

Examining Control Strategies for Cholera Incorporating Spatial Dynamics

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March 27, 2019 @ 5:31

Abstract

We solve everything because we're really smart

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1 Introduction

1.1 Biology of Cholera

Although it is listed as one of the oldest known diseases, cholera remains a major public health concern in areas with poor water sanitation with an estimated 1.3-4 million cases every year (WHO, 2019). Cholera is an infectious disease caused by the bacterium *Vibrio cholerae*. The bacterium survives and reproduces in aquatic environments, and is capable of colonizing small intestines (Codeco, 2001). The disease is not airborne, but can be transmitted through contaminated food or water and can survive in some aquatic environments from months to years (Tien & Earn, 2010). The bacterium produces enterotoxins responsible for the symptoms of cholera infection which are severe diarrhea, vomiting and nausea (Kaper *et al.*, 1995). Dehydration thickens the blood, causing circulation problems that can lead to death within a few hours. Since dehydration is the main problem, rehydration with clean water and minerals (such as ORS packages) is the most effective treatment (WHO, 2019). Current improvements in public health and sanitation largely decrease the likelihood of a cholera outbreak (WHO, 2019).

Four major outbreaks of cholera in the 19th century devastated the London population, resulting in tens of thousands of deaths. One of the early theories believed to be the cause of spread of cholera was the Miasma theory, suggesting that cholera is an airborne disease and that impurities in the air induced the spread (Paneth *et al.*, 1998). Thus, the suggested solution in 1848 was to discard the contents of cesspools and raw sewage pits into the River Thames. Since Thames was the drinking source of many, the misunderstanding about the method of transmission resulted in heightened number of infected individuals, severely worsening the epidemic (Paneth *et al.*, 1998). Early studies on cholera, such as the work of Jon Snow in the mid 19th century, have been pivotal in the development of modern epidemiology. However, the abundance of more recent studies using mathematical frameworks that try to model a framework for anticipating outbreaks of cholera and planning for interventions is the reason for our focus on this particular disease.

1.2 Transmission Dynamics of Cholera

Before introducing a simple model to simulate the temporal spread of cholera, we must discuss the processes we plan to analyze. The model should include the entire population, which for simplicity we will assume is comprised of only three groups: the susceptible, the infected (or infectious), and the recovered. The only area still remaining that has a major impact on the epidemic is the environment, or in this case the water. We assume that only Infectious individuals can contaminate the water sources by shedding the pathogen into the water. The halting remedy suggested increased the rate of water contamination drastically, which in turn increased the transmission rate from individuals coming into contact with the infected water. This is a plausible explanation for why maximum weekly deaths in London increase more than two-fold in the 1848 epidemic compared to the 1832 epidemic (Tien, Poinar, *et al.*, 2011). The main treatment strategies for cholera outbreaks are vaccination, antibiotic treatment, and water sanitation. We can incorporate these into our model to simulate the effect that each of these strategies has on the disease dynamics.

1.3 SIRW Model Construction

Our model has five distinct departments, a susceptible, exposed, infectious, removed, and water compartment. Susceptible: contains the proportion of the population that is capable of becoming infected. Individuals are born into compartment S at a rate of μ . Individuals leave the compartment in one of two ways, they die at a rate μ , or come into contact with the pathogen and move into the Infectious compartment. Interactions of susceptible and infected individuals from the I compartment yields new infected individuals at a rate of β_I , and interactions of susceptible individuals with the water compartment W yields new infected individuals at a rate β_w .

Infectious: Contains the proportion of individuals that enter from the S compartment in the manner discussed above. Individuals in this compartment are capable of infecting susceptible individuals during interactions at a rate of β_i . They are also capable of contributing to the choleric load of the water compartment by shedding the pathogen at a rate ξ . Individuals in this compartment recover at a rate γ , and move to the R compartment, else they leave this compartment as they die (not from Cholera) at a rate μ and from Cholera at a rate α .

Recovered: Contains the proportion of individuals that are neither infectious or susceptible to the pathogen. They leave this compartment as they die at a rate μ . Water: The w term is proportional to the concentration of Cholera in the environment. More bacteria enters the compartment as infected individuals shed the pathogen at a rate ξ , and the pathogen dies at a rate σ .

2 Single Patch Models

2.1 Model Introduction And Parameters

In this paper, we consider the cholera SIWR model as outlined by Tien and Earn Tien & Earn, 2010 with the addition of death rate by cholera σ . The tables below summarize the variables and parameters involved.

	Description	Units
S	Susceptible individuals	individuals
I	infected individuals	individuals
R	recovered individuals	individuals
W	Bacterial concentration in water	cells ml ⁻³
N	Total number of individuals	individuals

	Description	Units	Estimate
μ	Natural death/birth rate	day ⁻¹	
b_i	Person-person transmission/contact rate	cells ml ⁻³ day ⁻¹	
b_w	water reservoir-person transmission/contact rate	cells ml ⁻³ day ⁻¹	
β_i	scaled Person-person transmission/contact rate	day ⁻¹	0.25
β_w	scaled water reservoir-person transmission/contact rate	day ⁻¹	1×10^{-5} to 1
$\frac{1}{\gamma}$	Infectious period	day	2.9 to 14
σ	Bacterial decay/removal from reservoir	day ⁻¹	$\frac{1}{3}$ to $\frac{1}{41}$
ξ	Person to water reservoir shedding rate	cells ml ⁻³ day ⁻¹ individuals ⁻¹	0.01 to 10
α	Death rate by cholera	day ⁻¹	0.01 to 0.6

Parameter estimates are taken from Grad *et al.*, 2012, Codec0, 2001 and Tien, Poinar, *et al.*, 2011. The natural death rate is dependent on various factors such as city or location and year or era of interest.

2.2 Single Patch SIR Model With A Water Compartment

$$\begin{aligned}
\frac{dS}{dt} &= \mu N - \mu S - \beta_i SI - \beta_w SW \\
\frac{dI}{dt} &= \beta_i SI + \beta_w SW - I(\gamma + \mu + \alpha) \\
\frac{dR}{dt} &= \gamma I - \mu R \\
\frac{dW}{dt} &= \xi I - \sigma W
\end{aligned}$$

- μ = natural death rate
- β_i = transmission rate between S and I class
- β_w = transmission rate between I and W class
- γ = recovery rate (I to R class)
- α = death rate from cholera
- ξ = Shedding rate of cholera from I to W class
- σ = Removal rate of cholera from W class (depends on what we define as our water source)

The assumptions for this model are

- The population is homogenously susceptible to Cholera infection

- No waning immunity; once you recover from cholera you cannot return to the susceptible class
- The transmission rate between water the susceptible class is exponentially distributed

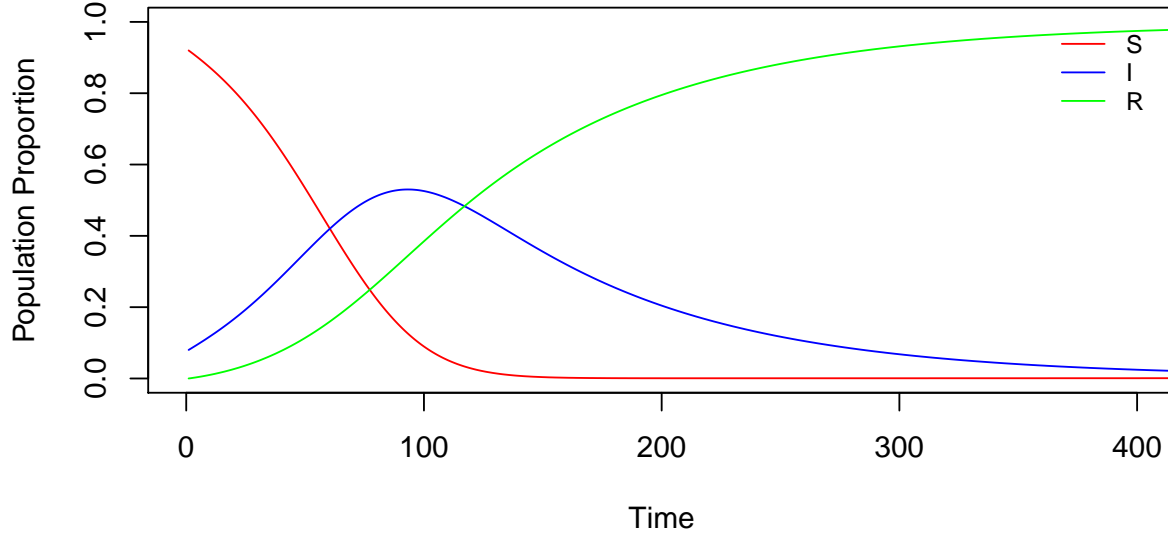


Figure 1: Plot of the SIRW model for a single patch. Parameters are $\mu = 0$ $\beta_i = 0.02$ $\gamma = 0.14$ $\sigma = 0.04$ $\beta_w = 0.5$ $\alpha = 0$ $\xi = 10$. Further the initial conditions for the model were $S_0 = 0.92$ $I_0 = 0.08$ $R_0 = 0$

2.3 Equilibrium and \mathcal{R}_0 Of The Single Patch Model

The basic reproductive number \mathcal{R}_0 is defined as the number of secondary infections as a result of a single infective during a time step. \mathcal{R}_0 can be computed as the spectral radius (i.e. the eigenvalue with the largest absolute value) of the next generation matrix at the disease free equilibrium. The next generation matrix FV^{-1} , where the entry F_{ij} of the matrix F is the rate at which infected individuals in compartment j produce new infections in compartment i , and the entry of V_{ij} of the matrix V is the mean time spent in compartment j after moving into j from compartment k . For our model, we have

$$F = \begin{pmatrix} \beta_i & \beta_w \\ 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{1}{\gamma + \mu + \alpha} & 0 \\ \frac{1}{\gamma + \mu + \alpha} & \frac{1}{\theta} \end{pmatrix}$$

The basic reproductive number is computed as the spectral radius of FV^{-1} as seen in Tien & Earn, 2010, which is

$$\begin{aligned}\mathcal{R}_0 &= \rho(FV^{-1}) \\ &= \frac{\beta_i + \beta_w}{\gamma + \mu}\end{aligned}$$

This single patch model has a disease-free equilibrium at $(S, I, R) = (1, 0, 0)$ when $\mathcal{R}_0 < 1$. It also has an endemic-equilibrium when $\mathcal{R}_0 > 1$

2.4 Single Patch With Low And High Shedding Compartments

$$\begin{aligned}\frac{dS}{dt} &= \mu N - \mu S - \beta_L S I_L - \beta_H S I_H - \beta_w S W \\ \frac{dI_L}{dt} &= \beta_i S (I_L + I_H) + \beta_w S W - I_L (\mu + \delta + \alpha_L) \\ \frac{dI_H}{dt} &= \delta I_L - I_H (\gamma + \mu + \alpha_H) \\ \frac{dR}{dt} &= \gamma I_H - \mu R \\ \frac{dW}{dt} &= \xi_L I_L + \xi_H I_H - \sigma W\end{aligned}$$

This model assumes that you start off with low intensity symptoms (lower rate of shedding) and the symptoms reach a high intensity with a greater rate of shedding. The assumptions for the single patch model apply here as well.

- α_i = death rate by cholera in low or high intensity
- δ = rate at which symptoms increase in severity

3 Multi Patch Model

The following equations represent the SIRW model for a single patch i in the multi patch model.

$$\begin{aligned}\frac{dS_i}{dt} &= \mu N - \mu S_i - \beta_i S_i I_i - \phi \beta_i S_i \sum_j^n I_j - \beta_w S_i W_i - \psi \beta_w S_i \sum_j^n W_j \\ \frac{dI_i}{dt} &= \beta_i S_i I_i + \beta_i \phi S_i \sum_j^n I_j + \beta_w S_i W_i + \beta_i \psi S_i \sum_j^n W_j - I_i (\gamma + \mu + \alpha) \\ \frac{dR_i}{dt} &= \gamma I_i - \mu R_i \\ \frac{dW_i}{dt} &= \xi_i I_i + \beta_i \psi I_i \sum_j^n W_j - \sigma W_i\end{aligned}$$

Where the set n in the set is all neighbours (i.e. adjacent and directly diagonal patches) of the patch i .

- μ = natural death rate
- ϕ = person to person contact rate between neighbouring patches
- ψ = person to water contact rate between neighbouring patches
- β_i = transmission rate between S and I class
- β_w = transmission rate between I and W class
- γ = recovery rate (I to R class)
- α = death rate from cholera
- ξ = Shedding rate of cholera from I to W class
- σ = Removal rate of cholera from W class (depends on what we define as our water source)

The assumptions for the single patch model apply here as well the following.

- No dispersal of individuals between patches
- infected individuals in patch i can infect susceptible individuals in the neighbouring patch j
- All patches neighbouring i have the same transmission rate to patch i

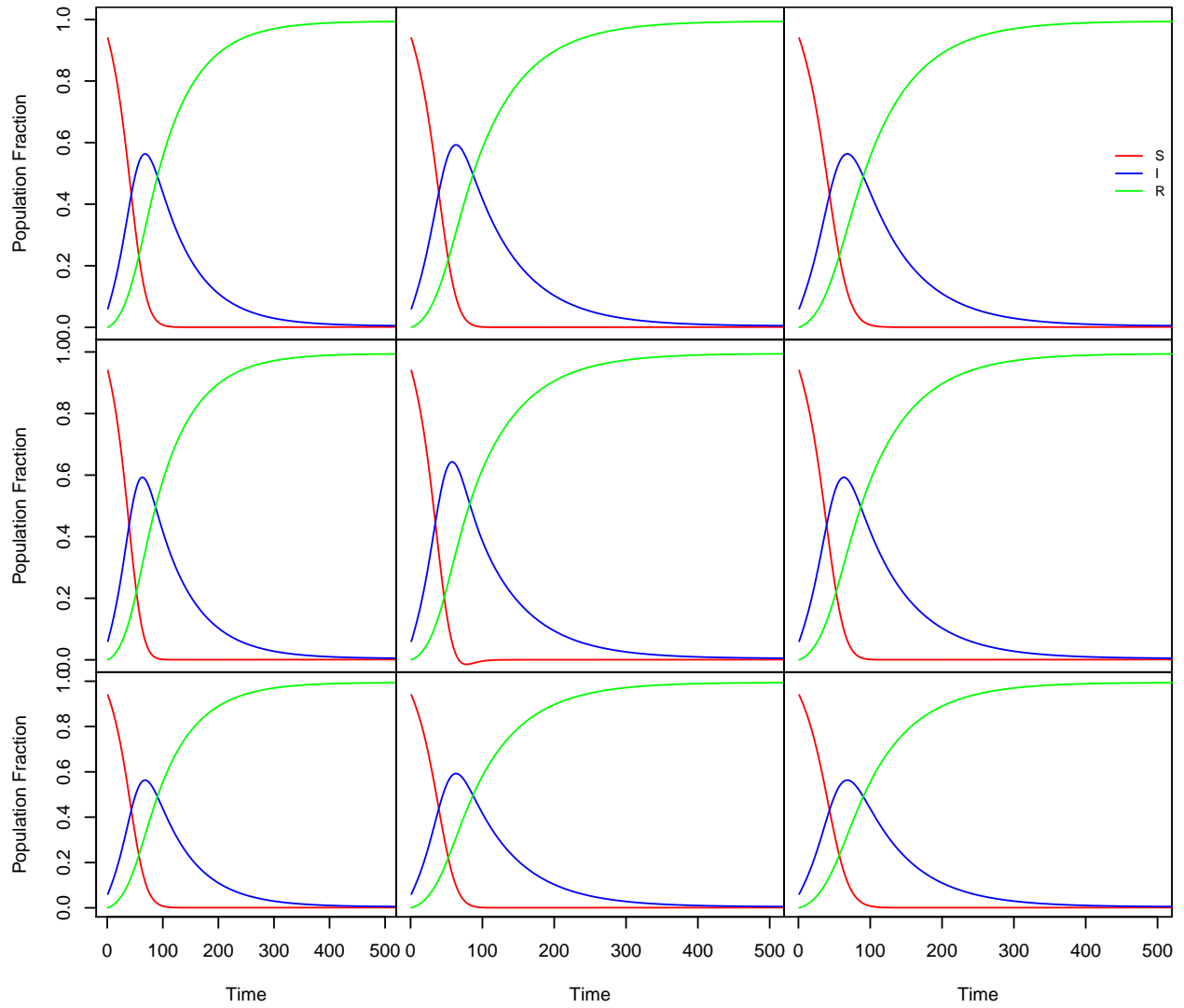


Figure 2: Plot of the SIRW model for all patches in a multi patch model. Parameters are $\mu = 0$ $\beta_i = 0.02$ $\gamma = 0.14$ $\sigma = 0.04$ $\beta_w = 0.5$ $\alpha = 0$ $\xi = 10$ $\phi = 0.01$ $\psi = 0.05$. The initial conditions for the model were $S_0 = 0.94$ $I_0 = 0.06$ $R_0 = 0$.

4 Treatment Strategies For Cholera

4.1 Treatment Plan 1: Sanitation of water over time

One of main ways control strategies for cholera is to treat the water directly (eg. with chlorine). This would essentially have the effect of increasing the rate of bacteria removal from water (defined as σ in the base single patch model). This can be modeled by incorporating a new term ρ in the base model:

$$\begin{aligned}\frac{dS}{dt} &= \mu N - \mu S - b_i SI - b_w SW \\ \frac{dS}{dt} &= b_i SI + b_w SW - I(\gamma + \mu + \alpha) \\ \frac{dR}{dt} &= \gamma I - \mu R \\ \frac{dW}{dt} &= \xi I - \sigma W - \rho W\end{aligned}$$

This new term ρ can be either a constant which is implemented right at the start, or can be a function of time, or can be implemented at a certain threshold depending on the bacterial concentration (W) or the proportion of infecteds (I). For example:

$$\rho(I) = \begin{cases} \rho & I \geq 0.1 \\ 0 & 0 \leq I \leq 0.1 \end{cases}$$

This represents the sanitation rate of ρ , implemented at certain threshold of infected (in this case the threshold is based on $I = 0.1$ but can be based on W (i.e. testing water levels for cholera).

4.2 Treatment Plan 2: Vaccinations on Base Model

$$\begin{aligned}\frac{dS}{dt} &= \mu N - \mu S - \beta_i SI - \beta_w SW - \nu S \\ \frac{dI}{dt} &= \beta_i SI + \beta_w SW - I(\gamma + \mu + \alpha) \\ \frac{dR}{dt} &= \gamma I - \mu R + \nu S \\ \frac{dW}{dt} &= \xi I - \sigma W\end{aligned}$$

- ν = is vaccination rate on S class

4.3 Treatment Plan 3: Antibiotics on Base Model

$$\begin{aligned}\frac{dS}{dt} &= \mu N - \mu S - \beta_i SI - \beta_w SW \\ \frac{dI}{dt} &= \beta_i SI + \beta_w SW - I(\gamma + \eta + \mu + \alpha) \\ \frac{dR}{dt} &= (\gamma + \eta)I - \mu R \\ \frac{dW}{dt} &= \xi I - \sigma W\end{aligned}$$

- η = is antibiotic rate on I class

5 Comparing Treatment Strategies For Cholera

5.1 Numerical Simulations and Phase Portraits For The Single Patch Model

The following are numerical simulations and phase portraits for the base model with vital dynamics, and the three treatment models.

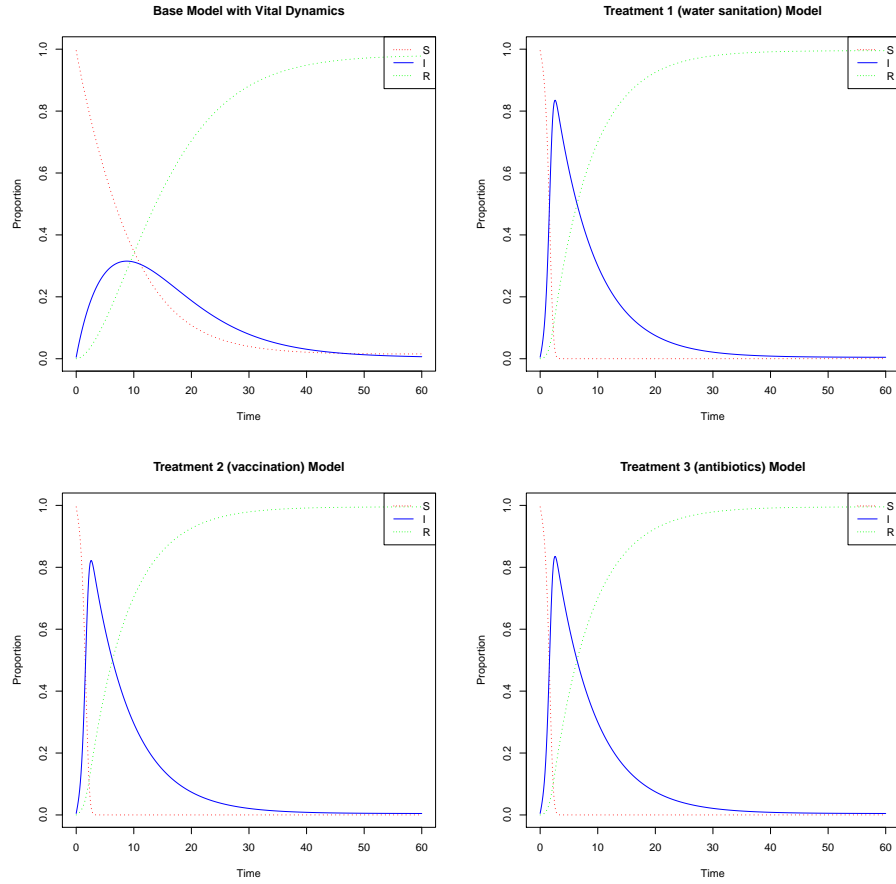


Figure 3: Plot of the SIRW model for a single patch and various treatment models. Parameters are $\mu = 0.15$ $\beta_i = 0.06$ $\gamma = 0.14$ $\sigma = 0.07$ $\beta_w = 0.15$ $\xi = 10$ $\rho = 0.005$ $\nu = 0.01$ $\eta = 1e - 04$ $\alpha = 0$. The initial conditions for the model were $S_0 = 0.995$ $I_0 = 0.005$ $R_0 = 0$

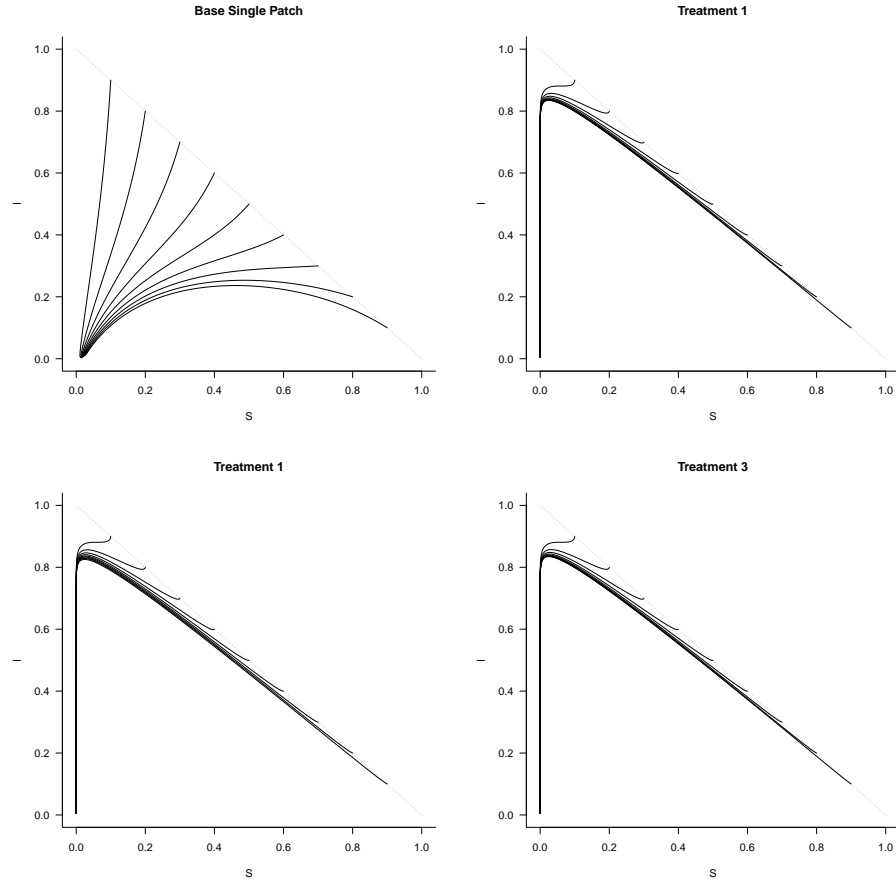


Figure 4: Phase portraits for a single patch and various treatment models. Parameters are $\mu = 0.15$ $\beta_i = 0.06$ $\gamma = 0.14$ $\sigma = 0.07$ $\beta_w = 0.15$ $\xi = 10$ $\rho = 0.005$ $\nu = 0.01$ $\eta = 1e-04$ $\alpha = 0$. The initial conditions for the model were $S_0 = 0.995$ $I_0 = 0.005$ $R_0 = 0$

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