
Implementing Complex Valued Neural Networks

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Abstract

Popular neural network frameworks like PyTorch and Keras with TensorFlow provide limited support for CVNNs. In this paper, we present an implementation of CVNNs using custom layers and activation functions in Keras with TensorFlow, taking advantage of TensorFlow's support for complex tensors. We evaluate the implementation on several benchmark datasets, including MNIST, Fashion MNIST, and audio classification tasks. Our experiments show that CVNNs can achieve competitive performance with real-valued neural networks, while offering the potential for improved efficiency and applicability in domains where complex values are prevalent.

GitHub:

<https://github.com/DJMacFarlane/Complex-Valued-Neural-Nets>

1 Introduction

Complex valued neural networks (CVNNs) offer several potential advantages over real valued neural networks (RVNNs). By incorporating both phase and magnitude in each value, CVNNs allow for a richer representation of data. This increased information content in each input and parameter can lead to a reduction in the number of parameters, subsequently lowering the likelihood of exploding and vanishing gradients while also reducing the need for regularization. Furthermore, some types of data are naturally suited for representation using complex numbers.

CVNNs hold great promise in domains where complex values are already extensively utilized, such as quantum computing and signal processing. Outputs from Fourier transforms and other complex representations can be directly fed into the network, eliminating the need to separate or remove information from each value as required with RVNNs. Additionally, certain complex transforms and filters can be applied to images, thereby reducing the need for convolutions in image classification tasks¹.

Despite these advantages, many popular neural network frameworks, such as PyTorch and Keras with TensorFlow, offer limited support for complex valued neural networks by default. Nevertheless, TensorFlow does provide support for complex tensors, which enables the implementation of CVNNs by defining custom layers. In this project, we take advantage of this functionality to explore the potential of CVNNs further.

2 Related Works

Akira Hirose (2012). *Complex-Valued Neural Networks, 2nd Edition*

This book provides an overview of complex valued neural networks and some applications. Much of the implementation is based on concepts from this author's works.

¹See Ko et al. 2022

36 **Manny Ko et al. (2022). *CoShNet: A Hybrid Complex Valued Neural Network using Shearlets*.**
37 **arXiv: 2208.06882 [cs.CV]**

38 In this paper they explore shearlets and CVNNs in image processing to reduce the need for
39 convolution, increase efficiency and improve performance. This is an excellent demonstration that
40 CVNNs are worth investigating.

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43 **Ryan Yu et al. (2022). *Biologically Plausible Complex-Valued Neural Networks and Model***
44 ***Optimization*. Ed. by Ilias Maglogiannis et al. Cham**

45 The primary motivation for this project was the potential of CVNNs to more accurately approximate
46 biological networks. The paper investigates CVNNs, which are designed to be more similar to
47 biological neural networks than their real-valued neural network (RVNN) counterparts, demonstrating
48 superior performance in certain tasks. However, the paper's main drawback lies in its reliance on
49 gradient descent as a training method, as this is likely an unrealistic learning mechanism for biological
50 systems (Hinton 2022).

51 **3 Description**

52 In order to implement complex valued neural networks we defined the following custom layers and
53 activations for keras. Keras and TensorFlow do support Wirtinger derivatives so there is no need to
54 modify the backpropagation methods.

55

56 **3.1 Layers**

- 57 1. **ComplexDense**: A dense layer that takes complex or real inputs and outputs complex
58 outputs.
- 59 2. **ComplexConv2D**: A 2D complex convolution layer that takes complex or real inputs and
60 outputs complex outputs.
- 61 3. **ComplexConv1D**: A 1D complex convolution layer that takes complex or real inputs and
62 outputs complex outputs.
- 63 4. **ComplexDropout**: A complex dropout layer that takes complex inputs and performs dropout
64 separately on the real and imaginary parts.
- 65 5. **ComplexMaxPool2D**: A complex max-pooling layer that takes complex inputs and outputs
66 complex outputs.
- 67 6. **ComplexAvgPool2D**: A complex average-pooling layer that takes complex inputs and
68 outputs complex outputs.
- 69 7. **ComplexLayerNormalization**: A complex layer normalization layer that takes complex
70 inputs and outputs complex outputs.
- 71 8. **ComplexUpSampling2D**: A complex upsampling layer that takes complex inputs and
72 outputs complex outputs.

73 **3.2 Activations**

- 74 1. **abs_softmax(x)**: This function computes the softmax of the absolute values of the input
75 tensor x . The softmax function is applied to the absolute values of the elements, normalizing
76 them to create a probability distribution.
- 77 2. **real_softmax(x)**: This function computes the softmax of the real parts of the input tensor x ,
78 which has complex elements. The softmax function is applied to the real parts, normalizing
79 them to create a probability distribution.
- 80 3. **imag_softmax(x)**: This function computes the softmax of the imaginary parts of the input
81 tensor x , which has complex elements. The softmax function is applied to the imaginary
82 parts, normalizing them to create a probability distribution.

- 83 4. **polar_softmax(x)**: This function computes the softmax of the angles (phases) of the input
84 tensor x , which has complex elements. The softmax function is applied to the angles,
85 normalizing them to create a probability distribution.
- 86 5. **cmplx_rrelu(x)**: This function applies the Rectified Linear Unit (ReLU) activation function
87 only to the real parts of the input tensor x , which has complex elements. The imaginary
88 parts are left unchanged.
- 89 6. **cmplx_crelu(x)**: This function applies the Rectified Linear Unit (ReLU) activation function
90 to both the real and imaginary parts of the input tensor x , which has complex elements.
- 91 7. **polar_relu(x)**: This function applies the Rectified Linear Unit (ReLU) activation function
92 to the magnitudes (absolute values) of the input tensor x , which has complex elements. The
93 phases (angles) are left unchanged.

94 4 Experiments

95 4.1 MNIST

96 To test the implementation we do a trial run on MNIST data set to verify the implementation is
97 functioning. For the real neural network we use 32 filter 4 by 4 kernels with softmax output layer.
98 The complex network has 16 filters of 4 by 4 kernels to halve the number of parameters. CVNNs do
99 not offer much benefit in a simple case like this, but is a convenient test set.

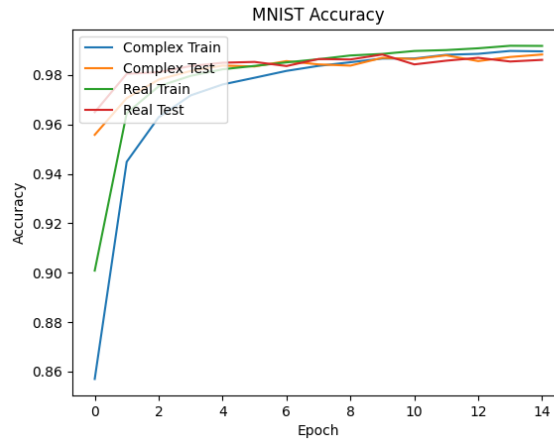


Figure 1: As expected we get similar performance.

100 4.2 Fashion MNIST

101 We also tested training on the 2D fourier transform of Fashion MNIST data set (Xiao, Rasul, and
102 Vollgraf 2017)

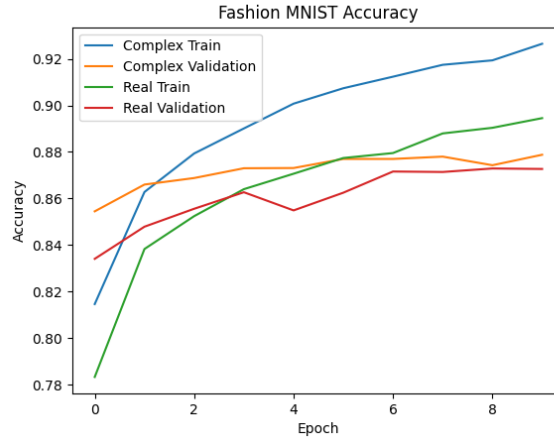


Figure 2: For complex case we used fourier transform values directly.

103 4.3 Audio

104 4.3.1 Dog versus Cat Audio

105 After training on images we downloaded a simple dataset of cat sounds vs dog sounds using complex
106 valued outputs of `tf.signal.stft(waveform)` as inputs to a convolutional neural network.

107 Data set here <https://www.kaggle.com/datasets/mmoriaux/audio-cats-and-dogs>

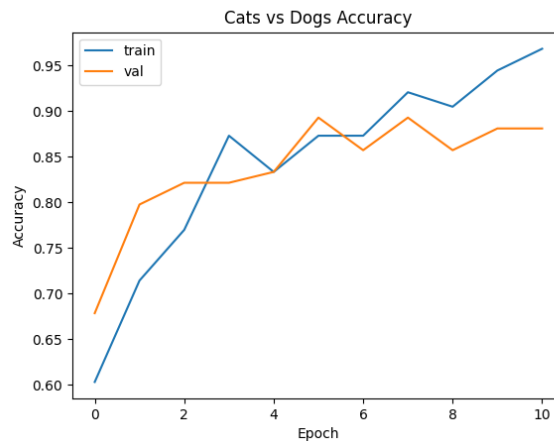


Figure 3: Given the small size of the data set we overfit quickly.

108 **4.3.2 Emotion Classification**

109 Using a subset of the Toronto emotional speech set (Pichora-Fuller and Dupuis 2020) we trained a
 110 complex valued neural network on the spectrogram (complex valued) to categorize audio into classes
 111 based on predicted emotion of the speaker.

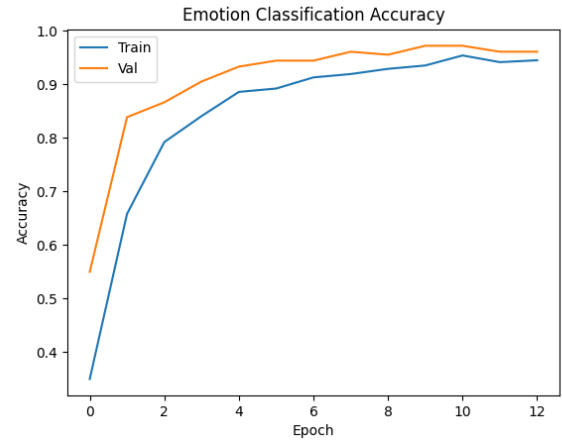


Figure 4: We achieved very quick convergence.

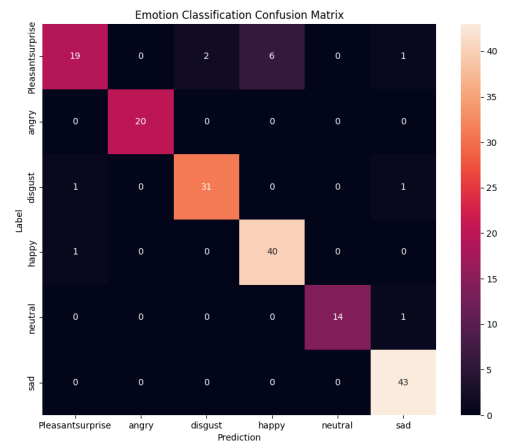


Figure 5: The model sometimes confused happy with pleasant surprise.

5 Discussion

Our experiments on various datasets, including MNIST, Fashion MNIST, and audio classification tasks, demonstrate the potential of CVNNs as a viable alternative to traditional RVNNs. In the MNIST experiment, we observed that the CVNNs with fewer parameters performed competitively with the RVNNs. Furthermore, when applying CVNNs directly to the 2D Fourier transform of the Fashion MNIST dataset, we achieved satisfactory results, showcasing the ability of CVNNs to handle complex input data effectively.

In the audio classification tasks, we tested CVNNs on both cat and dog sounds and emotion classification using a subset of the Toronto emotional speech set. In both cases, we observed rapid convergence and competitive performance. However, overfitting occurred quickly in the cat and dog audio dataset due to its small size. In the emotion classification task, we got excellent results.

The results of our experiments indicate that CVNNs can achieve competitive performance with RVNNs and, in some cases, provide improved efficiency and applicability in domains where complex values are prevalent. By implementing custom layers and activation functions in Keras with TensorFlow, we have taken advantage of TensorFlow’s support for complex tensors, allowing us to explore CVNNs more in the future. Though we did not have enough time to experiment with complex valued outputs or complex transformers.

Future work on CVNNs may focus on optimizing the architecture, incorporating more sophisticated regularization techniques, and exploring additional applications in areas such as quantum computing, signal processing, and image processing. Additionally, it would be interesting to investigate the potential of CVNNs in approximating biological networks more accurately, as alluded to in the Biologically Inspired Complex-Valued Neural Networks paper (Yu et al. 2022). As complex numbers may allow for easier encoding of frequency and phase of spiking patterns of neurons.

References

- Hinton, Geoffrey (2022). *The Forward-Forward Algorithm: Some Preliminary Investigations*. arXiv: 2212.13345 [cs.LG].
- Hirose, Akira (2012). *Complex-Valued Neural Networks, 2nd Edition*.
- Ko, Manny, Ujjawal K. Panchal, Héctor Andrade-Loarca, and Andres Mendez-Vazquez (2022). *CoShNet: A Hybrid Complex Valued Neural Network using Shearlets*. arXiv: 2208.06882 [cs.CV].
- Pichora-Fuller, M. Kathleen and Kate Dupuis (2020). *Toronto emotional speech set (TESS)*. Version DRAFT VERSION. DOI: 10.5683/SP2/E8H2MF. URL: <https://doi.org/10.5683/SP2/E8H2MF>.
- Xiao, Han, Kashif Rasul, and Roland Vollgraf (2017). “Fashion-MNIST: a Novel Image Dataset for Benchmarking Machine Learning Algorithms.” *CoRR* abs/1708.07747. arXiv: 1708.07747. URL: <http://arxiv.org/abs/1708.07747>.
- Yu, Ryan, Andrew Wood, Sarel Cohen, Moshick Hershcovitch, Daniel Waddington, and Peter Chin (2022). *Biologically Plausible Complex-Valued Neural Networks and Model Optimization*. Ed. by Ilias Maglogiannis, Lazaros Iliadis, John Macintyre, and Paulo Cortez. Cham.