

From our proportion equations, setting $\frac{dI}{dt} = 0$ we get:

$$\begin{aligned}\beta SI - \gamma I - \mu I &= 0 \\ I(\beta S - \gamma - \mu) &= 0 \\ \beta S &= \gamma + \mu \\ \hat{S} &= \frac{1}{R_0}\end{aligned}$$

Adding the proportion equations of $\frac{dS}{dt} = 0$ and $\frac{dI}{dt} = 0$ we also get:

$$\begin{aligned}\mu - \mu S - \gamma I - \mu I &= 0 \\ 1 - S - I &= \frac{\gamma}{\mu} I \\ 1 - S &= \frac{\mu + \gamma}{\mu} I \\ 1 - S &= \frac{1}{\epsilon} I \\ I &= \epsilon(1 - S) \\ I &= \epsilon(1 - \frac{1}{R_0})\end{aligned}$$

Therefore $(\hat{S}, \hat{I}) = (\frac{1}{R_0}, \epsilon - \frac{\epsilon}{R_0})$. Both equilibria are biologically relevant as long as $R_0 \geq 1$, since values of S and I outside of the range $[0,1]$ are not meaningful as proportions.