A STUDY OF THE GIANT DIPOLE RESONANCE OF VIBRATIONAL NUCLEI IN THE 103 $\leq A \leq$ 133 MASS REGION

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Abstract: This paper presents a set of experimental data concerning the giant dipole resonance of nuclei (GDR) in the $103 \le A \le 133$ mass region. The cross sections $\sigma(\gamma, n)$ and $\sigma(\gamma, 2n)$ were obtained in the energy region 8-30 MeV by means of a monochromatic photon beam produced by annihilation in flight of positons. This paper attempts also to give an interpretation of the experimental behaviour of the GDR for vibrational nuclei in the $103 \le A \le 133$ mass region in terms of the simple dynamic collective model. In particular it is shown that the width of the GDR increases as β increases and as E_{2+} decreases and that the theoretically predicted spreading of the dipole strength is confirmed by our experimental results. As to the characteristic behaviour of the GDR above its peak value at E_0 , we come to the conclusion that the actual state of the art in (γ, xn) research does not allow one to make an unambiguous choice between isospin splitting or electric quadrupole absorption. Finally the numerical evaluations of the different sum rules are given and some empirical results concerning the average energy of the GDR as a function of A are also presented.

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NUCLEAR REACTIONS $103 \le A \le 133$ (γ, n) , $(\gamma, 2n)$, E = 8-30 MeV; measured. $\sigma(\gamma, n)$, $\sigma(\gamma, 2n)$; deduced $\sigma(\gamma, tot)$, integrated σ , Lorentz line parameters, intrinsic quadrupole moments Q_0 . Natural and enriched targets.

1. Available nuclear models

Calculations of the positions and relative strengths of the dipole states of the GDR based on Brown and Bolsterli's 1) microscopic schematic model, where the initial components are taken to be the simple 1p-1h states, do not exist for heavy nuclei commonly considered to be vibrational (100 < A < 130) although they are available for the doubly magic nucleus 208 Pb [refs. 2,3)]. On the other hand, numerical evaluations of sum rules, such as the one of Thomas, Reiche and Kuhn giving the integrated cross section for dipole absorption 4), were correctly predicted to within a corrective factor taking exchange forces into account.

However a set of predictions concerning the distribution of the dipole strength, capable of giving one some idea as to the evolution of the form of the GDR in the

100 < A < 130 mass region, can be obtained from the two following models: (i) the finite Fermi liquid model (FLM); (ii) the dynamic collective model (DCM).

The predictions of the DCM are not only available for most nuclei but are also much easier to apply than those of the FLM since they depend mainly on well known low energy parameters of the nuclei. The DCM was therefore used as the only guide in the interpretation of our experimental results and a brief comparison between the two models will summarize the reasons for this choice.

A study of the interaction between nuclei and electromagnetic fields led Migdal ⁵) to propose the finite Fermi liquid theory which was first presented in the form of a semi-classical approximation ⁶). Using the same approach, Bunatyan ⁷) was able to improve the accuracy of the evaluations of the E1 and E2 resonances in heavy spherical nuclei such as ²⁰⁸Pb and ¹²⁰Sn and Kamerdzhiev ⁸) has recently performed the same calculations for the 112, 120 and 124 tin isotopes. These calculations indicate that predicted spreading of the dipole strength depends strongly on the values of the constants f^{nn} and f^{np} , where these constants in turn are given by the velocity dependent part of the interaction of identical and different quasi particles, and hence are hardly known experimentally. Moreover, Kamerdzhiev's calculations also show that the evaluated spreading of the dipole strength of ¹¹²Sn and ¹²⁴Sn depends to a considerable degree on the inclusion or exclusion of spin-spin forces.

On the other hand, the dynamic collective model (DCM), developed essentially by Le Tourneux 9) and by Danos and Greiner and their collaborators 10, 11), simply connects the spreading of the dipole strength to the low energy collective properties which characterize the quadrupole surface vibrations of nuclei:

$$R = R_0 [1 + \sum_{\mu} \alpha_{2\mu}(t) Y_{2\mu}(\theta, \phi)]. \tag{1}$$

These vibrational characteristics β and E_{2+} are now experimentally accessible if one measures first the root mean square amplitude of the surface vibrations β where

$$\beta^{2} = \sum_{\mu} \langle \text{ground state} | \alpha_{2\mu}^{2} | \text{ground state} \rangle,$$

$$\beta^{2} = \left[\frac{3ZR_{0}^{2}}{4\pi} \right]^{-2} B(E2),$$
(2)

and then the energy E_{2+} of the first 2^+ excited level corresponding to a one-phonon vibration. The coupling of the dipole oscillations ($H_{\rm dip}$. Hamiltonian) and the quadrupole surface vibrations ($H_{\rm quad}$. Hamiltonian) is then evaluated in the adiabatic approximation by diagonalizing the total Hamiltonian of the system

$$H = H_{\rm dip.} + H_{\rm quad.} + H_{\rm dip.\,quad.}$$

in the function space of $H_0 = H_{\rm dip.} + H_{\rm quad.}$, and it can be shown that the non-diagonal matrix elements due to the coupling term $H_{\rm dip.\,quad.}$ impart a spreading of the dipole strength into several intermediate collective states. An exhaustive study

of just these intermediate collective states as a function of β and E_{2+} , but considering simple harmonic vibrations only, has been undertaken by Huber ¹¹), Arenhövel ¹²) and Urbas *et al.* ¹³). Their results indicate that the spreading ought to increase whenever β increases and E_{2+} decreases.

However, there exists no nucleus which is truly a good harmonic spherical vibrator. In particular, recent electron scattering studies of vibrator spectra ¹⁴) have actually connected the form factor observed for ¹¹⁰Pd, ¹¹⁴Cd and ¹¹⁶Sn to the anharmonic vibrator model proposed by Lightbody ¹⁵). A recent improvement of the DCM by Rezwani *et al.* ¹⁶) enabled them to describe any collective behaviour of nuclei in terms of a potential energy surface. This "improved DCM" predicted successfully ¹⁷) the behaviour of the form of the GDR for transition nuclei (142 < A < 150), represented by the neodymium isotopes, and recently investigated at Saclay ¹⁸). However, such improved evaluations are not yet available for $100 \leq A \leq 130$ nuclei and this is why, in subsects. 1.1 and 1.2 below, we summarize the predictions concerning the GDR of such $100 \leq A \leq 130$ nuclei as obtained from the original simple DCM when taking harmonic vibrations only into account.

1.1. THE TIN ISOTOPES

For these nuclei, the parameters β and E_{2+} are practically constant, as can be seen in table 1, and β is rather small. Hence it does not come as a surprise that Arenhövel's ¹²) calculations predicted a small but essentially constant spreading of the GDR for all tin isotopes as can be seen in fig. 1.

	¹¹⁶ Sn	¹¹⁸ Sn	¹²⁰ Sn	¹²² Sn	¹²⁴ Sn
β	0.113	0.116	0.112	0.118	0.108
E_{2+} (MeV)	1.293	1.230	1.175	1.140	1.131

However, these predictions can not be said to fit the experimental data recently published on the subject and represented in fig. 2 where Fultz et al. ¹⁹) find a constant GDR width Γ of 4.8 ± 0.1 MeV for ¹¹⁸Sn, ¹²⁰Sn and ¹²⁴Sn but observes a sudden decrease for ¹¹⁶Sn where $\Gamma=4.2$ MeV. He concludes that although ¹¹⁶Sn does not have a closed neutron shell (N=66), the result of adding valency neutrons to ¹¹⁶Sn becomes evident through the broadening of the GDR, an effect which is not predicted by the DCM for the Sn isotopes. But Fultz's results are contradicted by a set of data also shown in fig. 2 and recently obtained by Sorokin et al. ²⁰), who used bremsstrahlung photons, and who observed a continuous decrease in Γ from 6.2 MeV for ¹¹⁶Sn to $\Gamma=4.4$ MeV for ¹²⁴Sn.

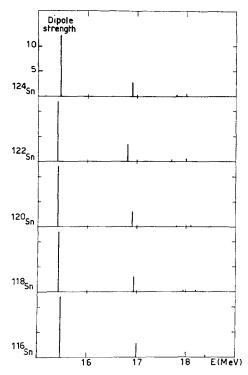


Fig. 1. Spreading of the dipole strength for the tin isotopes as predicted by Arenhövel 12).

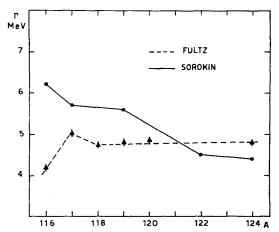


Fig. 2. Experimentally observed GDR widths obtained by Fultz 19) and Sorokin 20).

1.2. THE Te, Cd AND Pd NUCLEI

A study of the GDR of vibrational nuclei close to the Sn isotopes and for which β and E_{2+} are different from the values observed for these Sn isotopes seems also very interesting. Meaningful results, showing a possible correlation with the predictions

Table 2

Mean values $\overline{\beta_0}$ and $\overline{E_{2+}}$ of the low energy parameters β and E_{2+} for Te, Cd and Pd nuclei

	Те	Cd	Pd
$\overline{oldsymbol{eta_0}}$	0.145 ± 0.018	0.190 ± 0.010	0.230 ± 0.020
$\overline{E_{2}}$ +	0.750 ± 0.090	0.580±0.070	0.460 ± 0.080
$E_1 = 78A^{-\frac{1}{3}} (\text{MeV})$	15.4	16.0	16.3
$eta'_0 = \overline{eta_0}(\frac{1}{18}E_1)$	0.125	0.168	0.208

The β'_0 and E_1 notation is explained in the text.

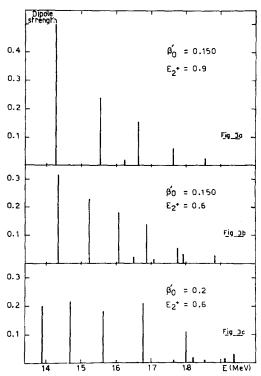


Fig. 3. Theoretical spreading of the dipole strength as computed by Huber ¹¹) for three typical vibrational nuclei.

of the DCM, can be expected for Te, Cd and Pd nuclei, even with natural targets. For these nuclei, the values of the parameters β and E_{2+} for the main isotopes change very little with respect to the mean values $\overline{\beta}_0$ and \overline{E}_{2+} presented in table 2.

Since Huber's calculations on the spreading were performed for $E_1 = 18 \text{ MeV}$ and since (β, E_1) is the parameter which contributes most to the spreading, due to the $H_{\text{dip. quad.}}$ coupling, one is obliged to use $\beta'_0 = \overline{\beta_0}(\frac{1}{18}E_1)$ so as to find theoretical

cases as close as possible to the required Te, Cd and Pd nuclei. We thus show in fig. 3 a selection of possible theoretical cases illustrating the spreading of the GDR, each representing as closely as possible, one of the real nuclei Te, Cd and Pd. In particular, fig. 3a shows the results, obtained with $\beta'_0 = 0.150$ and $E_{2+} = 0.9$ MeV, a case which closely resembles tellurium where

$$\frac{\beta_0'}{E_{2+}} = \frac{0.125}{0.750} = \frac{1}{6} = \frac{0.15}{0.90}.$$

In fig. 3b the result shown is for $\beta'_0 = 0.150$ and $E_{2+} = 0.6$ MeV which resembles the cadmium case but with a somewhat smaller theoretically predicted spreading.

In fig. 3c we used $\beta'_0 = 0.2$ and $E_{2+} = 0.6$ MeV, a case which is very similar to palladium but again with a somewhat smaller predicted spreading. If one now compares the prognostications of fig. 1 with those of fig. 3, one observes that about 90% of the dipole strength is concentrated in the following energy intervals ΔE :

 $\Delta E = 1.5$ MeV for any of the Sn isotopes;

 $\Delta E = 2.2 \text{ MeV for Te (fig. 3a)};$

 $\Delta E = 2.7 \text{ MeV for CD (fig. 3b)};$

 $\Delta E = 4$ MeV for Pd (fig. 3c).

Let us now assume that the damping widths, associated with each of the dipole states which characterize the intermediate collective strengths displayed in figs. 1 and 3, are constant. It then is to be expected that the experimentally observed GDR widths Γ will show the following relations:

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\Gamma (any Sn isotopes) = constant = \Gamma(Sn);

\Gamma(Nat Te) \approx \Gamma(Sn)+0.7 MeV;

\Gamma(Nat Cd) \approx \Gamma(Sn)+1.2 MeV;

\Gamma(Nat Pd) \approx \Gamma(Sn)+2.5 MeV.
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Using the quasi-monochromatic photon beam, obtained by means of the annihilation-in-flight technique of monochromatic positons created with the 65 MeV linear accelerator at Saclay ²¹), we therefore attempted to verify these predictions for all nuclei discussed in sect. 2 below.

Photoneutrons are detected with a gadolinium loaded liquid scintillator having a high and variable neutron detecting efficiency $40\% < \varepsilon < 60\%$ enabling one to measure the partial photoneutron cross sections $[\sigma(\gamma, n) + \sigma(\gamma, pn)]$ and $\sigma(\gamma, 2n)$ separately and simultaneously so that the total photoabsorption cross section $\sigma(\gamma, tot) = \sigma(\gamma, n) + \sigma(\gamma, pn) + \sigma(\gamma, 2n)$ becomes directly accessible ²²).

2. Experimental investigation of the GDR width

2.1. THE TIN ISOTOPES

Characteristics of the samples used are given in table 3.

In fig. 4 we show the partial photoneutron cross sections $[\sigma(\gamma, n) + \sigma(\gamma, pn)]$ and $\sigma(\gamma, 2n)$ for the different tin isotopes. The error bars in all our experimental results

Table 3	
Target specifications of Sn	isotopes

	¹¹⁶ Sn	¹¹⁷ Sn	¹¹⁸ Sn	120Sn	¹²⁴ Sn
Mass (g)	50	48	50	50	44
Purity (%)	95.6	90.9	98.5	99.2	95.9
Impurities (%)	¹¹⁷ Sn, 1.9 ¹¹⁸ Sn, 1.3	¹¹⁸ Sn, 6.5			¹²⁰ Sn, 1.45

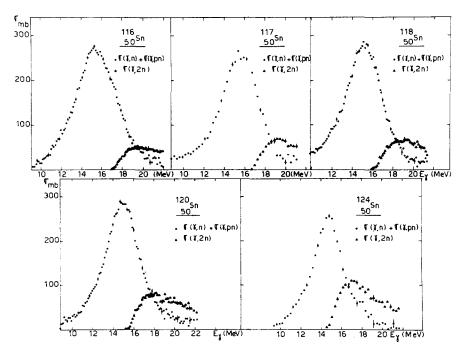


Fig. 4. Partial photoneutron cross sections $[\sigma(\gamma, n) + \sigma(\gamma, pn)]$ and $\sigma(\gamma, 2n)$ for the tin isotopes.

represent statistical errors only but if one takes possible errors in the calibration of the monochromatic photon flux also into account then the absolute error bars at the peak values of the respective GDR are $\approx \pm 20$ mb. One observes that the relative importance of $\sigma(\gamma, 2n)$ increases as one proceeds from ¹¹⁶Sn towards ¹²⁴Sn since its maximum value increases from $\sigma(\gamma, 2n) = 50$ mb to $\sigma(\gamma, 2n) = 110$ mb respectively.

This behaviour of $\sigma(\gamma, 2n)$ is to be expected in view of the fact that the threshold of the $(\gamma, 2n)$ exit channel decreases from $B_{2n} = 17.1$ MeV to $B_{2n} = 14.4$ MeV for 116 Sn and 124 Sn respectively, whereas the average energy E_1 of the position of the GDR, namely $E_1 = 78$ $A^{-\frac{1}{2}} = 15.5 \pm 0.2$ MeV, remains practically constant for all isotopes concerned. Under these circumstances one ought indeed to observe an

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	¹¹⁶ Sn	¹¹⁷ Sn	¹¹⁸ Sn	¹²⁰ Sn	124Sn
σ ₀ (mb)	270	255	278	284	275
Γ_0 (MeV)	5.21 ± 0.1	5.30 ± 0.1	4.99 ± 0.1	5.25 ± 0.1	4.96 ± 0.1
E ₀ (MeV)	15.57	15.67	15.44	15.38	15.29

TABLE 4

Lorentz line parameters for a single Lorentz line fit

Values of E_0 are given to within ± 0.1 MeV. Values of σ_0 are given to within ± 5 mb.

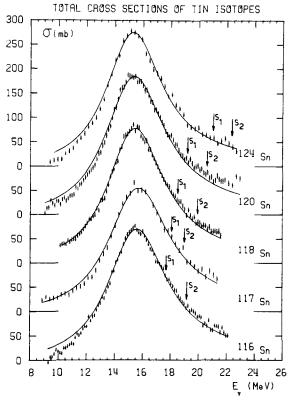


Fig. 5. Best single Lorentz line fit to $\sigma_{exp}(\gamma, tot) = \sigma(\gamma, n) + \sigma(\gamma, pn) + \sigma(\gamma, 2n)$ for the tin isotopes. Arrows S_1 and S_2 refer to isobaric analogue states as observed by Sugawara ³³) using (e, e'p)

increase in $\int_{B_{2n}}^{E_{max}} \sigma(\gamma, 2n) dE$ as B_{2n} decreases provided that, upon absorption of a photon, the nucleus in question emits one or two neutrons in cascade as predicted by the statistical model.

Since none of the Sn nuclei possesses a permanent deformation, one can now "fit", according to a standard procedure, a single Lorentz curve $\sigma_L(E)$ to the experimental

Table 5					
Lorentz line	parameters for a	two-Lorentz-line fit			

	E ₁ (MeV)	σ _i (mb)	Γ΄ (MeV)	E ₂ (MeV)	σ ₂ (mb)	$\frac{\sigma_1 \Gamma'}{(\sigma_2 + \sigma_1)\Gamma'}$	$\frac{\varSigma_1}{\varSigma_2 + \varSigma_1}$
¹¹⁶ Sn	15.26	222	4.54	16.72	67	0.77	0.795
¹¹⁸ Sn	15.13	230	4.37	16.60	68	0.775	0.775
¹²⁰ Sn	15.04	226	4.51	16.50	80	0.74	0.775
¹²⁴ Sn	14.98	230	4.10	16.44	70	0.765	0.770

Errors are the same as in table 4.

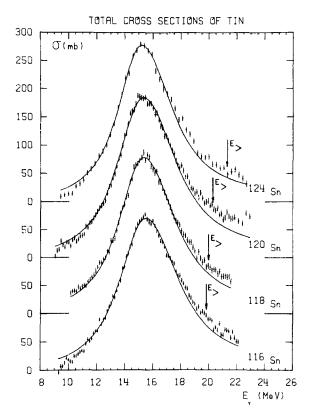


Fig. 6. Best two-Lorentz-line fit to $\sigma_{\rm exp}(\gamma, {\rm tot})$ for the tin isotopes where the spacing between the two lines was chosen according to Arenhövel's ¹²) computations. The $E_{>}$ arrows indicate the positions of the (T+1) states as computed by eq. (8).

 $\sigma(\gamma, \text{tot})$ results, where it should be kept in mind that a Breit-Wigner type of curve would just as well give satisfactory results ^{23, 24}):

$$\sigma_L(E) = \sigma_0 \frac{\Gamma_0^2 E^2}{(E^2 - E_0^2)^2 + E^2 \Gamma_0^2}.$$
 (3)

The resulting Lorentz line parameters for such a fitting process are given in table 4 and the corresponding curves are shown in fig. 5.

As can be seen from fig. 5, reasonable agreement exists in general between the experimental points and the above mathematical expression. However, the fit is not so good for E > 18 MeV in 120 Sn and for E < 13 MeV in 116 Sn, 120 Sn, and 124 Sn. In particular, one observes that the slope $d\sigma_L/dE < d\sigma_{exp}/dE$ from 12 to 15 MeV whereas $d\sigma_L/dE > d\sigma_{exp}/dE$ from 18 MeV onwards. This corresponds to a certain spreading of the dipole strength above E_0 which is partly accounted for by a too large Γ_0 value in the single Lorentz line fit. A better agreement can be obtained if one adjusts two Lorentz lines to the experimental $\sigma(\gamma, tot)$ points, where these two lines are now to be associated with the two main intermediate collective states as calculated by Arenhövel and for which one assumes a constant damping width Γ' :

$$\sigma'_{L}(E) = \sigma_{1} \frac{E^{2}(\Gamma')^{2}}{(E^{2} - E_{1}^{2})^{2} + E^{2}(\Gamma')^{2}} + \sigma_{2} \frac{E^{2}(\Gamma')^{2}}{(E^{2} - E_{2}^{2})^{2} + E^{2}(\Gamma')^{2}}.$$
 (4)

Results of such a fit are given in table 5 and fig. 6 where we imposed $E_1 - E_2 = 1.46$ MeV on the fitting process, this condition being predicted by the DCM. It is surprising to note that the average damping width $\Gamma' = 4.4$ MeV is very close to the computed value of 4.6 MeV obtained from the formula $\Gamma^{\perp} = 0.027$ $E^{1.9}$ established in ref. ²⁴).

As can be seen from figs. 5 and 6 one obtains a better adjustment between $\sigma_{\text{exp}}(\gamma, \text{tot})$ and σ'_{L} [eq. (4)] than between $\sigma_{\text{exp}}(\gamma, \text{tot})$ and σ_{L} [eq. (3)], in particular for the rising part of the GDR. This corresponds to the fact that the adjustment parameter P calculated over the n points of the fitting region, from 12 to 18 MeV,

$$P = \frac{1}{n} \sum_{i=1}^{n} \left[\sigma_{\exp}(E_i) - \sigma_L(E_i) \right]^2,$$

equals 68 for the ¹²⁴Sn fit of the GDR with one Lorentz line but only 35 for the two-Lorentz-line fit to eq. (4). Moreover, a remarkable agreement exists between the experimentally observed relative strength of the lower energy state $\sigma_1 \Gamma'/[\sigma_1 + \sigma_2]\Gamma'$ and the corresponding theoretical percentage $\Sigma_1/(\Sigma_1 + \Sigma_2)$ predicted by Arenhövel ¹²). It can also be seen that $\Gamma' < \Gamma_0$ and that the FWHM value Γ of the $\sigma_{\rm exp}(\gamma, {\rm tot})$ curve is given by $\Gamma \approx \Gamma_0 = 5.15 \pm 0.15$ MeV for all the Sn isotopes in discrepancy with the continuous decrease of Γ with Λ observed by Sorokin et al. ²⁰) and with the exceptionally low value Γ observed for ¹¹⁶Sn by Fultz et al. ¹⁹). Accepting our results and considering that we found Γ_0 to within ± 0.1 MeV, this can be taken to be a confirmation of the constant value of the spreading of the dipole strength within the framework of the DCM.

2.2. THE 45Rh, 46Pd, 47Ag, 48Cd AND 49In NUCLEI

All samples used for the A < 113 nuclei were composed of natural elements. In fig. 7 we present the partial photoneutron cross sections $[\sigma(\gamma, n) + \sigma(\gamma, pn)]$ and $\sigma(\gamma, 2n)$; the corresponding total cross sections $\sigma_{\rm exp}(\gamma, {\rm tot}) = \sigma(\gamma, n) + \sigma(\gamma, pn) + \sigma(\gamma, 2n)$ are shown in fig. 8. The parameters obtained from a single Lorentz line fit to the $\sigma_{\rm exp}(\gamma, {\rm tot})$ points are given in table 6.

In fig. 8 we again show the actual adjustments made between σ_L of eq. (3) and our experimental results $\sigma_{\rm exp}(\gamma,{\rm tot})$ for the above nuclei. It can be seen that ${\rm d}\sigma_L/{\rm d}E < {\rm d}\sigma_{\rm exp}/{\rm d}E$ for E < 14 MeV which indicates once again that a single Lorentz line fit does not constitute the ideal adjustment. This is probably due to the fact that the fitting process tries to take the spreading of the dipole strength around $E > E_0$ into account by giving an excessive weighing factor to σ_L for $E < E_0$.

The experimental value of the FWHM of the $\sigma_{exp}(\gamma, tot)$ curve for Ag is

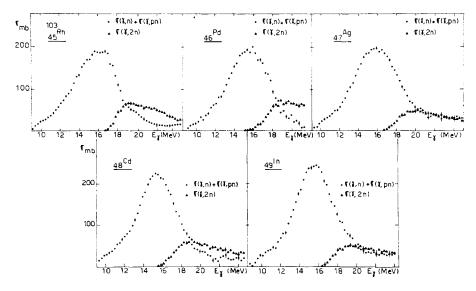


Fig. 7. Partial photoneutron cross sections $[\sigma(\gamma, n) + \sigma(\gamma, np)]$ and $\sigma(\gamma, 2n)$ for $_{45}$ Rh, $_{46}$ Pd, $_{47}$ Ag, $_{48}$ Cd and $_{49}$ In.

TABLE 6

Lorentz line parameters for a single Lorentz line fit

	¹⁰³ ₄₅ Rh	46Pd	47Ag	48Cd	115 49 In
σ_0 (mb)	191	199	198	226	243
Γ_0 (MeV)	7.4 ± 0.1	$\textbf{7.1} \!\pm\! \textbf{0.1}$	$7.7\!\pm\!0.1$	6.3 ± 0.1	6.1 ± 0.1
E_0 (MeV)	16.15	15.9	16.1	15.8	15.8

Errors are the same as in table 4.

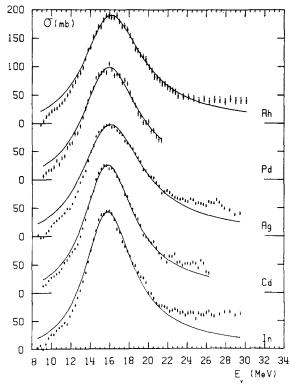


Fig. 8. Best single Lorentz line fit to $\sigma_{\exp}(\gamma, \text{tot}) = \sigma(\gamma, n) + \sigma(\gamma, np) + \sigma(\gamma, 2n)$ for $_{45}\text{Rh}, _{46}\text{Pd}, _{47}\text{Ag}, _{48}\text{Cd}$ and $_{49}\text{In}.$

 $\Gamma=7.5\pm0.15$ MeV which is slightly above the value of 7.0 MeV found by Berman et al. ²⁵) for ¹⁰⁷Ag, whereas our experimental value of $\Gamma=7.1\pm0.15$ MeV, obtained for Pd, is much smaller than the result 8 MeV $<\Gamma<10$ MeV which characterizes the Γ for the total cross section $\sigma(\gamma, n) + \sigma(\gamma, np) + \sigma(\gamma, 2n)$ obtained by Deague et al. ²⁶) with bremsstrahlung photon beams. But since this latter method does not allow a separate measurement of $\sigma(\gamma, 2n)$, this results in large experimental errors for $E>B_{2n}$ thus preventing a precise evaluation of the width Γ .

As to the rhodium case, Bogdankevitch et al. ²⁷) observed a pronounced splitting of the GDR into two parts, at $E_1=14.25$ MeV and at $E_2=17.5$ MeV respectively, by unfolding the bremsstrahlung yield data with the Penfold-Leiss method which is known to overemphasize the structures. They ascribed this splitting to the quadrupole deformation of rhodium. However, a two-Lorentz-line fit to our $\sigma_{\rm exp}(\gamma, {\rm tot})$ points, with the restriction $\sigma_2 \Gamma_2/\sigma_1 \Gamma_1=2$ corresponding to a prolate deformation imposed on the fitting process, gives an adjustment parameter P=13, for the set of parameters shown in table 7, whereas a one-Lorentz-line fit gave the parameters of table 6 with P=17.

Table 7

Lorentz line parameters for a two-Lorentz-line fit of the GDR of rhodium

σ ₁ (mb)	E ₁ (MeV)	Γ_1 (MeV)	σ ₂ (mb)	E ₂ (MeV)	Γ_2 (MeV)
88	15	4.94 ± 0.1	126	17.05	6.90±0.1

Errors are the same as in table 4.

We can now compute the intrinsic quadrupole moment Q_0 of $^{103}_{45}$ Rh by means of the classical formulas ⁴):

$$0.911d + 0.089 = E_2/E_1, (5)$$

$$Q_0 = \frac{2}{5}Z(R_0)^2 A^{\frac{2}{3}}(d^2 - 1)/d^{\frac{2}{3}},\tag{6}$$

which for $R_0 = 1.2$ fm gives $Q_0 = 1.7 \pm 0.2$ b, a value smaller than the calculated value $Q_0 = 2.2 \pm 0.1$ b obtained by Heydenburg *et al.*²⁸) from their experimental B(E2) values using the rotational interpretation.

2.3. THE 51Sb, 52Te AND 133Cs NUCLEI

In fig. 9 we present the partial photoneutron cross sections $[\sigma(\gamma, n) + \sigma(\gamma, pn)]$ and $\sigma(\gamma, 2n)$ with the corresponding total cross section $\sigma_{\exp}(\gamma, tot) = \sigma(\gamma, n) + \sigma(\gamma, pn) + \sigma(\gamma, 2n)$ shown in fig. 10. Lorentz line parameters obtained from a single Lorentz line fit to our $\sigma_{\exp}(\gamma, tot)$ data are given in table 8 and the best "fits" are also indicated in fig. 10.

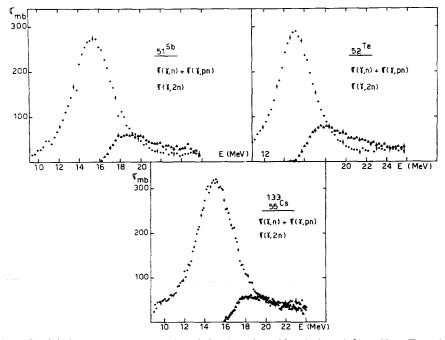


Fig. 9. Partial photoneutron cross sections $[\sigma(\gamma, n) + \sigma(\gamma, np)]$ and $\sigma(\gamma, 2n)$ for $_{51}$ Sb, $_{52}$ Te and $_{55}$ Sc.

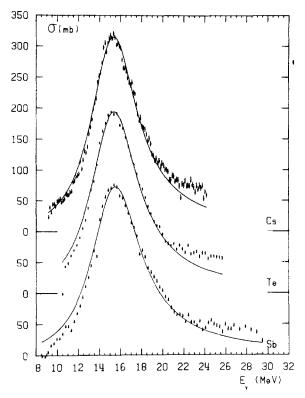


Fig. 10. Best single Lorentz line fit to $\sigma_{\exp}(\gamma, \cot) = \sigma(\gamma, n) + \sigma(\gamma, np) + \sigma(\gamma, 2n)$ for $_{51}$ Sb, $_{52}$ Te and $_{55}$ Cs.

TABLE 8

Lorentz line parameters for a single Lorentz line fit

51Sb	₅₂ Te	¹³³ Cs
273	294.5	315.7
5.75 ± 0.1	5.49 ± 0.1	5.39 ± 0.1
15.51	15.37	15.32
	273 5.75±0.1	273 294.5 5.75±0.1 5.49±0.1

Errors are the same as in table 4.

As can be seen from fig. 10 such fits are clearly not very good in general since an excess of experimentally observed dipole strength for 16 < E < 20 MeV is "fitted" only at the expense of a bad adjustment at $E < E_0$. In particular this is quite obvious for the Sb case, whose two isotopes are known to possess oblate deformations and hence possess negative spectroscopic quadrupole moments $Q_s = -0.5$ b and $Q_s = -0.7$ b for $^{121}_{51}$ Sb $(I_0 = \frac{5}{2}^+)$ and $^{123}_{51}$ Sb $(I_0 = \frac{7}{2}^+)$ respectively. Hence we tried

to fit our data with two Lorentz lines, where the fitting process is constrained by the imposed characteristic oblate ratio $\sigma_2 \Gamma_2 / \sigma_1 \Gamma_1 = 0.5$. The parameters thus obtained are given in table 9 where it should be noted that the adjustment parameter for such a two-line fit is P = 51 whereas P = 104 for the Lorentz line parameters given in table 8 and obtained from a single line fit.

TABLE 9

Lorentz line parameters for a two-Lorentz-line fit of the GDR of Sb

σ ₁ (mb)	Γ_1 (MeV)	E ₁ (MeV)	σ ₂ (mb)	Γ_2 (MeV)	E ₂ (MeV)
217	4.17±0.1	15	96	4.72±0.1	16.9

Errors are the same as in table 4.

Using eqs. (5) and (6) we can now evaluate the intrinsic quadrupole moment of $_{51}$ Sb to be $Q_0 = -1.8 \pm 0.4$ b, a result which shows a surprising agreement with the Q_0' values -1.4 b and -1.5 b obtained for 121 Sb and 123 Sb respectively and deduced from the simple rotational model formula:

$$Q_0' = \frac{(I_0 + 1)(2I_0 + 3)}{I_0(2I_0 - 1)}Q_s. \tag{7}$$

This could be connected to the fact that Joshi et al. ²⁹) observed a rather strong coupling between the core and the motion of the odd proton for ¹²¹Sb and ¹²³Sb.

2.4. COMMENTS ON THE WIDTH OF THE GDR FOR 100 < A < 133 NUCLEI

The above experimental results concerning the width of the GDR for quasivibrational nuclei allow us to draw the following conclusions;

(a) It is never possible to obtain a very good single Lorentz line fit to $\sigma_{exp}(\gamma, tot)$ but a similar two-line fit gives better results.

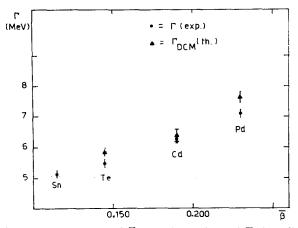


Fig. 11. Comparison between theoretical Γ_{DCM} and experimental Γ giant dipole resonance widths for Sn, Te, Cd and Pd.

- (b) The two main dipole states, computed by Arenhövel within the framework of the DCM for the tin isotopes, are in excellent agreement with the "two-Lorentz-line fits" to our experimental data.
- (c) It is possible to find a good two-Lorentz-line fit, corresponding to a prolate and an oblate nuclear deformation for the ¹⁰³Rh and Sb nuclei respectively.

Finally, it should be noted that the predictions concerning the spreading of the dipole strength, already discussed in sect. 1, agree to a remarkable degree with the experimentally observed evolution of the width of the GDR, and this in spite of the oversimplified spherical harmonic vibrator model used for such predictions.

In fig. 11 we show the theoretical spreading Γ_{DCM} of the GDR of the Te, Cd, and Pd nuclei normalized with respect to the GDR of the Sn isotopes, a result which is in good agreement with our experimental Γ -values.

3. Characteristic behaviour of GDR for $E > E_0$

If one fits the central part of the experimentally obtained GDR with either a single [eq. (3)] or two [eq. (4)] Lorentz lines, one always observes that $\sigma_{\rm exp}(\gamma, {\rm tot}) > \sigma_{\rm L}$ for the high energy tail $E > E_0$ of the GDR. However, the precise amount of such an experimental excess of dipole strength depends to a considerable extent on:

- (a) The exactness of $\sigma_{\rm exp}(\gamma, {\rm tot})$ which decreases with increasing photon energy because it depends on the accuracy of $\sigma(\gamma, n)$ and $\sigma(\gamma, 2n)$, where the former can become rather bad while the latter remains good in this energy region.
- (b) The choice of $\sigma_L[eq. (3)]$ or $\sigma'_L[eq. (4)]$ for the fitting process and hence the extrapolation of the GDR to the 20-25 MeV region. By comparing the GDR tails in figs. 5 and 6, one observes indeed that $[\sigma_{exp} \sigma_L]$ differs appreciably according to the above choice.

Since it is well known that such an excess dipole strength is to be attributed either to the isospin splitting of the dipole absorption ³⁰) or to an electric quadrupole interaction ^{13, 31}), we shall now try to identify the physical nature of this experimentally observed excess.

3.1. POSSIBLE (T+1) DIPOLE STATES

Dipole states with an isospin (T+1), created when a nucleus with isospin T absorbs a photon by electric dipole absorption E1, can be interpreted, within the schematic representation proposed by Ejiri et al. 32), as being isolated analogue states which interfere with the dipole states of the GDR. Using (e, e'p) reactions, Sugawara et al. 33) observed two such isobaric analogue states (IAS) for each of the Sn isotopes at the energies given in table 10, and referred to by arrows S_1 and S_2 in fig. 5.

But, contrary to the "structure" observed for $A \approx 90$ nuclei ³⁴), no similar effect can be clearly distinguished at the corresponding energies of our $\sigma_{\rm exp}(\gamma, {\rm tot})$ curves for the Sn isotopes.

The collective dipole mode of isospin (T+1) for A=60 and A=90 has been

Table 10					
Energies of isobaric analogue states observed by Sugawara ³³) in Sn isotopes, using (e, e'p) reactions					
116Sn	¹¹⁷ Sn	¹¹⁸ Sn	¹²⁰ Sn	124Sn	

	¹¹⁶ Sn	¹¹⁷ Sn	¹¹⁸ Sn	¹²⁰ Sn	¹²⁴ Sn
Energy of 1st IAS (MeV)	17.7	18	18.4	19.2	21
Energy of 2nd IAS (MeV)	19.3	19	19.8	20.6	22.3

predicted ³⁵) and observed ³⁶) at an energy ΔE above the mean energy E_0 of the GDR ($\Delta T = 0$), where we have

$$\Delta E \approx 60T/A \text{ MeV},$$
 (8)

with a strength obtained from the ratio of the squares of the matrix elements of the dipole transition ³⁰) given by

$$\frac{|\langle D \rangle_{T+1}|^2}{|\langle D \rangle_T|^2} = \frac{\sigma_{-1}(T+1)}{\sigma_{-1}(T)} = \frac{1}{T+1} \left(1 - \frac{3T}{2A^{\frac{3}{2}}} \right). \tag{9}$$

For $A \approx 90$, eq. (9) gives ≈ 0.10 , which allowed us to observe the $\Delta T = 1$ contribution to the GDR for such nuclei. For the Sn isotopes however, eq. (9) varies from 0.05 to 0.02 for ¹¹⁶Sn and ¹²⁴Sn respectively, and hence the observed differences between $\sigma_{\rm exp}(\gamma, {\rm tot})$ and the best Lorentz fits shown in figs. 5 or 6 are very small at the energies $E_{>}$ predicted by formula (8). Using (γ, p) reactions how-

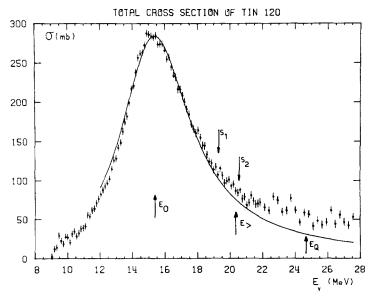


Fig. 12. Typical one-Lorentz-line fit for 120 Sn showing the inherent ambiguity in identifying analogue states S_1 , S_2 and $E_>$ or the E2 state E_Q .

	103Rh	Pd	Ag	Cd	¹¹⁵ In
E _M (MeV)	30	21.3	29.7	26.5	29.7
σ_0 (MeV · b)	1.75	1.38	1.90	1.75	2.1
0.06 NZ/A (MeV · b)	1.53	1.567	1.59	1.65	1.69
$\sigma'_{0}/(0.06 NZ/A)$	1.45	1.41	1.50	1.35	1.39
σ_{-1} (mb)	96	88.2	109	107	121
σ_{-2} (mb· MeV ⁻¹)	6.2	5.8	6.6	6.8	7.4
$\sigma_{-1}/A^{\frac{4}{3}}$	0.212	0.175	0.215	0.200	0.220
$\sigma_{-2}/A^{\frac{5}{3}}$	2.83×10^{-3}	2.44×10^{-3}	2.78×10^{-3}	2.68×10^{-3}	2.81×10^{-3}

TABLE 11
Integrated cross sections

The average errors $\Delta \sigma_0$ and $\Delta \sigma_0$ are ± 0.1 MeV · mb.

ever, these (T+1) dipole states have recently been observed in ¹²⁴Sn by Sorokin ³⁷) at an energy of 21.6 MeV and with an intensity of 14 MeV · mb as predicted by eqs. (8) and (9) respectively.

3.2. POSSIBLE QUADRUPOLE STATES E2

Results obtained at Livermore ¹⁹) were interpreted by Urbas and Greiner ¹³) in terms of two dipole states and five quadrupole states. As can be seen from fig. 12, where ¹²⁰Sn is given as an example, the excess strength $(\sigma_{\rm exp}(\gamma, {\rm tot}) - \sigma_{\rm L})$ observed at $E_Q \approx 1.6$ $E_0 \approx 24.7$ MeV might be interpreted in the above terms, but the experimental accuracies of the real photon absorption technique will have to be improved very much indeed before one can expect a proper identification of these E2 states as has been recently achieved by (e, e') reactions ³⁸⁻⁴⁰).

4. Integrated cross sections and sum rules

In table 11 we give the different integrated cross sections defined by

$$\sigma_0 = \int_{E_{\rm Th}}^{E_{\rm M}} \sigma_{\rm tot}(E) dE \quad {\rm MeV \cdot b},$$

$$\sigma_{-1} = \int_{E_{\rm Th}}^{E_{\rm M}} (E^{-1}) \sigma_{\rm tot}(E) dE \quad {\rm mb},$$

$$\sigma_{-2} = \int_{E_{\rm Th}}^{E_{\rm M}} (E^{-2}) \sigma_{\rm tot}(E) dE \quad {\rm mb \cdot MeV^{-1}},$$

$$\sigma_0' = \frac{1}{2} \pi \sigma_1 \Gamma_1 = {\rm area \ under \ a \ single \ Lorentz \ line}.$$

Once again we observe that σ_0 exceeds the classical TRK sum rule $(0.06 \, NZ/A)$ as soon as the upper integration limit $E_{\rm M}$ attains ≈ 24 MeV. Furthermore the value of the ratio $(\sigma'_0A/0.06 \, NZ) = 1.3 \pm 0.15$ confirms once more previously obtained experimental data for heavy nuclei obtained in this laboratory $^{41, \, 42}$), even if the

as defined in the text							
116Sn	¹¹⁷ Sn	¹¹⁸ Sn	¹²⁰ Sn	¹²⁴ Sn	Sb	Те	¹³³ Cs
29.7	21.6	23	29.7	23	29.7	26	24.1
1.86	1.57	1.69	2.14	1.62	2.1	2.05	2.15
1.71	1.72	1.73	1.75	1.79	1.78	1.85	1.93
1.29	1.25	1.25	1.34	1.17	1.39	1.37	1.38
113	102	107	128	104	125	125	136.9
7.15	6.80	7.0	8.1	6.9	7.9	7.9	9.05
0.203	0.181	0.188	0.220	0.171	0.210	0.198	0.205
2.67×10^{-3}	2.51×10^{-3}	2.55×10^{-3}	2.86×10^{-3}	2.31×10^{-3}	2.72×10^{-3}	2.52×10^{-3}	2.70×10^{-3}

single Lorentz line fit to the $\sigma_{\rm exp}(\gamma, {\rm tot})$ points does not give an excellent fit. Similarly we found that $\sigma_{-1} \approx (0.2 \pm 0.02) A^{\frac{1}{3}}$ mb, a value which again agrees with our previous results ^{41, 42}) but disagrees with the value 0.35 $A^{\frac{1}{3}}$ or 0.3 $A^{\frac{1}{3}}$ mb as computed by Levinger ⁴³) for an isotropic harmonic oscillator potential and for a finite square potential respectively. Finally, the observed average value $\sigma_{-2} \approx (2.6 \pm 0.2) A^{\frac{1}{3}}$ mb·MeV⁻¹ is also confirmed by our previous data and agrees with the theoretical prediction $\sigma_{-2} \approx 2.25 A^{\frac{1}{3}}$ as proposed by Migdal ⁴³).

5. The average energy $E_{\rm D}$ for E1 photon absorption

Within the framework of Steinwedel and Jensen's ⁴⁴) interpretation of the collective model proposed by Goldhaber and Teller ⁴⁵) applied to the GDR, the restoring force is to be associated with the symmetry energy term $K(N-Z)^2/A$ appearing in the semi-empirical mass formula. This leads one to the following expression for the average E1 absorption energy:

$$E_{\rm D} = \frac{2.08}{r_0 A^{\frac{3}{4}}} \hbar c \sqrt{\frac{8KNZ}{M^* C^2 A^2}} \approx 78A^{-\frac{1}{4}}, \tag{10}$$

where

 $M^* = \text{effective nucleon mass} = M/(1+\alpha);$

 $r_0 A^{\frac{1}{2}} \approx 1.2 A^{\frac{1}{2}}$ fm = radius of A(N, Z) nucleus;

 $\alpha = 0.3 =$ coefficient of the exchange term.

The earliest Goldhaber and Teller interpretation, however, supposed the restoring force to be proportional to the nuclear surface area $A^{\frac{2}{3}}$ and since the nuclear mass is proportional to A they found the average El absorption energy to be:

$$E_{\rm D} = C(A^{\frac{2}{3}}/A)^{\frac{1}{2}} = CA^{-\frac{1}{6}}.$$
 (11)

In fact, eq. (10) does indeed agree reasonably well with the observed evolution of the average energy of the E1 absorption for A > 100 nuclei, although eq. (11) is

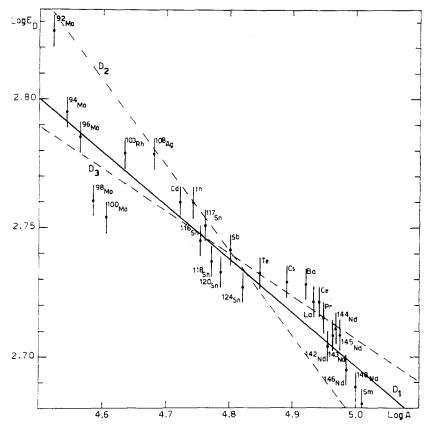


Fig. 13. Experimental results of the average energy E_D of the GDR versus the mass number A with the best fit $E_D = 41.8 \ A^{-1/4.8}$ (line D_1). Two fits with $A^{-\frac{1}{2}}$ and $A^{-\frac{1}{6}}$ laws are also shown as lines D_2 and D_3 respectively.

better suited to represent the behaviour of the energy of the maximum of the GDR E_{max} for relatively light nuclei as can be seen from the following example:

$$\frac{E_{\text{max}} \text{ for }^{124}\text{Sn}}{E_{\text{max}} \text{ for }^{28}\text{Si}} = \frac{15.3}{20} \approx \left(\frac{124}{28}\right)^{-\frac{1}{6}}$$

A new approach to the hydrodynamic model of the GDR, taking the properties of the nuclear surface and in particular its thickness into account, has been proposed by Bach and Werntz ⁴⁶), who noted that the Goldhaber-Teller model provides a simple phenomenological expression for the transition density of collective states. This model shows also that an $A^{-\frac{1}{6}}$ behaviour is to be expected for E_D . In fig. 13 we present recent data obtained at Saclay for 100 < A < 150 nuclei, where we have taken the following first approximations as being appropriate for the energy of the

El absorption:

 $E_{\rm D}=E_1$ for a single Lorentz line fit,

 $E_{\rm D} = \frac{1}{3}(E_1 + 2E_2)$ for a two-Lorentz-line fit on a prolate nucleus such as ¹⁵⁰Nd.

The best fit for all nuclei making up our experimental data is given by $E_D = 41.8$ $A^{-1/4.8}$, designated by the straight line (D_1) in fig. 13, which lies between $A^{-\frac{1}{3}}$ of eq. (10) and $A^{-\frac{1}{4}}$ of eq. (11). This intermediate result could possibly be connected to the recent "three-fluid hydrodynamic model" of the GDR proposed by Mohan et al. ⁴⁷) in which deviations from the $A^{-\frac{1}{3}}$ law should be expected if the A-dependence of the parameter K, defining the restoring energy densities, is also taken into account.

By imposing restraints $A^{-\frac{1}{4}}$ and $A^{-\frac{1}{4}}$ on the above fitting process one obtains the straight lines D_2 and D_3 described respectively by

$$E_{\rm D} = 76.8A^{-\frac{1}{3}} \,{\rm MeV},$$

$$E_{\rm D} = 34.5 A^{-\frac{1}{6}} \,{\rm MeV},$$

and which are also shown in fig. 13. Finally one can also see, from fig. 13, that for each of the three isotope sets Sn, Nd and Mo, the $A^{-\frac{1}{3}}$ law is closer to the experimental data than the $A^{-\frac{1}{4}}$ expression.

We are greatly indebted to Dr. H. Arenhövel for his calculations on the Sn isotopes and we also wish to thank the USSR Bureau of Isotopes for lending us the isotopes concerned.

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