{toBoolean}(v_1) \end{array}$	$\frac{\text{EVALORFALSE}}{E \vdash e_1 e_2 \Downarrow v_2} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ \text{false} = \text{toBoolean}(v_1) \\ E \vdash e_2 \Downarrow v_2 \end{array}$	
$\frac{\text{EVALPRINT}}{E \vdash \text{console.log}(e_1) \Downarrow \text{undefined}} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ v_1 \text{ printed} \end{array}$		$\frac{\text{EVALIFTRUE}}{E \vdash e_1 ? e_2 : e_3 \Downarrow v_2} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ \text{true} = \text{toBoolean}(v_1) \\ E \vdash e_2 \Downarrow v_2 \end{array}$	
$\frac{\text{EVALIFFALSE}}{E \vdash e_1 ? e_2 : e_3 \Downarrow v_3} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ \text{false} = \text{toBoolean}(v_1) \\ E \vdash e_3 \Downarrow v_3 \end{array}$		$\frac{\text{EVALCONST}}{E \vdash \text{const } x = e_1; e_2 \Downarrow v_2} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ E[x \mapsto v_1] \vdash e_2 \Downarrow v_2 \end{array}$	
$\frac{\text{EVALCALL}}{E \vdash e_1(e_2) \Downarrow v'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ v_1 = (x) \Rightarrow e' \\ E \vdash e_2 \Downarrow v_2 \\ E[x \mapsto v_2] \vdash e' \Downarrow v' \end{array}$			
$\frac{\text{EVALCALLREC}}{E \vdash e_1(e_2) \Downarrow v'} \quad \begin{array}{l} E \vdash e_1 \Downarrow v_1 \\ v_1 = x_1(x_2) \Rightarrow e' \\ E \vdash e_2 \Downarrow v_2 \\ E[x_1 \mapsto v_1][x_2 \mapsto v_2] \vdash e' \Downarrow v' \end{array}$			

Figure 4: Big-step operational semantics of JAVASCRIPTY (with dynamic scoping).

toNumber(n)	$\stackrel{\text{def}}{=}$	n
toNumber(true)	$\stackrel{\text{def}}{=}$	1
toNumber(false)	$\stackrel{\text{def}}{=}$	0
toNumber(undefined)	$\stackrel{\text{def}}{=}$	NaN
toNumber(str)	$\stackrel{\text{def}}{=}$	parse str
toNumber($p(x) \Rightarrow e_1$)	$\stackrel{\text{def}}{=}$	NaN
toBoolean(n)	$\stackrel{\text{def}}{=}$	false if $n = 0$ or $n = \text{NaN}$
toBoolean(n)	$\stackrel{\text{def}}{=}$	true otherwise
toBoolean(b)	$\stackrel{\text{def}}{=}$	b
toBoolean(undefined)	$\stackrel{\text{def}}{=}$	false
toBoolean(str)	$\stackrel{\text{def}}{=}$	false if $str = ""$
toBoolean(str)	$\stackrel{\text{def}}{=}$	true otherwise
toBoolean($p(x) \Rightarrow e_1$)	$\stackrel{\text{def}}{=}$	true
toString(n)	$\stackrel{\text{def}}{=}$	string of n
toString(true)	$\stackrel{\text{def}}{=}$	"true"
toString(false)	$\stackrel{\text{def}}{=}$	"false"
toString(undefined)	$\stackrel{\text{def}}{=}$	"undefined"
toString(str)	$\stackrel{\text{def}}{=}$	str
toString($p(x) \Rightarrow e_1$)	$\stackrel{\text{def}}{=}$	"function"

Figure 5: Conversion functions. We do not specify explicitly the parsing or string conversion of numbers. The conversion of a function to a string deviates slightly from JavaScript where the source code of the function is returned.

			$E \vdash e \Downarrow v$
$\frac{\text{EVALTYPEERREQUALITY} \quad E \vdash e_1 \Downarrow p_1(x_1) \Rightarrow e'_1 \quad E \vdash e_2 \Downarrow p_2(x_2) \Rightarrow e'_2 \quad bop \in \{==, !=\}}{E \vdash e_1 \text{ bop } e_2 \Downarrow \text{typeerror}}$			$\frac{\text{EVALTYPEERRORCALL} \quad E \vdash e_1 \Downarrow v_1 \quad v_1 \neq p(x) \Rightarrow e'}{E \vdash e_1(e_2) \Downarrow \text{typeerror}}$
$\frac{\text{EVALPROPAGATEUNARY} \quad E \vdash e_1 \Downarrow \text{typeerror}}{E \vdash uope_1 \Downarrow \text{typeerror}}$	$\frac{\text{EVALPROPAGATEBINARY}_1 \quad E \vdash e_1 \Downarrow \text{typeerror}}{E \vdash e_1 \text{ bop } e_2 \Downarrow \text{typeerror}}$	$\frac{\text{EVALPROPAGATEBINARY}_2 \quad E \vdash e_2 \Downarrow \text{typeerror}}{E \vdash e_1 \text{ bop } e_2 \Downarrow \text{typeerror}}$	
$\frac{\text{EVALPROPAGATEPRINT} \quad E \vdash e_1 \Downarrow \text{typeerror}}{E \vdash \text{console.log}(e_1) \Downarrow \text{typeerror}}$	$\frac{\text{EVALPROPAGATEIF} \quad E \vdash e_1 \Downarrow \text{typeerror}}{E \vdash e_1 ? e_2 : e_3 \Downarrow \text{typeerror}}$	$\frac{\text{EVALPROPAGATECONST} \quad E \vdash e_1 \Downarrow \text{typeerror}}{E \vdash \text{const } x = e_1; e_2 \Downarrow \text{typeerror}}$	
$\frac{\text{EVALPROPAGATECALL}_1 \quad E \vdash e_1 \Downarrow \text{typeerror}}{E \vdash e_1(e_2) \Downarrow \text{typeerror}}$		$\frac{\text{EVALPROPAGATECALL}_2 \quad E \vdash e_2 \Downarrow \text{typeerror}}{E \vdash e_1(e_2) \Downarrow \text{typeerror}}$	

Figure 6: Big-step operational semantics of JAVASCRIPTY: Dynamic type error rules.

Such a run-time error is known as a dynamic type error. Languages are called *dynamically typed* when they allow all syntactically valid programs to run and check for type errors during execution.

In our Scala implementation, we will not clutter our Expr type with a typeerror marker. Instead, we will use a Scala exception DynamicTypeError:

```
case class DynamicTypeError(e: Expr) extends Exception
```

to signal this case. In other words, when your interpreter discovers a dynamic type error, it should throw this exception using the following Scala code:

```
throw DynamicTypeError(e)
```

The argument should be the input expression to eval where the type error was detected. One advantage of using a Scala exception for typeerror is that the marker does not need to be propagated explicitly as in the inference rules in Figure 6. In particular, your interpreter will implement the EVALTYPEERROR rules explicitly, but the EVALPROPAGATE rules are implemented implicitly with Scala's exception propagation semantics.

Note in rule EVALEQUALITY_1 and EVALEQUALITY_2 , we disallow equality and disequality checks (i.e., $==$ and $!=$) on function values. If both arguments to a equality or disequality check are function values, then we consider this a dynamic type error. We make this choice to depart from JavaScript semantics with a run-time error, as matching JavaScript in this case is unnecessarily complicated.

- (a) First, write some JAVASCRIPTY programs and execute them as JavaScript programs. This step will inform how you will implement your interpreter and will serve as tests for your interpreter.

Write-up: Give one test case that behaves differently under dynamic scoping versus static scoping (and does not crash). Explain the test case and how they behave differently in your write-up.

- (b) Then, implement

```
def eval(env: Env, e: Expr): Expr
```

that evaluates a JAVASCRIPTY expression e in a value environment env to a value according to the evaluation judgment $E \vdash e \Downarrow v$.

You will again want the following helper functions for converting values to numbers, booleans, and strings:

```
def toNumber(v: Expr): Double
def toBoolean(v: Expr): Boolean
def toStr(v: Expr): String
```

We suggest the following step-by-step process:

1. Bring your Lab 2 implementation into Lab 3. Make sure your previous test cases work as expected and check your Lab 2 implementation against the inference rules in Figure 4.

2. Extend your implementation with non-recursive functions. On function calls, you need to extend the environment for the formal parameter but not for the function itself. Do not worry yet about dynamic type errors.
3. Add support for checking for dynamic type errors.
4. Check that your interpreter, unfortunately, implements dynamic scoping instead of static scoping.
5. Modify your implementation to support recursive functions.

3. JavaScripty Interpreter: Substitution and Evaluation Order.

In this question, we will do two things. First, we will remove environments and instead use a language semantics based on substitution. This change will “fix” the scoping issue, and we will end up with static, lexical scoping.

As an aside, substitution is not the only way to “fix” the scoping issue. Another way is to represent function values as *closures*, which is a pair of the function with the environment when it is defined. Substitution is a fairly simple way to get and specify lexical scoping, but in practice, it is rarely used because it is not the most efficient implementation.

The second thing that we do is move to implementing a small-step interpreter. A small-step interpreter makes explicit the evaluation order of expressions. These two changes are orthogonal, that is, one could implement a big-step interpreter using substitution or a small-step interpreter using environments.

(a) Implement

```
def iterate(e0: Expr)(next: (Expr, Int) => Option[Expr]): Expr
```

that iterates calling the callback `next` until `next` returns `None`. The callback `next` takes an `Expr` to transform and the iteration number, which is initially `(e0, 0)`.

This function is used by the interface method `iterateStep` to repeatedly call `step` to reduce a JAVASCRIPTY expression to a value.

(b) Implement

```
def substitute(e: Expr, v: Expr, x: String): Expr
```

that substitutes value `v` for all *free* occurrences of variable `x` in expression `e`. We advise defining `substitute` by recursion on `e`. The cases to be careful about are `ConstDecl` and `Function` because these are the variable binding constructs. In particular, `substitute` on expression

```
a; (const a = 4; a)
```

with value 3 for variable "a" should return

```
3; (const a = 4; a)
```

not

```
3; (const a = 4; 3)
```

This function is a helper for the `step` function, but you might want to implement all of the cases of `step` that do not require `substitute` first.

(c) Implement

```
def step(e: Expr): Expr
```

that performs one-step of evaluation by rewriting the input expression e into a “one-step reduced” expression. This one-step reduction should be implemented according to the judgment form $e \longrightarrow e'$ defined in Figures 7, 8, and 9. We write $e[\nu/x]$ for substituting value ν for all free occurrences of the variable x in expression e (i.e., a call to `substitute`).

(d) **Write-up:** Explain whether the evaluation order is deterministic as specified by the judgment form $e \longrightarrow e'$.

It is informative to compare the small-step semantics used in this question and the big-step semantics from the previous one. In particular, for all programs where dynamic scoping is not an issue, your interpreters in this question and the previous should behave the same. We have provided the functions `evaluate` and `iterateStep` that evaluate “top-level” expressions to a value using your interpreter implementations.

Hint: We strongly recommend that you implement interpreters (including this one) incrementally by growing the language. That is, use test-driven development with incrementally more complex subsets of the language adding the cases that you need across all functions rather than working a function-at-a-time. In this case, we strongly recommend incrementally implementing `step` and `substitute` together as you grow the language that you handle. There is no one correct way to navigate the growing the language, though adding types of values one-by-one is generally a good strategy. For example, start with numbers, add variable binding, add booleans, undefined, strings, and finally functions. Growing the language is much more effective than, say, implementing `substitute` for the entire language before starting on `step`.

4. Evaluation Order.

Consider the small-step operational semantics for JAVASCRIPTY shown in Figures 7, 8, and 9. What is the evaluation order for $e_1 + e_2$? Explain. How do we change the rules obtain the opposite evaluation order?

5. Short-Circuit Evaluation.

In this question, we will discuss some issues with short-circuit evaluation.

- (a) **Concept.** Give an example that illustrates the usefulness of short-circuit evaluation. Explain your example.
- (b) **JAVASCRIPTY.** Consider the small-step operational semantics for JAVASCRIPTY shown in Figures 7, 8, and 9. Does $e_1 \ \&\& \ e_2$ short circuit? Explain.

$$e \longrightarrow e'$$

$\frac{\text{DOnEG} \quad n' = -\text{toNumber}(v)}{-v \longrightarrow n'}$	$\frac{\text{DoNOT} \quad b' = \neg \text{toBoolean}(v)}{!v \longrightarrow b'}$	$\frac{\text{DoSEQ}}{v_1, e_2 \longrightarrow e_2}$
$\frac{\text{DoPLUSNUMBER} \quad n' = \text{toNumber}(v_1) + \text{toNumber}(v_2)}{v_1 + v_2 \longrightarrow n'}$	$\frac{\text{DoPLUSSTRING}_1 \quad str' = str_1 + \text{toString}(v_2)}{str_1 + v_2 \longrightarrow str'}$	
$\frac{\text{DoPLUSSTRING}_2 \quad str' = \text{toString}(v_1) + str_2}{v_1 + str_2 \longrightarrow str'}$	$\frac{\text{DoARITH} \quad n' = \text{toNumber}(v_1) \text{ bop } \text{toNumber}(v_2) \quad bop \in \{-, *, /\}}{v_1 \text{ bop } v_2 \longrightarrow n'}$	
$\frac{\text{DoINEQUALITYNUMBER}_1 \quad b' = \text{toNumber}(v_1) \text{ bop } \text{toNumber}(v_2) \quad bop \in \{<, <=, >, >=\}}{v_1 \text{ bop } v_2 \longrightarrow b'}$		
$\frac{\text{DoINEQUALITYNUMBER}_2 \quad b' = \text{toNumber}(v_1) \text{ bop } \text{toNumber}(v_2) \quad bop \in \{<, <=, >, >=\}}{v_1 \text{ bop } v_2 \longrightarrow b'}$		
$\frac{\text{DoINEQUALITYSTRING} \quad b' = str_1 \text{ bop } str_2 \quad bop \in \{<, <=, >, >=\}}{str_1 \text{ bop } str_2 \longrightarrow b'}$	$\frac{\text{DoEQUALITY}_1 \quad v_1 \neq p_1(x_1) \Rightarrow e_1 \quad b' = (v_1 \text{ bop } v_2) \quad bop \in \{==, !=\}}{v_1 \text{ bop } v_2 \longrightarrow b'}$	
$\frac{\text{DoEQUALITY}_2 \quad v_2 \neq p_1(x_2) \Rightarrow e_2 \quad b' = (v_1 \text{ bop } v_2) \quad bop \in \{==, !=\}}{v_1 \text{ bop } v_2 \longrightarrow b'}$	$\frac{\text{DoANDTRUE} \quad \mathbf{true} = \text{toBoolean}(v_1)}{v_1 \&\& e_2 \longrightarrow e_2}$	
$\frac{\text{DoANDFALSE} \quad \mathbf{false} = \text{toBoolean}(v_1)}{v_1 \&\& e_2 \longrightarrow v_1}$	$\frac{\text{DoORTRUE} \quad \mathbf{true} = \text{toBoolean}(v_1)}{v_1 e_2 \longrightarrow v_1}$	$\frac{\text{DoORFALSE} \quad \mathbf{false} = \text{toBoolean}(v_1)}{v_1 e_2 \longrightarrow e_2}$
$\frac{\text{DoPRINT} \quad v_1 \text{ printed}}{\mathbf{console.log}(v_1) \longrightarrow \mathbf{undefined}}$	$\frac{\text{DoIFTRUE} \quad \mathbf{true} = \text{toBoolean}(v_1)}{v_1 ? e_2 : e_3 \longrightarrow e_2}$	$\frac{\text{DoIFFALSE} \quad \mathbf{false} = \text{toBoolean}(v_1)}{v_1 ? e_2 : e_3 \longrightarrow e_3}$
$\frac{\text{DoCONST}}{\mathbf{const} x = v_1; e_2 \longrightarrow e_2[v_1/x]}$	$\frac{\text{DoCALL} \quad v_1 = (x) \Rightarrow e_1}{v_1(v_2) \longrightarrow e_1[v_2/x]}$	$\frac{\text{DoCALLREC} \quad v_1 = x_1(x_2) \Rightarrow e_1}{v_1(v_2) \longrightarrow e_1[v_1/x_1][v_2/x_2]}$

Figure 7: Small-step operational semantics of JAVASCRIPTY: DO rules.

$e \longrightarrow e'$

$$\begin{array}{c}
\text{SEARCHUNARY} \\
\frac{e_1 \longrightarrow e'_1}{uope_1 \longrightarrow uope'_1}
\end{array}
\quad
\begin{array}{c}
\text{SEARCHBINARY}_1 \\
\frac{e_1 \longrightarrow e'_1}{e_1 \text{ bop } e_2 \longrightarrow e'_1 \text{ bop } e_2}
\end{array}
\quad
\begin{array}{c}
\text{SEARCHBINARY}_2 \\
\frac{e_2 \longrightarrow e'_2}{v_1 \text{ bop } e_2 \longrightarrow v_1 \text{ bop } e'_2}
\end{array}$$

$$\begin{array}{c}
\text{SEARCHPRINT} \\
\frac{e_1 \longrightarrow e'_1}{\text{console.log}(e_1) \longrightarrow \text{console.log}(e'_1)}
\end{array}
\quad
\begin{array}{c}
\text{SEARCHIF} \\
\frac{e_1 \longrightarrow e'_1}{e_1 ? e_2 : e_3 \longrightarrow e'_1 ? e_2 : e_3}
\end{array}$$

$$\begin{array}{c}
\text{SEARCHCONST} \\
\frac{e_1 \longrightarrow e'_1}{\text{const } x = e_1; e_2 \longrightarrow \text{const } x = e'_1; e_2}
\end{array}
\quad
\begin{array}{c}
\text{SEARCHCALL}_1 \\
\frac{e_1 \longrightarrow e'_1}{e_1(e_2) \longrightarrow e'_1(e_2)}
\end{array}
\quad
\begin{array}{c}
\text{SEARCHCALL}_2 \\
\frac{e_2 \longrightarrow e'_2}{(p(x) \Rightarrow e_1)(e_2) \longrightarrow (p(x) \Rightarrow e_1)(e'_2)}
\end{array}$$

Figure 8: Small-step operational semantics of JAVASCRIPTY: SEARCH rules.

$e \longrightarrow e'$

$$\begin{array}{c}
\text{TYPEERREOREQUALITY} \\
\frac{\text{bop} \in \{==, !=\}}{(p_1(x_1) \Rightarrow e_1) \text{ bop } (p_2(x_2) \Rightarrow e_2) \longrightarrow \text{typeerror}}
\end{array}
\quad
\begin{array}{c}
\text{TYPEERRORCALL} \\
\frac{v_1 \neq p_1(x_1) \Rightarrow e_1}{v_1(e_2) \longrightarrow \text{typeerror}}
\end{array}$$

$$\begin{array}{c}
\text{PROPAGATEUNARY} \\
\frac{}{uoptypeerror \longrightarrow \text{typeerror}}
\end{array}
\quad
\begin{array}{c}
\text{PROPAGATEBINARY} \\
\frac{}{\text{typeerror bop } e_2 \longrightarrow \text{typeerror}}
\end{array}
\quad
\begin{array}{c}
\text{PROPAGATEBINARY} \\
\frac{}{v_1 \text{ bop } \text{typeerror} \longrightarrow \text{typeerror}}
\end{array}$$

$$\begin{array}{c}
\text{PROPAGATEPRINT} \\
\frac{}{\text{console.log}(\text{typeerror}) \longrightarrow \text{typeerror}}
\end{array}
\quad
\begin{array}{c}
\text{PROPAGATEIF} \\
\frac{}{\text{typeerror} ? e_2 : e_3 \longrightarrow \text{typeerror}}
\end{array}$$

$$\begin{array}{c}
\text{PROPAGATECONST} \\
\frac{}{\text{const } x = \text{typeerror}; e_2 \longrightarrow \text{typeerror}}
\end{array}
\quad
\begin{array}{c}
\text{PROPAGATECALL}_1 \\
\frac{}{\text{typeerror}(e_2) \longrightarrow \text{typeerror}}
\end{array}
\quad
\begin{array}{c}
\text{PROPAGATECALL}_2 \\
\frac{}{v_1(\text{typeerror}) \longrightarrow \text{typeerror}}
\end{array}$$

Figure 9: Small-step operational semantics of JAVASCRIPTY: Dynamic type error rules.