

Differential Cross-Section Calculations & Angular Distributions at the SE-SPS

Bryan Kelly

April 10, 2023

How to Calculate a Cross-Section

Studying cross sections at the SE-SPS is vital for analyzing data of excited states of interest in a nuclear reaction. The total cross-section is given by,

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega, \quad [0.1]$$

where $\frac{d\sigma}{d\Omega}$ is the differential-cross section and is integrated over the solid angle $d\Omega$ (typically $\sin\theta d\theta d\phi$ in spherical coordinates). Because of the narrow acceptance given by the slit geometry in most experiments, a total cross-section cannot be determined. Rather, the differential cross-section is what is found, thus the term cross-section used henceforth will always imply the differential cross-section. A cross-section is a measurement of probability that a reaction will occur with your target nuclei given an effective area of material. The example calculations that will follow are for a $^{47}\text{Ti}(d, p)$ reaction.

To calculate cross-sections we must know certain aspects of our experimental setup, such as the number beam particles incident, the target thickness, our acceptance leaving the scattering chamber, and the number of outgoing particles into the focal plane of the SE-SPS. The equation formed with these values is given by,

$$\frac{d\sigma}{d\Omega} = \frac{N_p}{N_b \cdot F_{target} \cdot \Delta\Omega}, \quad [0.2]$$

where N_p is the number of outgoing particles (in this case protons) from the scattering chamber, N_b is the number of incident beam particles, F_{target} is the target thickness, and $\Delta\Omega$ is the outgoing beam area.

To start, the thickness of your target must be known. This will affect the number of target nuclei that an incident beam may impinge upon, thus a thicker target would lead to a higher probability of a target nucleus being ‘hit’ by the beam. A target’s thickness is usually stated by a manufacturer (whether outsourced or made in-house), such as in our case shown in **FIG. 1**.

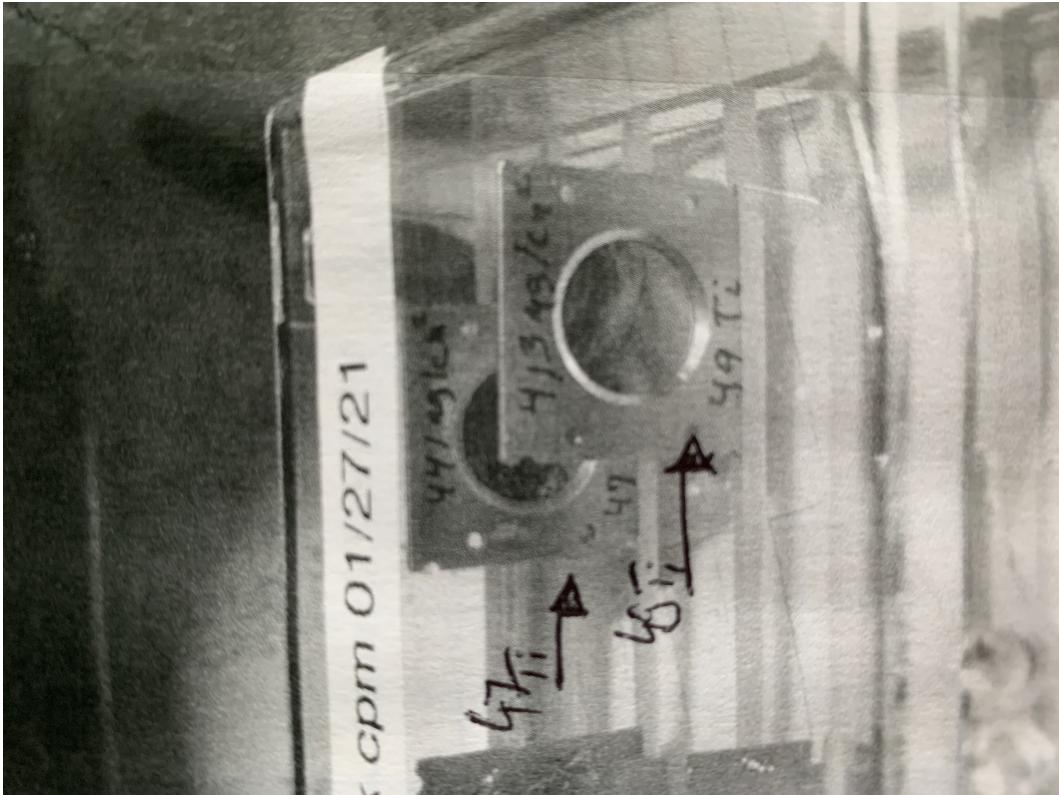


Figure 1: Both ^{47}Ti and ^{49}Ti targets with their respective thickness values, ρt , on the top side of the frame. Target thickness values are typically stated in $\mu\text{g}/\text{cm}^2$.

Due to the area of cross-sections being on such a small scale, a unit defined as a barn ($1 \text{ barn} = 1 \cdot 10^{-24} \text{ cm}^2$) is commonly used as a final unit of measure for area. Additionally, we need to know the number of target nuclei present on the target itself, therefore we can use,

$$F_{\text{target}} = \frac{\rho t \cdot N_A}{M_{\text{target}}} \quad [0.3]$$

where Avogadro's number, the target thickness, and the molar mass M_{target} for ^{47}Ti was used. This conversion gives a value of $F_{\text{target}} = 5.6552 \cdot 10^{-6} \frac{1}{\text{b}}$.

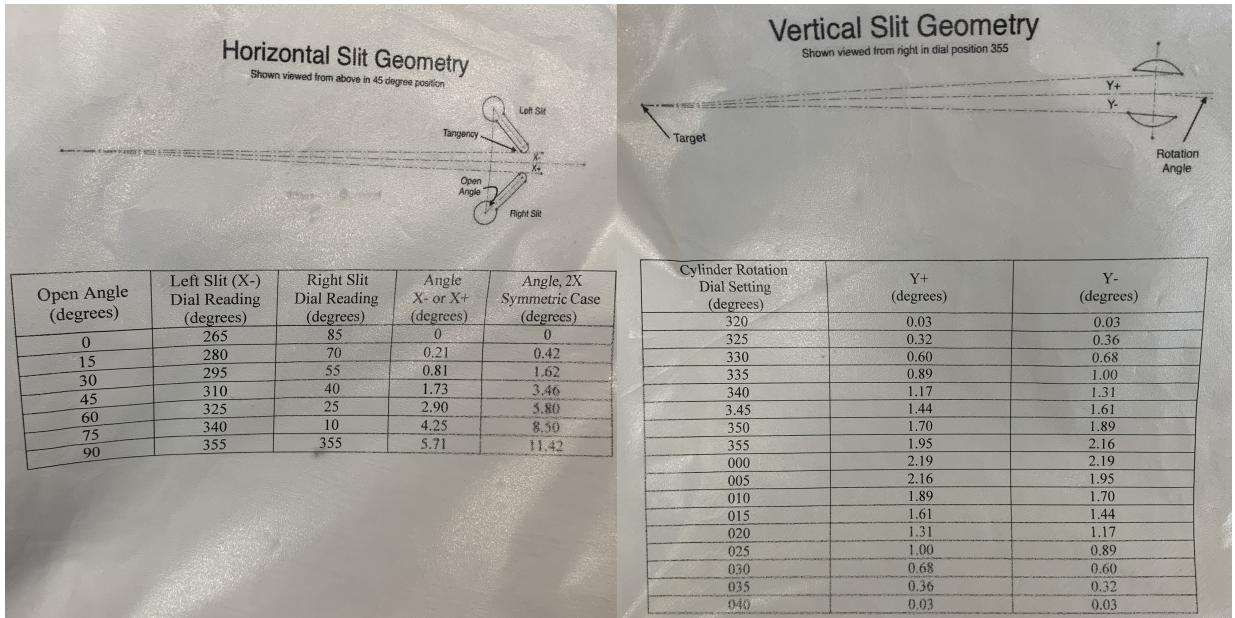
Next, the cross-sectional area from the scattering chamber into the spectrograph is determined by the vertical and horizontal slit-geometry, which can vary from experiment to experiment. Both **FIG.** 2 & 3 show the slit dials as well as the tables that are used to convert from the slit settings into the angles opened in degrees. Note that opening the slits to a wider setting would allow more rate into the SE-SPS but would decrease the resolution at the focal plane.

Therefore, with our slit settings we obtain an angular coverage of 3.46° in the x-plane and 4.38° in the y-plane. Therefore the solid angle that is covered is 15.1548 squared degrees, and converting to steradians by $(\frac{\pi}{180})^2$, gives a value for $\Delta\Omega = 0.00462 \text{ sr}$, or 4.62 msr .



(a) The horizontal slit dials which sit just in front of the SE-SPS.
(b) Vertical slit dial and its setting shown by matching to the black stripe on the copper plate.

Figure 2: The slit dials that allow you to control the solid angle coverage into the SE-SPS.



(a) Horizontal slit geometry conversion table

(b) Vertical slit geometry conversion table

Figure 3: These slit-geometry conversion tables (found on the beam line near the scattering chamber) can be used to convert from the dial settings to the angles created by the slits.

The last values to be determined are the number of incident beam particles, N_b , and the number of ejectiles (protons), N_p in the focal plane. The latter is more easily determined. We expect our ejected particles detected by the focal plane detectors to take on a Gaussian-like curve. Therefore integrating a Gaussian function over the region of an excited state will give you the number of particles detected at that state. Most programs we used in the lab (such as ROOT) have built in functions that will do this for you for a given excited state of interest.

To determine the number of incident beam particles requires further work, and we will start by the following,

$$N_b = \frac{Q_b}{Z_b \cdot e}, \quad [0.4]$$

where Q_b is the total charge of all beam particles incident to your target, Z_b is the proton number of your beam, and e is the elementary unit of charge. In the case for a deuteron beam, Z_b will simply be equal to 1. To obtain the total charge of all incident beam particles, we use scalar values that are inherent to our experimental setup and are to be recorded during the experiment. The Beam Current Integrator, or BCI, allows us to monitor the current of our beam down the beam-line in our facility. The two scalar values we need to determine the total charge of incident particles is the scale of BCI, which may vary from run to run, and the total number of events the BCI sees. The scale of the BCI is to be recorded during each run, and the BCI counts are read in to a digitizing board during our runs and can be found in the data files generated. Therefore, to get the total charge of particles incident,

$$Q_b = \frac{\beta \cdot \text{BCIscale}}{100\text{Hz}}, \quad [0.5]$$

where β is the total number of BCI events read, and the 100 Hz factor is the sampling rate of the BCI itself. **It is important** to know that the BCI scale is usually somewhere on the order of nano-amps, e.g. for a 30 nA scale you should use the value in amps ($30 \cdot 10^{-9} \text{ A}$), to get the correct units of charge to cancel in **Eq. 0.4** (assuming the SI unit of charge $1.6 \cdot 10^{-19} \text{ C}$ for e). Thus, combining **Eq. 0.3-0.5**, as well our calculated value for the solid angle $\Delta\Omega$, and substituting into **Eq. 0.2** we can obtain a value for a cross-section measurement. Obtaining cross-sections over a sweeping set of angles (typically between $10^\circ - 60^\circ$ at our setup) for a specific excited state of interest will give you an angular distribution, and will be discussed in the following section. Another note, cross-sections are typically stated in millibarns/steradian, therefore keeping your units in order is extremely vital.

Angular Distributions and Their Use

This section will be slightly brief as I will not outline how to calculate theoretical curves for angular distributions (typically done through using Distorted Wave Borne Approximation) but rather what angular distributions are and why they are important.

In the previous section we learned how to calculate a cross-section, which again, is a probability that your nuclear reaction of interest will occur given some effective area of your target nuclei. Studying the angular distributions of these cross-sections, that is, plotting the cross-section measurements across all angles studied, will give insight of the structure of your excited states of interest. Many areas of research in nuclear physics rely heavily on understanding the nuclear structure of your system. The foundation of the structure is built on the principles of quantum mechanics such as angular momentum addition, selection rules, and so on. Very useful information can be found in Appendices A & B in "Nuclear Physics of Stars" by Iliadis.

Understanding the ground state spin and parity of your target is essential. This allows you to determine the range of possible states you expect to see from your reaction. Typically a state will be denoted by a J^π value, where J is the total angular momentum $J = l + s$ and the parity, $\pi = (-1)^l$, where l is the orbital angular momentum and s is the spin angular momentum. From wave-particle duality, we know that we can map our particles as waves that have corresponding wave functions which describe their dynamics in both space and time. From solving the Schrodinger equation in a central potential, we know that the angular portions of a wave function are proportional to Legendre Polynomials (assuming azimuthal symmetry). The first few Legendre Polynomials are,

Legendre Polynomials
$P_0(\cos \theta) = 1$
$P_1(\cos \theta) = \cos \theta$
$P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$
$P_3(\cos \theta) = \frac{1}{2}(5 \cos^3 \theta - 3 \cos \theta)$
$P_4(\cos \theta) = \frac{1}{8}(35 \cos^4 \theta - 30 \cos^2 \theta + 3)$
$P_5(\cos \theta) = \frac{1}{8}(63 \cos^5 \theta - 70 \cos^3 \theta + 15 \cos \theta)$

Table 1: A table of first six Legendre Polynomials in spherical coordinates.

Thus, an overlap in the wave functions (and by extension the Legendre Polynomials that they are described by) are what create angular distributions in a reaction where some orbital angular momentum l is transferred from the initial wave function to the final, which is described by some matrix element $\langle f | V | i \rangle$ where V is a central potential. Theoretical curves generated from the use of partial wave expansion of a particle's wave function in the Borne Approximation (known as Distorted Wave Borne Approximation), can be generated. The amplitude of these curves can be scaled by the spectroscopic factor, thus giving detail into the single-particle

structure of the initial state's wave function. What this means is there are different nuclear reactions that can lead you to the same final state, say $^{47}Ti(d, p)^{48}Ti$ and $^{49}Ti(p, d)^{48}Ti$. Both lead you to the same final nucleus and thus a DWBA curve for an excited state, say a state with $J^\pi = 2^+$. In each case the final state will have the same spin and parity, but the spectroscopic factor is a key piece of information of how 'pure' the state is in relation to the shell model of a nucleus. The overall goal of an angular distributions is extracting the spectroscopic factor, as well as assigning spin & parity values to your state.

An example of an angular distribution is given in **FIG. 4**.

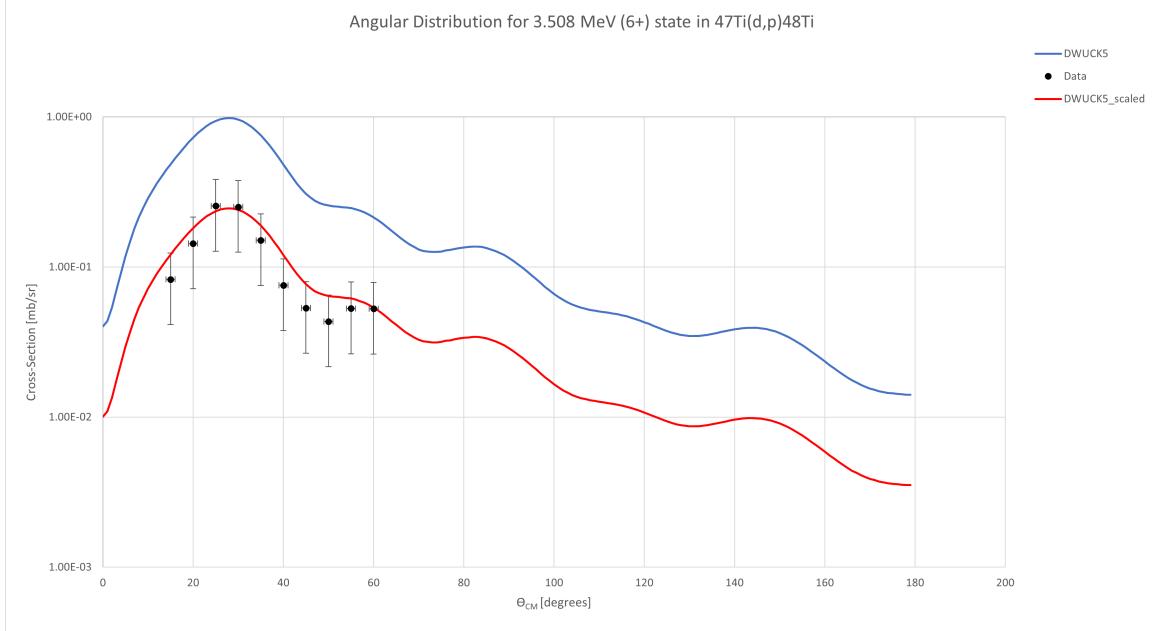


Figure 4: Angular Distribution for the 3.508 MeV excited state in ^{48}Ti . The experimental data is shown in black, and the DWUCK5 DWBA code is also plotted. The blue curve represents the raw curve generated by the DWUCK5 program, and the red curve is the scaled curve, which gives your spectroscopic factor (S), here S=0.25.