Assignment11

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Assignment 11: Build a binary classifier based on k random features for each digit against all the other digits at MNIST dataset. dataset

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```
In [1]: import numpy as np
        import pandas as pd

In [2]: #get train data
        train = pd.read_csv("mnist_train.csv")

In [3]: #get test data
        test = pd.read_csv("mnist_test.csv")

In [4]: train = np.array(train)
        test = np.array(test)

In [5]: tr_ans = train.T[0]
        ts_ans = test.T[0]

In [6]: tr_data = train.T[1:].T
        ts_data = test.T[1:].T

In [7]: #get traing_set samples - feature matrix
        A_tr = np.zeros((len(tr_data), (28*28)))
        for i in range((28*28)):
            for j in range(len(tr_data)):
                A_tr[j][i] = tr_data[j][i]

In [8]: #get traing_set samples - feature matrix
        A_ts = np.zeros((len(ts_data), (28*28)))
        for i in range((28*28)):
            for j in range(len(ts_data)):
                A_ts[j][i] = ts_data[j][i]

In [9]: #make a label of each answer
        Y_tr = np.zeros((10, len(tr_ans)))
        for j in range(10):
            for i in range(len(tr_ans)):
```

```
            if tr_ans[i] == j:
                Y_tr[j][i] = 1
            elif tr_ans[i] != j:
                Y_tr[j][i] = -1
```

In [10]:
```
#A function that make random vector

def make_random_vector(k):
    random_vector = np.zeros((k, 28*28))
    for i in range(k):
        random_vector[i] = np.random.normal(0, 1, 28*28)


    return random_vector
```

In [11]:
```
#make a random vector

random_vector = make_random_vector(256)
```

In [12]:
```
random_vector
```

Out[12]:
```
array([[ 0.54870165,  1.12766529, -0.27699064, ..., -1.48480725,
        -0.10300457, -0.87199142],
       [-0.33577638, -0.15156703,  1.80802044, ..., -0.80857839,
         0.08739667,  1.19556634],
       [ 2.95636195,  0.30883941, -0.99559995, ...,  0.08278613,
        -0.4975703 ,  0.56519748],
       ...,
       [ 2.64163253, -1.04876144, -0.88058121, ..., -0.41628193,
         0.77941655,  1.29828029],
       [ 0.60578067,  1.47866025,  0.91206855, ..., -3.15188598,
         1.03591192,  0.89351609],
       [-1.1707442 , -0.5401223 , -2.53516925, ...,  1.16423444,
        -1.55792439,  0.00327155]])
```

In [14]:
```
temp_random_A_tr = np.dot(A_tr, random_vector.T)
```

In [16]:
```
temp_random_A_tr
```

Out[16]:
```
array([[-2833.28483025, -1154.99022912, -1361.09250435, ...,
        -3202.49897467, -2278.81361201, -1458.55421086],
       [  203.74082727,  -998.07278413, -3156.47585379, ...,
        -2070.16712274, -3791.81903608,  1561.07736772],
       [ -153.37177308,   114.96003362,  2388.3284299 , ...,
        -2339.36553202,  -955.7505433 ,  -199.40335125],
       ...,
       [  468.42595313,    80.00806737,   -44.27421658, ...,
        -1856.27090864, -4482.25654911,  5641.30171757],
       [-1601.96432011,  -877.55900419,  1880.92748682, ...,
```

```
                1152.17906222, -3478.40394125,  2607.19238643],
              [ 1068.27653866,  -778.34372258,  -912.72572259, ...,
                1158.82852579, -3160.9534617 ,  1067.852894  ]])
```

In [17]:
```
random_A_tr = np.zeros((len(tr_data), 257), dtype='float')
for i in range(257):
    for j in range(len(tr_data)):
        if i == 0:
            random_A_tr[j][i] = 1
        else:
            random_A_tr[j][i] = temp_random_A_tr[j][i-1]
```

In [19]:
```
random_A_tr
```

Out[19]:
```
array([[ 1.00000000e+00, -2.83328483e+03, -1.15499023e+03, ...,
         -3.20249897e+03, -2.27881361e+03, -1.45855421e+03],
        [ 1.00000000e+00,  2.03740827e+02, -9.98072784e+02, ...,
         -2.07016712e+03, -3.79181904e+03,  1.56107737e+03],
        [ 1.00000000e+00, -1.53371773e+02,  1.14960034e+02, ...,
         -2.33936553e+03, -9.55750543e+02, -1.99403351e+02],
        ...,
        [ 1.00000000e+00,  4.68425953e+02,  8.00080674e+01, ...,
         -1.85627091e+03, -4.48225655e+03,  5.64130172e+03],
        [ 1.00000000e+00, -1.60196432e+03, -8.77559004e+02, ...,
          1.15217906e+03, -3.47840394e+03,  2.60719239e+03],
        [ 1.00000000e+00,  1.06827654e+03, -7.78343723e+02, ...,
          1.15882853e+03, -3.16095346e+03,  1.06785289e+03]])
```

In [20]:
```
#do a QR composition

q, r = np.linalg.qr(random_A_tr)
r_inverse = np.linalg.pinv(r)
temp_z = np.dot(r_inverse, q.T)
```

In [22]:
```
temp_z
```

Out[22]:
```
array([[ 3.07224468e-05,  9.17152216e-05,  4.17815256e-05, ...,
          4.59001282e-05,  1.14725489e-04,  1.01331885e-04],
        [-1.77147811e-08,  3.50461035e-08, -8.80864491e-08, ...,
          1.03429964e-08, -6.66485935e-09,  2.23748797e-08],
        [ 5.46330833e-09,  6.34368116e-08, -9.04780604e-09, ...,
          1.29598235e-08, -7.73219327e-09, -5.63143398e-08],
        ...,
        [-4.00024558e-09, -3.92498252e-08, -1.80226617e-08, ...,
         -6.40710566e-08,  3.82386165e-08,  2.79156956e-08],
        [-2.50038449e-08, -6.87845787e-08, -2.80195612e-08, ...,
          1.55298562e-08, -1.49806586e-09, -1.10590615e-08],
        [-2.73903669e-08,  4.35280336e-08, -4.77416426e-08, ...,
          3.59503809e-08,  3.95549321e-08,  5.83624436e-09]])
```

1. Compute an optimal model parameter using the training dataset for each classifier f_d(x, w)

```
In [23]: #obtain co-efficients
         z = np.zeros((10, 257))
         for i in range(10):
             z[i] = np.dot(temp_z, Y_tr[i])
             print("Optimal model parameter of ", i, " classifier: ")
             print(z[i])
             print('\n\n')
```

```
Optimal model parameter of  0  classifier:
[-7.15886737e-01  5.85992135e-06 -2.63025417e-05 -2.23100894e-05
 -7.21698996e-06  1.24677896e-05 -9.41763602e-06  3.51994259e-06
 -4.84347805e-06 -4.87307199e-06 -3.64935485e-06  1.57969402e-05
 -1.17600882e-05  8.41908155e-06 -4.25734931e-06 -5.67264920e-07
 -4.58246224e-06  1.81441553e-05 -2.59266189e-05 -6.50807581e-06
  1.13492817e-05  1.35355724e-06  7.06636937e-07  2.67606063e-05
 -1.56263044e-06 -3.44556298e-05  4.75017910e-06 -8.40373211e-07
  6.17256756e-06 -1.45639191e-05 -1.66357720e-06 -3.38274804e-06
 -1.66785081e-05  1.81988774e-05  1.88951298e-05  1.84904642e-05
 -1.55295194e-05  6.28631324e-06 -1.09809532e-05  7.26771331e-07
 -9.07210372e-06  7.53195241e-06 -1.20967215e-05 -2.56082656e-06
 -1.15899852e-05 -5.30395993e-06  5.89051558e-06  1.16899585e-05
 -6.58792546e-07 -6.75379619e-06  1.49283610e-05 -1.34220842e-05
 -1.17633504e-05  1.86908872e-06 -9.50341808e-06  1.01866343e-05
  2.47022237e-05  1.11198185e-05  1.37403387e-05  7.24162508e-06
 -1.22395332e-05 -9.91830293e-06  4.12159625e-06  1.70496921e-06
 -1.56284054e-05  7.05631308e-06  1.93642480e-05 -1.53563594e-05
  1.33957365e-05 -3.02361804e-06 -1.52387913e-05  3.02069689e-06
 -9.90025637e-07  2.01519396e-05  2.85745498e-05 -7.57831301e-06
 -5.13288458e-06  9.11923246e-07 -2.66753366e-05  1.40731272e-05
 -2.34639037e-05 -5.38716055e-06 -4.63131383e-06 -2.77379504e-05
 -1.35478926e-05  1.57938018e-05  5.66921498e-06 -2.51699239e-05
  2.11209563e-05  2.76908480e-05 -7.48098382e-06  3.54534288e-06
  5.36656596e-06  1.37356658e-06  1.09616302e-05  2.30045050e-05
  2.54786088e-05  5.32889845e-06 -1.34827557e-06  1.15664162e-05
  2.53636414e-05 -8.01567771e-06 -1.05214119e-05  2.89349702e-05
 -1.38997882e-06  1.61162961e-05  6.07696060e-07 -5.36308657e-06
  1.00623000e-05 -1.94555851e-05  7.50711859e-06 -1.20987399e-05
 -1.16829237e-05 -3.04524008e-05 -1.32160655e-05  2.20944722e-06
  1.60602233e-05  3.63644112e-06  7.91038246e-06  5.79553639e-06
 -3.80508921e-06 -1.18351490e-05 -1.45668696e-06  5.87075351e-06
 -4.83257548e-07  2.95614969e-05  2.05621273e-05 -1.14418994e-05
 -2.32861331e-05  9.79979175e-06 -6.64364339e-06  2.11456964e-06
  2.18142415e-05 -2.33096417e-05 -1.18013062e-05  4.94817096e-06
 -2.36458722e-05  4.25411659e-06 -2.04457312e-05  7.46632240e-06
  2.14845484e-06  2.44865149e-05  3.19490069e-05 -2.33058982e-05
  3.73151831e-06 -1.40708630e-05  7.95598792e-06 -2.41203797e-06
```

```
  6.04754869e-06  9.70422165e-06  2.93590040e-05 -8.70566115e-06
  4.66191119e-06 -1.89009095e-05 -1.84122450e-05  2.43486997e-05
  1.07316984e-05 -8.77402211e-06 -2.64808545e-07  3.01812770e-05
 -6.98185954e-07  1.98904584e-05  1.29448190e-05 -6.92259661e-06
  7.89566600e-06 -1.87564403e-05  1.35171974e-05 -2.01936456e-05
  1.81664705e-05  1.30858755e-05  1.29745494e-05  9.72816663e-07
  2.62987344e-05  1.04022087e-06 -2.33371754e-06  7.13336765e-06
  2.63965750e-05 -4.40528119e-06 -1.53178399e-05 -1.02218518e-05
  1.51617396e-05  5.04245472e-06  3.11249753e-06 -7.18650642e-06
 -1.42353553e-05  9.29322877e-06  1.48560482e-05  3.15600636e-05
  1.33007310e-07  7.43943908e-07  4.18806419e-06  9.33680626e-06
 -6.38744962e-06  6.87318973e-06  2.02001466e-05  1.01012913e-05
 -1.27375434e-05  7.51491888e-06 -1.37298248e-05 -3.37574110e-05
  1.75581763e-05  9.33067183e-06  1.64705759e-05  1.72861820e-05
 -5.51399061e-06  1.51824534e-05 -1.76406386e-05  1.98422200e-05
  2.26162180e-05  7.62210491e-06 -1.17043553e-06  1.41908438e-05
 -4.92022167e-06  8.32693974e-07 -2.32479155e-06  5.48964331e-06
  1.00394190e-05 -1.12358193e-05 -1.25797340e-05 -1.11322516e-05
 -1.52171600e-05  1.55012009e-05 -8.31652290e-06  7.21647772e-07
 -1.06736598e-05  7.84840343e-06 -4.34762214e-06  5.40347083e-06
 -1.10933464e-05 -1.51825004e-05 -2.22022849e-05 -6.19333826e-06
  4.23674615e-05 -1.16611042e-05 -2.38584085e-06  9.74474196e-07
 -5.96321113e-06  4.46482660e-06 -9.70004940e-06 -1.12625950e-06
  1.58003281e-05 -1.32129037e-05 -1.10855617e-05  2.49359507e-05
 -5.22501874e-06  2.05709101e-05  1.17828595e-06 -1.37056062e-05
 -1.29298107e-05 -1.23469581e-05 -1.19578258e-05  1.27637841e-05
 -1.37780651e-05  1.15432356e-05 -7.85108300e-07 -1.34701506e-05
 -3.42363747e-05]



Optimal model parameter of  1  classifier:
[-5.13794454e-01 -2.74766141e-05 -1.61668573e-05  2.44414796e-05
  2.22793510e-05  4.21648828e-06  6.76308944e-06 -4.74834639e-06
 -2.71508253e-06  7.32685098e-07  2.78185821e-05  1.07566422e-05
 -5.76233059e-06 -1.28472528e-05  8.95358961e-06 -1.74317130e-05
 -2.08348779e-06 -6.90956133e-06  9.57931605e-06  1.53303983e-05
 -1.03299274e-05  1.09692597e-05 -1.64759216e-05  1.13320163e-05
 -8.68246933e-06  8.51892168e-06 -3.52413687e-08  5.52837635e-06
 -8.93522949e-07 -7.21416568e-06 -1.22673397e-05  7.75128090e-06
 -1.82114881e-06  2.40196562e-05  2.47098073e-05  2.47298785e-05
  1.69011739e-05 -1.23171541e-07  5.46123505e-07  2.88374174e-06
 -3.69598816e-05 -2.30503908e-05  2.16690032e-05  1.81922973e-06
 -4.63623043e-05  2.63549695e-05  1.62773347e-05  3.40906323e-06
 -3.36452402e-05 -1.71078615e-05  3.07699565e-06  1.74670947e-05
 -2.56963678e-05 -7.78820035e-06  2.91739873e-06 -3.91331593e-06
 -1.26812416e-05  4.49375433e-06 -5.25397076e-06  1.43224928e-06
  1.80294324e-05 -6.53608239e-06  1.26337398e-05 -2.95235118e-06
```

```
 5.57482957e-06  1.72068547e-05  3.11182329e-05 -5.68427318e-06
 7.19937813e-06 -2.08713500e-05 -1.20196883e-05 -9.15859896e-07
-2.78815514e-06  5.39977809e-06  2.87376738e-05 -2.20882975e-05
-4.25041965e-06  3.81298189e-05  7.42206004e-06  2.36079466e-05
 5.34649085e-06 -1.06415102e-05  2.71671455e-06  6.09403952e-06
-2.02106071e-05  3.23593661e-05  6.79994477e-06 -6.50215551e-06
 1.23283535e-05 -5.90365563e-06  1.68511899e-05  9.91779157e-06
 1.78852408e-05 -1.73493690e-06  3.14059261e-05 -1.87497501e-05
 2.11641147e-05  2.27265755e-05  9.77890255e-06  1.33655128e-05
 1.65772199e-05 -2.22715795e-05  9.05024913e-08 -3.72891992e-06
 2.75093502e-05 -1.91288135e-05  9.49281656e-06  1.00936493e-05
 8.97414428e-06  2.95845939e-06 -2.39460941e-05 -1.22866990e-05
-2.78299699e-06 -1.45048649e-05  1.00649440e-05  5.07453466e-06
-2.74057273e-06  1.86019561e-05 -2.61617115e-05  6.31810624e-06
 5.30927319e-06  2.22287604e-05  2.31610326e-05  4.44398938e-06
-1.86579791e-05  1.47295481e-05  2.02927411e-05 -7.37184252e-07
-2.85630597e-05  1.51025247e-06 -5.49676030e-06 -5.04422147e-06
-2.22733963e-05 -6.63140193e-06  6.62381685e-07 -1.18030739e-05
-2.49628333e-05  1.56360371e-05  1.14147203e-05  1.59956113e-05
-7.67848910e-06 -1.25866370e-05  4.45710996e-05 -1.05702277e-05
 4.21388788e-06 -1.39942087e-05 -9.07898133e-07 -1.89105719e-05
 1.53343371e-05  4.13186537e-06  3.80429060e-06  1.22338455e-05
-5.72480888e-06 -2.86027207e-06  2.27418806e-05  2.20081904e-05
-6.61769374e-06  2.54067161e-06  2.37826577e-05 -6.21645641e-06
-1.02315326e-05 -1.32828979e-05  5.84981386e-06 -1.31474572e-05
 2.52540353e-06  1.35477064e-05  2.62996840e-05 -4.80763633e-06
-2.78363203e-06  3.51997313e-05  8.08834549e-06  1.10843898e-07
 2.34880313e-05  3.13980385e-06  1.99969797e-05 -5.01112852e-06
 3.52992233e-06 -1.52319267e-05 -3.08517039e-06 -1.54783990e-05
-1.53114830e-05 -5.38754862e-07  1.54109695e-05 -1.53279443e-05
 2.17728002e-05  5.91231488e-06 -2.82256660e-06  9.24005028e-06
-3.76926500e-06  3.64334493e-05 -5.53639792e-06  1.30656318e-05
-2.61506622e-05  1.09918602e-05 -7.30363086e-07  5.21999787e-07
 8.78709354e-06  1.78793965e-05 -1.58680172e-05 -4.97083478e-06
-7.52978835e-06  1.52742851e-05 -1.16834254e-06 -1.75928631e-05
-9.99131550e-06 -3.77465552e-06 -9.59192914e-07 -2.50525027e-05
-9.03470364e-06 -4.58794538e-06 -1.04852994e-05  2.62080674e-05
 2.62992251e-06 -1.63616430e-05  1.09614088e-06  1.46162255e-05
-3.05149247e-05 -1.48274912e-05 -1.87328267e-05  1.64507535e-05
 1.37204992e-05  4.30656253e-06  8.52178007e-06 -6.44799136e-06
 1.55025340e-05  7.49652236e-06 -1.71649118e-06 -6.21771709e-06
 1.31710797e-05 -1.56162242e-05 -9.49927047e-06 -1.34927104e-05
 3.17052228e-05  8.39249977e-06 -1.60838986e-05  1.15871324e-05
-1.02771440e-05 -1.43415499e-05  1.33161432e-05 -2.61095389e-05
-1.29768095e-05 -1.24772438e-05  9.64038624e-06  1.40240853e-05
-9.51230092e-06  4.89073725e-06 -2.28387563e-06 -8.91982033e-06
-1.50117420e-05 -1.58037836e-05 -3.51744798e-05  1.32215990e-05
-1.68897404e-05  2.44658048e-07 -5.08488850e-06 -5.02359592e-07
```

```
 -2.90272702e-05]



Optimal model parameter of  2  classifier:
[-9.14635868e-01  1.71795892e-05  1.00647090e-05 -1.44479648e-05
  3.25149299e-06  2.25060567e-06 -1.45680214e-05 -2.04426325e-07
  1.63098674e-05  1.63383104e-05 -1.14216317e-05 -1.62101091e-05
 -1.99220234e-05  2.05733953e-05  1.50332154e-05  2.19838955e-05
  5.29831405e-06 -1.63964233e-05 -3.41111875e-05 -1.59166372e-05
 -1.90929922e-05 -9.87144780e-06 -2.70304569e-05  7.58666823e-06
 -1.32402762e-05 -2.00091401e-05 -1.48289963e-05  1.01703440e-05
  2.90437463e-06  1.51580953e-05  6.48908792e-06  1.49762231e-05
  3.26852171e-06 -9.66227420e-06 -1.62658597e-05  1.26381113e-06
  2.68330479e-06  1.61894018e-05 -9.56395561e-06 -2.19637137e-05
  3.70311651e-05 -1.49114572e-05 -2.60419706e-05 -1.58846594e-05
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  4.38872589e-06  1.55927569e-05  5.24181822e-07 -8.49704924e-06
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  6.10117350e-06 -2.11308787e-05 -5.28191501e-06 -7.37707174e-06
 -1.75094014e-05 -5.76266890e-05  3.56760040e-06  5.94368441e-06
  4.07739544e-05 -1.05951416e-05 -1.13292474e-06 -6.31217657e-06
  2.66383426e-05  1.43297358e-05 -4.31330181e-06  7.02432084e-06
 -7.08002572e-06 -1.96526613e-06  2.62245103e-05  1.33706950e-05
 -2.99503199e-06 -3.38497650e-06 -2.38400727e-05  6.53580098e-06
 -1.17665615e-06 -2.46080957e-06  6.69623430e-06 -2.41530954e-05
  2.00158382e-05  1.80120135e-05  3.26435355e-06 -4.90925034e-06
  3.27061293e-06 -3.33320903e-05 -2.01827313e-05  1.18807047e-05
 -5.32739348e-06  3.69487380e-05  4.03932252e-05  2.75293224e-06
  1.89929342e-05 -4.48782625e-07 -8.63741164e-06 -1.26876684e-06
  1.50454792e-05 -3.69122584e-06 -3.40556889e-05 -2.80931438e-05
 -1.23248712e-06  9.96926255e-06  2.03345455e-05 -1.36487661e-05
 -8.65215850e-06  7.39626802e-06  5.64679158e-06 -2.62547749e-05
  2.68852205e-05 -5.06819617e-06  3.37148429e-06 -3.46337874e-06
  1.93799854e-05 -1.90649097e-05  1.21117314e-05 -8.40161945e-06
  2.02231637e-05  2.08287277e-05 -3.79165057e-06  1.35875709e-06
  1.49582380e-05  1.47164691e-05 -9.32957677e-07  1.70882317e-05
  1.62036979e-05 -2.04764620e-05 -1.36374556e-05  4.80094082e-06
 -3.12373079e-06]



Optimal model parameter of  3  classifier:
[-9.49564601e-01 -3.16493056e-06 -1.37856204e-05 -1.52061825e-05
  1.91324996e-05 -9.41720399e-06  4.15470430e-06  4.17435679e-06
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  2.12496968e-06 -1.21168400e-05  2.25142672e-05 -6.36887290e-06
  8.18445145e-06  1.46553992e-05  1.27203389e-05 -6.88077493e-06
 -1.44152929e-05  2.46465247e-07  1.18197857e-05  1.92073704e-05
 -9.17838018e-06 -8.00606159e-07 -8.78461284e-06  3.47941076e-06
  2.43951449e-05  1.64099787e-05 -3.81756773e-06  3.63462475e-06
 -8.78424570e-06 -2.38880411e-05  1.27366632e-05 -1.53097768e-05
 -8.38250456e-07 -6.17179558e-07 -3.24109700e-08  6.90129354e-06
 -2.31717160e-06 -1.19275466e-05 -3.84584069e-06 -2.07158149e-06
  4.44326975e-05 -8.52206273e-06  6.23530297e-06  1.68484711e-06
  8.65954124e-06  2.42464360e-06  4.51611912e-06 -6.83531221e-06
  1.44421536e-06  4.45070932e-06  4.25722820e-06  1.25495491e-05
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  3.30115856e-07 -1.00759970e-05 -4.30582488e-05  4.28399388e-06
  2.29239624e-05 -1.63450943e-05  1.95206043e-05 -2.95411917e-05
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 -2.05768615e-05  1.65831388e-05  2.28017443e-05  1.23537701e-05
 -2.71484214e-05 -3.49393760e-06 -9.36582024e-06  1.46611482e-05
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 -2.13372116e-05 -2.10569045e-05  1.91913285e-05  3.32657248e-05
 -2.49605515e-05 -1.94757795e-05 -3.41710046e-06 -1.00414982e-05
 -2.71702255e-06  1.58058808e-05  1.33454208e-05  1.26119157e-05
 -1.16535665e-05 -4.81259756e-06  4.73840995e-05 -2.31271076e-05
  9.85050933e-06  2.41964861e-05  7.17284832e-06 -6.70452634e-06
  2.21331159e-05  1.48606837e-05  1.67072083e-05  1.28638915e-05
  9.71407890e-06  5.70202563e-06  5.51556799e-06 -1.36527528e-05
 -1.57985482e-05  3.20436407e-05 -5.72489330e-06 -7.92975012e-06
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  3.88802305e-06  5.39425823e-06  2.42434680e-05 -3.14263333e-06
  2.76702211e-05]



Optimal model parameter of  4  classifier:
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 3.29677624e-05 -1.31713935e-05  1.33759822e-05  5.65913589e-06
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-1.58187115e-06  2.89175708e-05  2.44757297e-05  3.68300760e-06
-1.75822156e-05 -8.05394210e-06  1.20186540e-05  1.59901752e-05
 1.50444211e-05 -2.58791261e-06 -3.06054402e-06  2.41252377e-05
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 5.21952612e-06  8.98886003e-06  1.57923153e-05 -2.44073650e-07
-1.27256772e-05  1.55246571e-05  7.38921965e-06  3.04648520e-05
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Optimal model parameter of  5  classifier:
[-6.06466301e-01 -2.98121212e-05  2.12001594e-05  6.19819517e-06
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 -1.76702025e-05 -4.09199422e-05  2.29241420e-05  2.57859260e-05
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  3.28806004e-05  2.78699833e-05  5.41603481e-07 -1.74999114e-05
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 -2.06852670e-05 -6.40893187e-06 -5.03338740e-05  3.54262883e-05
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  4.87695729e-05 -4.26993191e-05  3.17878354e-05  3.28349597e-06
  4.75153805e-05 -2.60435801e-05  6.76211311e-06  1.70764807e-05
  4.92167249e-06  2.61947154e-05 -5.46702729e-06  2.13981974e-05
 -4.77429869e-06  9.94679143e-06  8.15387400e-06 -9.78473015e-06
 -1.26695119e-05 -1.13087504e-05 -2.25866341e-05 -2.29141587e-05
 -6.42963442e-06 -8.76586766e-06  5.24742669e-06 -8.85155605e-06
 -2.64548156e-05  8.79257265e-07  2.02041048e-05 -2.65346305e-05
 -1.03219553e-05  9.27727096e-06 -7.79957351e-06 -2.25873280e-05
  9.69580798e-06  5.62977043e-05 -3.60905555e-05 -4.41225534e-06
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 -1.05243505e-05  8.82678338e-07 -3.33131300e-05 -1.42309204e-05
  1.21878529e-05  5.16331443e-06 -1.80585440e-05  1.83270182e-05
  9.65305233e-06  5.71145654e-06  4.14719014e-05  1.35520941e-05
 -2.67013947e-05 -1.48926817e-05 -1.06657869e-05  3.39459586e-05
 -1.23113677e-05 -1.27168352e-05  5.06016081e-07 -4.18549580e-05
 -4.95368354e-05  5.27762813e-05  1.44515397e-05  4.36934332e-06
  2.19505993e-06 -7.14209683e-06  1.16635414e-05  2.47241722e-05
 -3.34424434e-05 -1.88857374e-05 -5.52482320e-05  1.63118711e-05
  7.78032144e-07  1.50149436e-05 -3.82053671e-06  3.73548583e-06
 -5.88380130e-06  1.69830106e-05 -3.19727785e-05  2.04589415e-05
 -2.44702305e-05  2.92622693e-05 -7.11115437e-06 -3.30636661e-05
  5.23845848e-06  1.96544092e-05 -2.56992854e-05 -2.85309198e-05
  1.62211841e-05 -1.59524589e-05  2.16254286e-05 -3.36354593e-06
  8.78475357e-06 -5.02226207e-06 -1.15640579e-06 -7.32625619e-06
 -2.00196878e-06 -2.25752666e-05 -8.24587095e-06  1.40793785e-05
 -3.71132858e-05 -2.27444460e-05  1.25528479e-05  3.18684294e-05
 -2.25689620e-05  1.04097558e-05  2.60527724e-05  1.61789619e-06
 -3.63362568e-05  5.76488230e-07  1.32064708e-05 -1.40536614e-05
  3.32364737e-05 -1.29648508e-06 -4.99524326e-05 -4.97824622e-05
 -2.10138716e-06  1.87206277e-05 -5.65934749e-06 -1.53562076e-05
  2.10933253e-05  2.69458468e-06 -6.44557635e-07 -1.95861824e-06
  3.01595961e-05 -2.47073344e-05 -2.01492213e-07  1.43517972e-05
 -4.26451176e-05 -3.17712357e-05 -1.25279389e-05 -1.82431083e-06
  2.24912954e-05 -4.87930208e-06  2.79239175e-05 -1.27294525e-05
  4.09288236e-06 -9.12988980e-06 -6.58416520e-06 -1.70575903e-05
  3.58844914e-05 -6.40625638e-06  6.33598291e-06  2.71487093e-06
 -2.95882509e-06 -1.66450106e-06  4.89748015e-06  5.16478132e-06
  7.99370718e-06 -3.47643559e-05  4.00866675e-05  9.00131675e-06
 -3.15862293e-05 -7.58509678e-06 -6.23494663e-06  1.79148402e-05
 -2.52567267e-05  2.68793587e-05  2.23600675e-05  3.62143453e-06
 -3.32726044e-05  2.43156219e-05  1.00198607e-05 -1.14722767e-05
  4.85180392e-06  1.50626264e-05 -1.26575474e-06 -4.99761966e-06
 -5.88335203e-06  1.51861492e-05  3.27791784e-06  6.86365347e-06
  8.80152361e-06 -4.98106376e-05 -7.09231980e-06 -7.12392359e-06
  1.10720646e-05  8.54212744e-06 -1.44045790e-05 -8.87551891e-06
  6.41729213e-06 -2.22837663e-05 -1.27542937e-05  9.11700300e-07
  3.42542297e-05]



Optimal model parameter of  6  classifier:
[-8.39080517e-01 -1.43796964e-05 -1.18703049e-05  1.37412047e-05
 -2.84367452e-05  1.58470924e-05  2.70647725e-06  1.32631641e-05
 -2.24230553e-05  7.58688239e-06 -1.05527512e-05  2.83801978e-05
  1.62611279e-05 -7.68922781e-06 -1.01708387e-06 -2.49177798e-05
  4.63450125e-06 -3.50256431e-05  4.53873053e-05 -1.23341409e-05
  1.73235803e-05 -3.06102813e-06  1.07473362e-05  3.26376299e-06
 -1.29860287e-05  2.22384986e-06  2.88408614e-05  2.12639460e-05
```

```
-2.24060114e-05 -1.25797583e-05 -1.12190950e-05 -9.91388855e-06
 1.68597493e-05  9.68573040e-06  2.27217271e-05 -2.66494039e-05
-1.29275460e-05 -2.16383327e-06  2.77202887e-05  2.80802889e-05
-2.64102345e-07  3.74291279e-05  2.76036809e-06  1.30033364e-06
-1.30186120e-05 -9.41272295e-06 -8.96441783e-06 -1.49346448e-05
-1.04346868e-05 -2.05935190e-06  8.23080139e-06 -6.43932317e-06
 1.93724256e-05 -4.27698698e-06  1.34032210e-05 -1.88038823e-06
-2.71187137e-05 -3.82548416e-05  1.79515209e-05 -4.91438739e-06
-6.78782282e-06 -2.01179219e-05  1.20686296e-05 -1.71504131e-05
 2.51524753e-05  3.68582663e-06 -3.82397325e-05 -7.28742509e-06
 2.33159124e-05 -3.06246804e-06 -2.07903293e-05  2.17709433e-05
-1.30620967e-05  7.78324389e-06 -7.25488361e-05 -1.86536218e-05
-1.63533445e-05  1.39497142e-05  3.06496818e-05 -1.10387699e-05
 7.66161813e-06  2.71367590e-05 -1.64683300e-05  1.20854134e-05
 1.76770898e-05 -3.60586502e-05 -2.05969560e-05  2.05577456e-05
-7.07474933e-05 -3.32093914e-05  2.47224467e-05  2.30447078e-05
 3.09873778e-05 -3.32391822e-06 -4.88585069e-05 -6.12364059e-07
-3.94580622e-05 -3.97281795e-06 -1.53711714e-05 -3.02978156e-05
-1.22945356e-05  2.56365332e-05  2.45487744e-05  5.56539710e-06
-6.59130285e-06  3.74488039e-05 -2.14885021e-05  2.45412907e-05
-5.05342245e-05 -2.96730147e-05 -3.29653636e-07 -8.44914400e-06
-6.62970125e-06 -2.95223704e-05 -6.39248692e-06  4.43532810e-06
 5.68601250e-06 -2.78406252e-05 -8.46353948e-06 -1.25517208e-05
-1.00554846e-05 -1.86838524e-05  1.92778777e-05  2.52849533e-05
-3.82606516e-05 -4.25842803e-05  1.30266808e-05  1.36310976e-05
 4.42368327e-05 -1.91331170e-05  1.20454093e-05  1.72201645e-05
 3.88896534e-05  5.49212442e-05 -3.80366695e-06  3.32763325e-06
 1.73728601e-06  4.19723627e-06 -3.56881057e-05 -3.18312965e-05
-2.93202946e-06  7.73715836e-06 -3.26939887e-05  2.19528649e-05
-1.54009179e-05  2.80273096e-05 -4.51730317e-07  8.01613105e-06
-3.68839939e-05 -3.20976600e-06 -1.41654596e-05  8.99710530e-06
-8.33931338e-06  2.01322708e-06  3.18714378e-06  2.47585422e-05
 7.29188715e-06 -1.21247016e-05  9.67335829e-06 -7.46699876e-06
-1.44165761e-06  4.72996092e-05 -1.20552422e-05 -9.69373838e-06
 6.34959596e-06  8.79739895e-06 -1.97045246e-05 -2.26999064e-05
 2.73164370e-05 -1.69230081e-07  4.48365677e-06 -8.80239152e-06
-9.87870675e-06  2.63075934e-05 -1.65244879e-05  1.47026813e-05
-3.46339654e-06  2.73665231e-05  3.68152850e-05 -1.35097077e-05
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-3.35362526e-06  2.11871108e-05  1.49095388e-05 -9.26247607e-06
-4.36370283e-05  6.58449068e-06 -1.59549911e-05 -1.14541823e-05
-3.82582686e-05 -1.46422343e-05 -8.23851291e-06 -1.97800275e-06
-4.66599978e-06 -2.01635314e-05  1.76242010e-05  1.55424769e-05
 3.43127662e-05  4.94446190e-05  2.54181469e-06  1.43111628e-05
 3.81132582e-06 -1.89806012e-05  4.94665817e-06  1.82763418e-05
-2.44338836e-05 -9.89729481e-06 -1.22507512e-05 -1.08585040e-05
-3.30971431e-06 -1.36197883e-06 -1.75427041e-06 -2.60188743e-05
-8.11288747e-06 -2.21243965e-05 -4.87840844e-05 -3.09203580e-05
```

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  3.51800863e-07 -3.38981661e-05  1.33125317e-05 -5.30125662e-06
  7.14895057e-06  1.87529281e-06  1.94765153e-05 -5.20425097e-06
 -2.77949903e-05  2.71886367e-06  1.10761099e-05  2.16016690e-05
 -2.76337009e-05  8.48276515e-06  2.10877671e-05  1.05231162e-05
 -2.71242640e-05  3.13530056e-06  2.04343674e-06 -7.68501304e-07
 -8.21270855e-07  2.97867561e-05  1.24111962e-05 -1.77625785e-05
 -1.44807737e-05 -2.32048845e-05  7.55246515e-06 -8.98595615e-06
 -2.70512313e-06  4.87747483e-05  1.73077773e-05 -1.35327809e-05
 -5.16604073e-07  1.17570515e-05  4.66971358e-06  8.54101101e-06
  2.00514093e-05]



Optimal model parameter of  7  classifier:
[-6.88866882e-01  3.00199913e-05  6.71334440e-06 -3.51680026e-06
 -1.23294138e-06 -1.66141436e-05  1.02401774e-05  8.31371115e-06
 -1.65543008e-05 -9.28599670e-06  8.36175485e-06  1.56102555e-05
 -6.49024254e-06 -8.57891558e-06  1.04387052e-05  1.47277123e-05
  3.44970329e-06  2.62202198e-06  3.42754358e-05 -2.52883640e-05
  1.56111484e-06  2.06502528e-06 -2.18956401e-05 -3.28556210e-05
  5.84746804e-06  2.41275019e-05  2.35578073e-05  8.59349274e-06
 -1.68314790e-05  1.92739817e-05  1.47267957e-05 -2.73843789e-06
  6.55870417e-06 -1.62368809e-05 -2.26400512e-05  2.97013420e-05
 -1.03022026e-05  2.50346275e-05  2.71819302e-05 -1.31907153e-06
  3.82156748e-06  2.91081874e-05  2.04009558e-05 -2.94187615e-05
  2.13400247e-06  9.96337247e-06 -1.57378772e-05 -1.99800377e-05
  7.16162400e-06  1.56026953e-05 -3.72595547e-05  2.47362035e-06
  7.67874839e-06 -3.04982526e-05 -1.45930746e-05 -3.46341096e-05
 -1.44796874e-05 -3.45149488e-06  5.61838898e-06  2.02896710e-05
  1.36297226e-05 -6.16157286e-06 -1.72392280e-05  1.24181115e-05
  5.03241881e-06  2.00086747e-06 -2.82540386e-05  1.57586149e-05
  1.79873364e-05  1.13368532e-06  1.16549166e-05 -1.96466603e-05
 -3.44268579e-05 -1.15751506e-05 -2.10116487e-05  3.43860320e-06
 -1.94767665e-05  5.04586076e-06 -1.62122737e-05 -1.43196728e-05
  6.93145418e-06  1.89273114e-05 -1.82259903e-05  3.51267618e-05
  3.13789822e-05 -2.95183468e-05 -6.52061368e-07  1.97318652e-05
 -1.22380497e-05 -1.54937650e-05 -2.09049319e-05 -2.23928066e-05
 -1.54673830e-05  2.58734047e-05 -1.87999922e-05  1.06962636e-05
 -1.48092060e-05  4.39502935e-05 -2.77841902e-07 -1.87393216e-05
 -1.40963034e-05  5.23283648e-06 -7.72912229e-07 -2.66819259e-05
 -1.07347387e-05 -9.99677487e-06 -1.89657012e-05  1.42721995e-05
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 -1.06961656e-05 -6.31437769e-06  1.88183723e-05  1.66215064e-06
 -2.51578685e-05 -2.94883780e-05  3.88342884e-05 -9.35120235e-06
 -3.50970482e-05 -1.12003555e-05 -3.09550717e-05 -2.19272025e-05
  1.42979288e-05  7.77870762e-06 -1.36736675e-05 -2.86871467e-06
  2.24945165e-05  6.26341895e-06  1.46005685e-05  3.03963542e-05
 -2.10133883e-05 -3.10128483e-05 -2.27754788e-05 -1.02606729e-05
```

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  3.23026003e-05 -1.28167808e-05 -3.71910196e-06 -2.14228931e-05
  1.59021201e-05  6.24756841e-06  1.02761602e-05  1.76425216e-05
 -1.65150681e-05  7.69542020e-06 -4.15533413e-05 -1.43542585e-06
 -1.47913707e-05 -2.92333807e-05  4.93596024e-06 -7.92438147e-06
  2.56286872e-05 -2.72253659e-06 -7.21498376e-06 -1.29361825e-05
 -1.66946560e-06  1.30893158e-06 -3.08680247e-06 -1.30516738e-05
  1.90872845e-05 -2.96901575e-05 -9.54073152e-08  2.96843348e-05
 -8.66355084e-06  6.14255237e-06 -2.47397776e-05  1.33283675e-05
 -2.91393475e-06 -1.70235900e-05 -1.50519347e-05 -5.61201450e-06
  9.39990713e-06 -8.44963959e-06  8.23930107e-06 -1.02449711e-05
 -2.80791982e-05  1.83818830e-05 -1.86405234e-05 -2.19904492e-05
 -1.75233253e-05 -1.13770027e-05 -1.42161974e-06  3.07012691e-05
  1.67226020e-05  1.01753370e-05  1.56502600e-05 -2.30088574e-05
 -2.33318720e-05 -6.55981685e-06  2.61544497e-05  2.36054354e-05
  2.03545245e-05 -2.93982980e-06 -2.30754448e-05 -2.21692137e-05
  3.40640825e-06 -3.23219941e-06  1.58180927e-05 -2.34536970e-05
 -5.13185109e-06 -2.70573478e-05  6.45394750e-06 -4.41695367e-06
 -1.05144229e-05 -3.90382807e-05 -1.24732702e-05 -1.85118951e-06
 -1.59094371e-05  6.87833845e-06  5.37937854e-06 -2.99803631e-05
  1.45907492e-05  3.32174280e-05  1.42939211e-05 -3.23434615e-05
 -2.03949123e-05  1.18695692e-05  7.90060182e-06  9.13215243e-06
  1.41135743e-05  7.94353594e-06 -8.49814599e-06 -1.56235027e-07
  3.71353022e-06 -3.32130090e-06 -4.80378540e-06 -1.61195366e-05
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 -1.11600720e-05 -3.64024597e-06 -1.45158944e-05 -6.51140574e-07
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 -1.30173742e-05 -1.38479460e-06  1.52107960e-05  1.19059295e-05
  1.81528619e-05 -1.44226471e-05  2.39566449e-05  3.45791400e-06
 -4.09591577e-06  1.79081980e-06 -1.37141518e-05  5.83337139e-06
  2.09050164e-05]



Optimal model parameter of  8  classifier:
[-1.25565861e+00  1.24553603e-05  1.56634016e-05  1.20231280e-05
 -1.92287416e-06 -1.09791356e-05  2.41696485e-06 -1.04950782e-05
  7.07654248e-06  5.96964027e-06 -1.62407088e-05 -3.11602836e-05
  1.45271859e-05 -1.18032149e-05 -7.18951134e-06  4.50601731e-06
  6.27780892e-06  2.08769500e-05 -4.31129176e-05  9.59095315e-06
  8.15792666e-06 -1.96927280e-05  2.25764968e-05  5.49607390e-06
  6.44576584e-06 -2.11038678e-06 -1.47154074e-05 -7.75143443e-07
 -5.03438958e-06 -7.06837909e-06  7.94235861e-06 -2.74146546e-05
  4.64564766e-06  1.08505194e-05 -1.07233347e-05  1.76919162e-06
 -1.01081463e-05 -2.42636468e-05 -1.13847228e-05 -3.48625123e-05
 -5.65104919e-06 -1.50249517e-05 -1.73925923e-05  1.98855563e-05
 -8.45423456e-07  7.37278435e-06 -1.53676310e-05 -8.14114035e-06
  2.27099622e-05 -4.08752404e-06 -7.09936840e-06  8.37824677e-06
```

```
 9.15347598e-06  1.06962464e-05  1.55779222e-05  5.68505149e-06
 1.44588222e-05 -8.39860584e-06 -1.22935488e-05 -2.74796578e-05
-1.32502309e-06  7.56712642e-07  2.73259154e-05  1.28476952e-05
-1.00258450e-05 -1.48613859e-05 -2.08184258e-06 -5.93561860e-06
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 9.08622385e-06 -2.34682324e-05 -3.08674281e-05  2.04545244e-06
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 1.06172468e-05 -3.54604226e-05  3.78951811e-06 -1.88187560e-05
 7.17420782e-06  2.66890367e-05 -2.00901604e-05  1.88399968e-05
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```

```
 -3.97039219e-06  1.75691904e-05 -1.07120999e-05 -1.23210679e-06
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  6.37474716e-07 -8.87572308e-07 -6.76144406e-06 -1.46825674e-05
 -1.81196973e-05]



Optimal model parameter of  9  classifier:
[-8.92538489e-01 -2.32007221e-06 -5.28909563e-06  9.61282309e-06
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 -2.67884214e-05 -1.78157065e-05  5.02692173e-06  8.37526782e-06
  1.22870594e-06  8.93130056e-06 -3.90454684e-05  1.86195036e-06
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  1.11895495e-05  1.44299573e-05 -2.35750258e-05 -3.43253046e-05
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 -1.81479682e-06 -3.09887471e-06 -3.33397952e-05  1.29712983e-05
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  2.10014331e-06 -1.45162248e-05  2.40846347e-05 -1.98974565e-05
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  1.29976651e-05  3.97352781e-06 -3.47620543e-06 -1.16938436e-05
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  4.12248533e-05 -9.16006655e-06 -1.26590323e-05 -1.62898929e-05
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```

```
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 1.70620066e-05  3.45056076e-05  6.10498453e-06  2.32740280e-05
-9.55381267e-06]
```

```
In [25]: temp_random_A_ts = np.dot(A_ts, random_vector.T)
```

```
In [26]: random_A_ts = np.zeros((len(ts_data), 257), dtype='float')
         for i in range(257):
             for j in range(len(ts_data)):
                 if i == 0:
                     random_A_ts[j][i] = 1
                 else:
                     random_A_ts[j][i] = temp_random_A_ts[j][i-1]
```

```
In [27]: A_mul_x_ts = np.zeros((10, len(ts_data)))
         A_mul_x_tr = np.zeros((10, len(tr_data)))
         label_tr = np.zeros(len(tr_ans))
         label_ts = np.zeros(len(ts_ans))
```

```
In [29]: for i in range(10):
             A_mul_x_ts[i] = np.dot(random_A_ts, z[i].T)
             A_mul_x_tr[i] = np.dot(random_A_tr, z[i].T)
```

```
        #get the label of input x (Argmax_d)
        for i in range(len(tr_data)):
            temp_max = max(A_mul_x_tr.T[i])
            max_index = np.where(A_mul_x_tr.T[i] == temp_max)
            label_tr[i] = max_index[0][0]

        for i in range(len(ts_data)):
            temp_max = max(A_mul_x_ts.T[i])
            max_index = np.where(A_mul_x_ts.T[i] == temp_max)
            label_ts[i] = max_index[0][0]
```

```
In [31]: #compare predict and answer
         Train_cnt = np.zeros((10,2))
         Test_cnt = np.zeros((10,2))
         for j in range(len(tr_ans)):
             if tr_ans[j] == label_tr[j]:
                 Train_cnt[tr_ans[j], 0] +=1
             elif tr_ans[j] != label_tr[j]:
                 Train_cnt[tr_ans[j], 1] +=1
         for j in range(len(ts_ans)):
             if ts_ans[j] == label_ts[j]:
                 Test_cnt[ts_ans[j], 0] +=1
             elif ts_ans[j] != label_ts[j]:
                 Test_cnt[ts_ans[j], 1] +=1
```

2. Compute (1) true positive rate, (2) error rate using (1) training dataset and (2) testing dataset.

```
In [33]: TP_ratio_tr = np.zeros((10))
         TP_ratio_ts = np.zeros((10))
         Error_ratio_tr = np.zeros((10))
         Error_ratio_ts = np.zeros((10))

         for i in range(10):
             TP_ratio_tr[i] = Train_cnt[i][0] / (Train_cnt[i][0] + Train_cnt[i][1])
             TP_ratio_ts[i] = Test_cnt[i][0] / (Test_cnt[i][0] + Test_cnt[i][1])
             Error_ratio_tr[i] = Train_cnt[i][1] / (Train_cnt[i][0] + Train_cnt[i][1])
             Error_ratio_ts[i] = Test_cnt[i][1] / (Test_cnt[i][0] + Test_cnt[i][1])

         Total_TP_tr_ratio = sum(Train_cnt.T[0]) / len(tr_data)
         Total_Error_tr_ratio = sum(Train_cnt.T[1]) / len(tr_data)
         Total_TP_ts_ratio = sum(Test_cnt.T[0]) / len(ts_data)
         Total_Error_ts_ratio = sum(Test_cnt.T[1]) / len(ts_data)
```

```
In [34]: print("Train Set: ")
         print("Number/ True Positive Ratio/ Error Rate: \n")

         for i in range(10):
             print(i, ' ', TP_ratio_tr[i], ' ', Error_ratio_tr[i])
         print("Total TP Ratio: ", Total_TP_tr_ratio)
```

```
        print("Total Error Ratio", Total_Error_tr_ratio)
        print('\n\n\n')
        print("Test Set: ")
        print("Number/ True Positive Ratio/ Error Rate: \n")
        for i in range(10):
            print(i, ' ', TP_ratio_ts[i], ' ', Error_ratio_ts[i])
        print("Total TP Ratio: ", Total_TP_ts_ratio)
        print("Total Error Ratio", Total_Error_ts_ratio)
```

```
Train Set:
Number/ True Positive Ratio/ Error Rate:

0   0.9503629917271653   0.04963700827283471
1   0.9694452684663305   0.030554731533669534
2   0.7962403491104397   0.20375965088956025
3   0.8313488827271245   0.16865111727287554
4   0.8885655597398151   0.11143444026018487
5   0.7134686346863469   0.28653136531365314
6   0.9266644136532612   0.07333558634673876
7   0.8758180367118915   0.12418196328810854
8   0.7480772517518373   0.2519227482481627
9   0.797949235165574   0.20205076483442597
Total TP Ratio:  0.8526642110701845
Total Error Ratio 0.1473357889298155
```

```
Test Set:
Number/ True Positive Ratio/ Error Rate:

0   0.9673469387755103   0.0326530612244898
1   0.9744493392070485   0.02555066079295154
2   0.7829457364341085   0.21705426356589147
3   0.8554455445544554   0.14455445544554454
4   0.8971486761710794   0.10285132382892057
5   0.7118834080717489   0.2881165919282511
6   0.9237995824634656   0.07620041753653445
7   0.8646543330087634   0.13534566699123662
8   0.7751540041067762   0.22484599589322382
9   0.8067393458870169   0.19326065411298315
Total TP Ratio:  0.8584858485848584
Total Error Ratio 0.1415141514151415
```

In [ ]:

In [ ]:

In [ ]:

In [ ]: