Examining Treatment Strategies for Cholera Incorporating Spatial Dynamics

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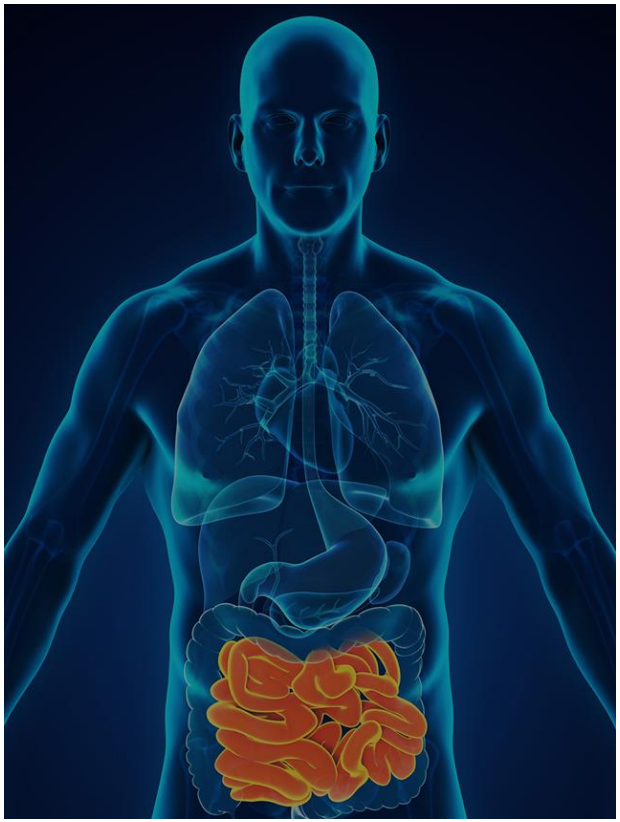
Introduction

- Treatments have not always gone as planned in history
- Cholera

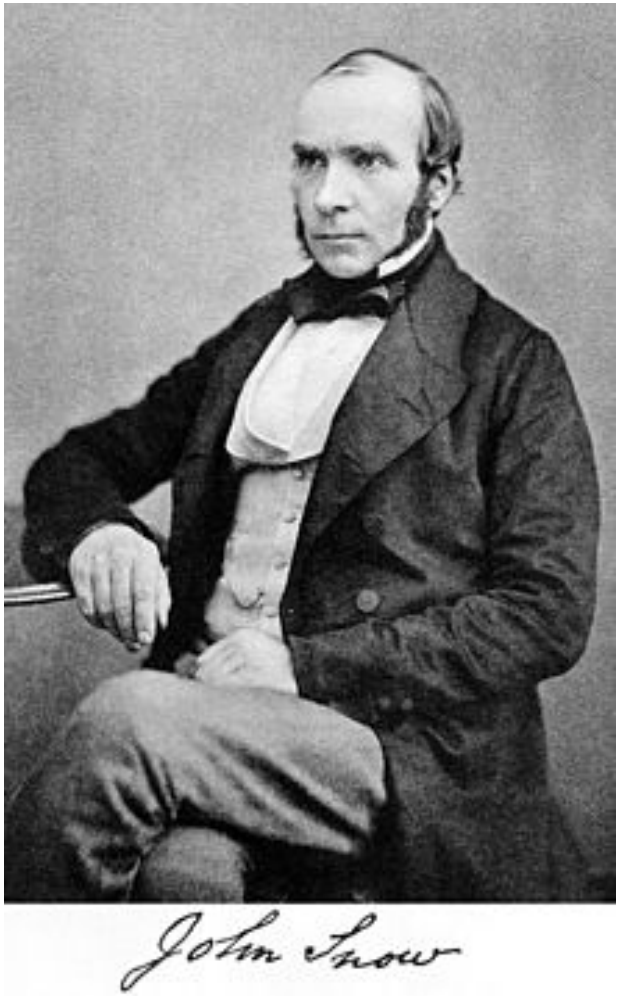
Some Biology on Cholera

- *Vibrio cholerae*
- Colonize the small intestines
- 10 percent of infected individuals develop symptoms
- Causes severe dehydration



Outbreaks in London (19^{th} Century)

- 1832, 1849, 1854, 1866
- Miasma Theory
- John Snow



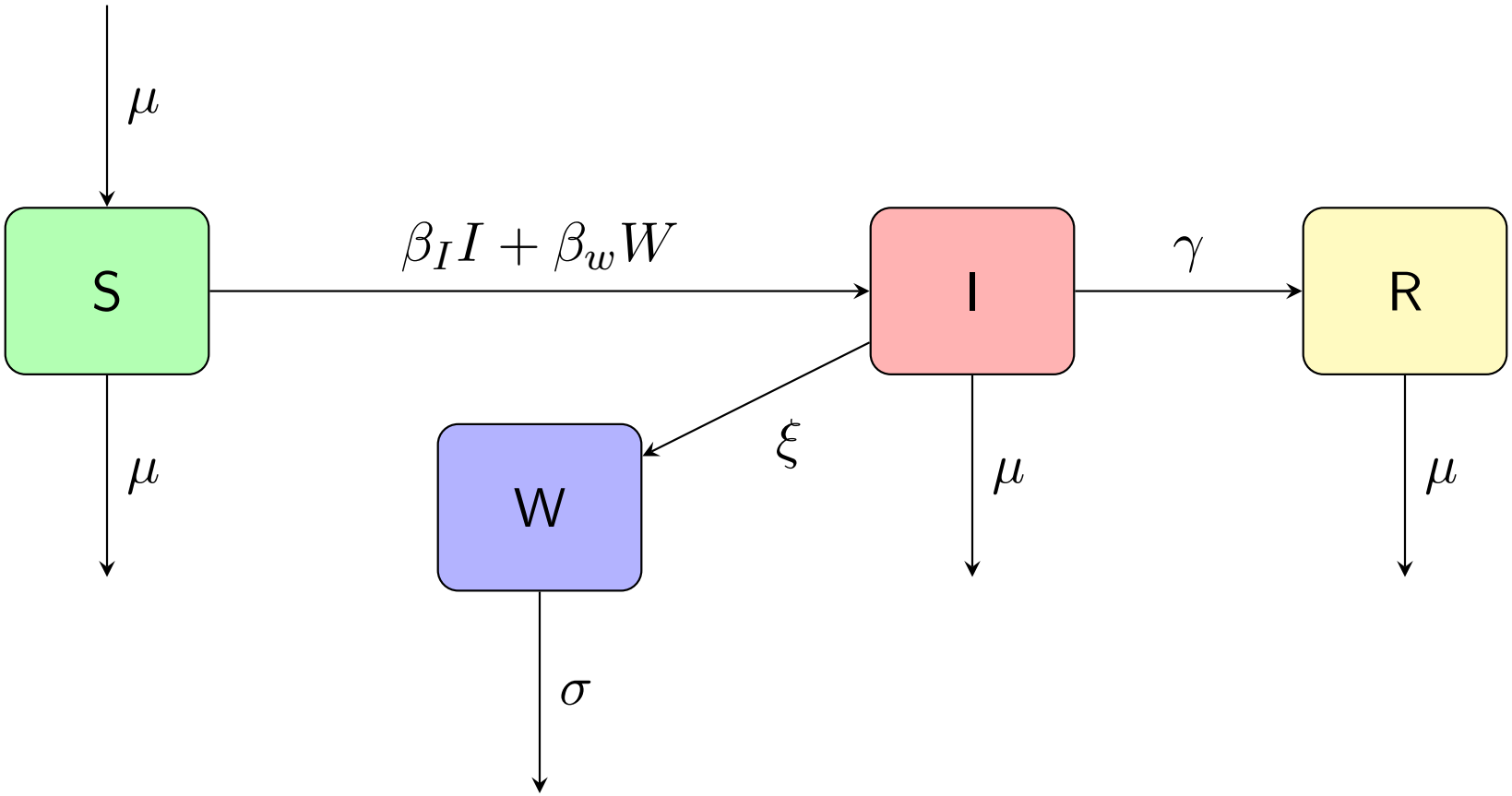
Developing a Single-Patch Model

- Entire population (N) included
- 3 Compartments : S, I, R
- Compartment values are proportional
- Environment (Water)

SIRW Model Assumptions

- Birth Rate = Natural Death Rate and is constant
- Homogenous susceptibility to cholera across population
- No waning immunity
- No latency period
- Only infected individuals can infect the water sources
- Water source is still

SIRW Model



R_0 Calculation

- Using the method of Next Generation Matrix (van den Driessche and Watmough, 2002)

$$F = \begin{pmatrix} \beta_i & \beta_w \\ 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{1}{\gamma+\mu} & 0 \\ \frac{1}{\gamma+\mu} & \frac{1}{\sigma} \end{pmatrix}$$

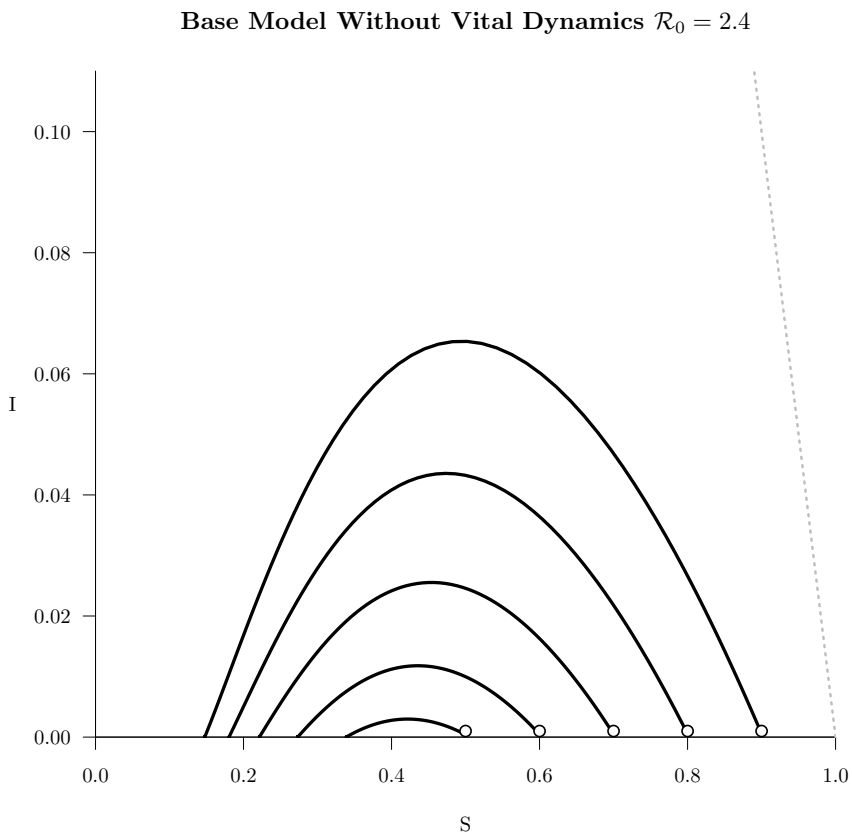
- R_0 is computed as the spectral radius of FV^{-1}:

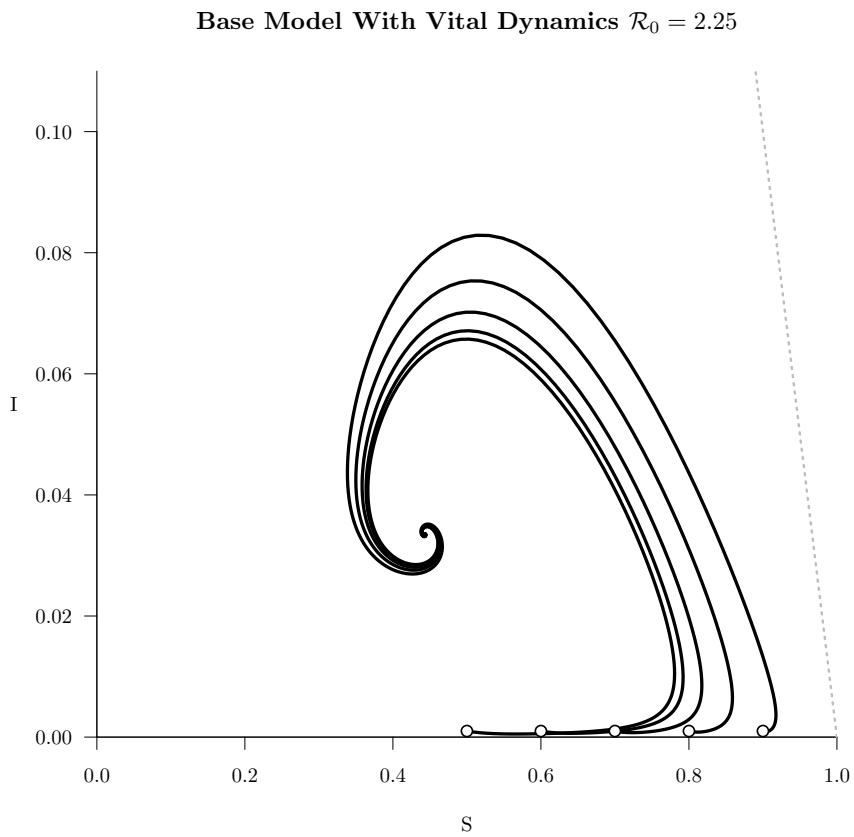
$$\begin{aligned} \mathcal{R}_0 &= \rho(FV^{-1}) \\ &= \frac{\beta_i + \beta_w}{\gamma + \mu} \approx 1.1 - 2.7 \end{aligned}$$

Equilibria and Stability

- Two equilibria:
  1. DFE: $(S, I, R) = (1, 0, 0)$
  2. EE: $(S^*, I^*, R^*) = (\frac{1}{R_0}, \frac{\mu}{\gamma+\mu}(1 - S^*), I^*)$
- The DFE is stable when $\mathcal{R}_0 < 1$.
- The EE is globally stable when $\mathcal{R}_0 > 1$ (Tien and Earn, 2010).

SIWR Model Phase Portrait

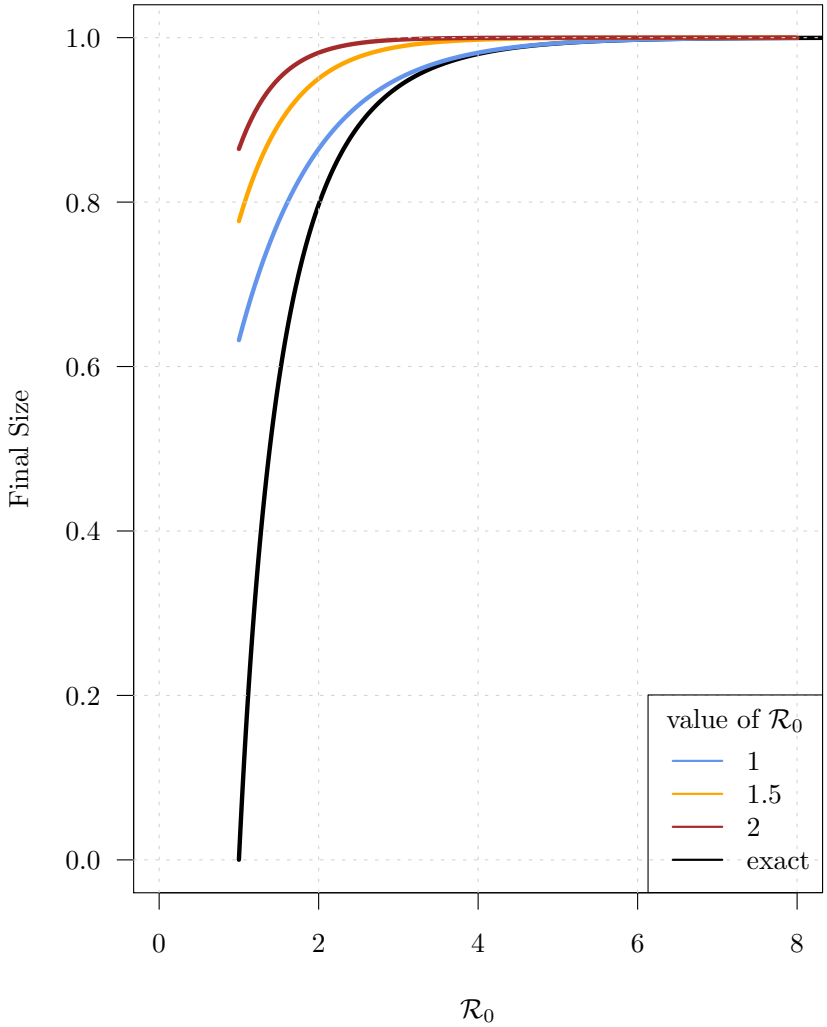




Final Size

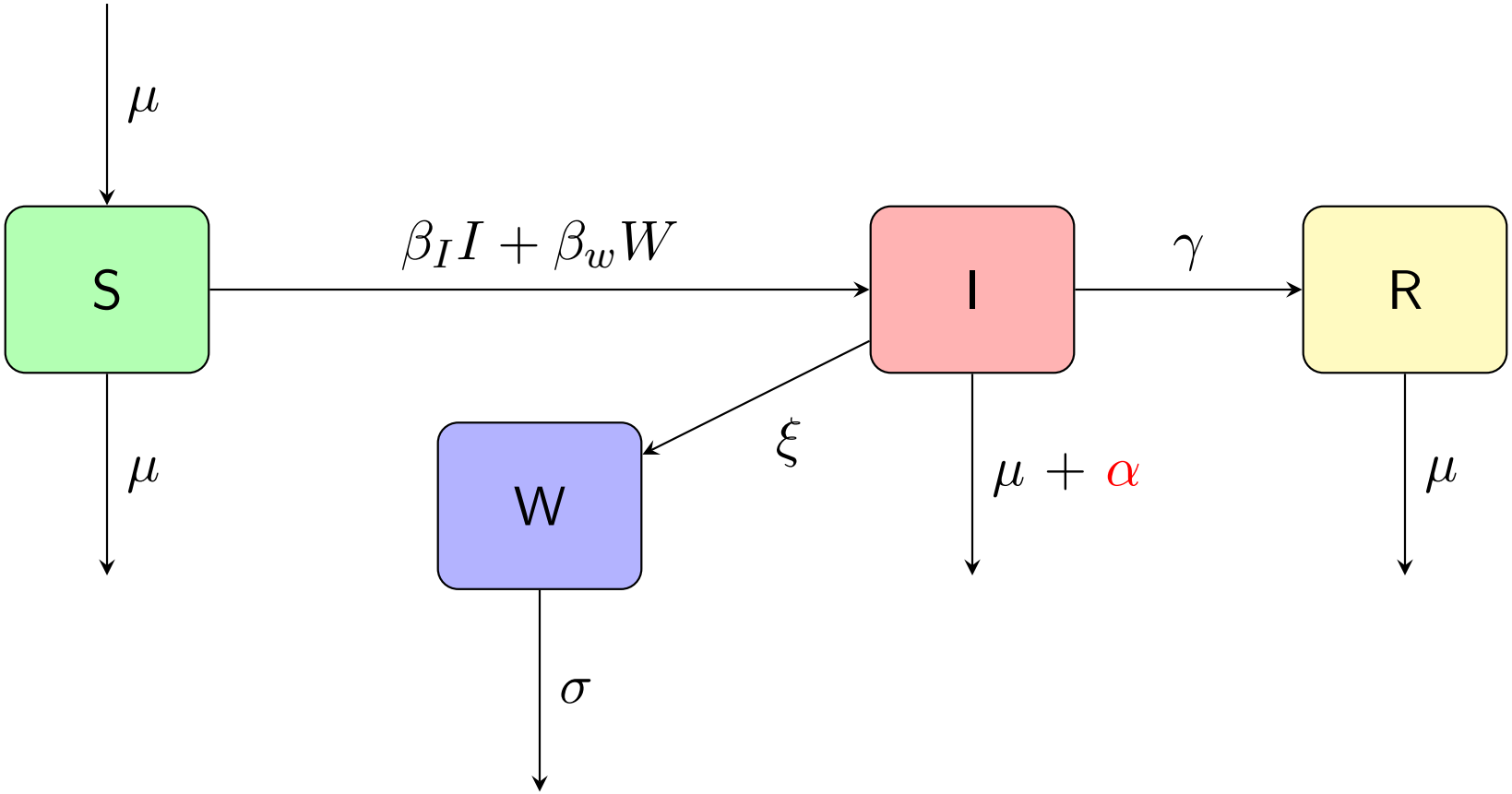
- Assuming $\mu = 0$ and $\mathcal{R}_0 > 1$, final size formula* still holds:

- $Z = 1 - \exp\left(-\mathcal{R}_0 Z - \frac{\beta_w}{\sigma} w_0\right)$



Effects of the 19th Century Treatments

- Added parameter death caused by cholera (α)



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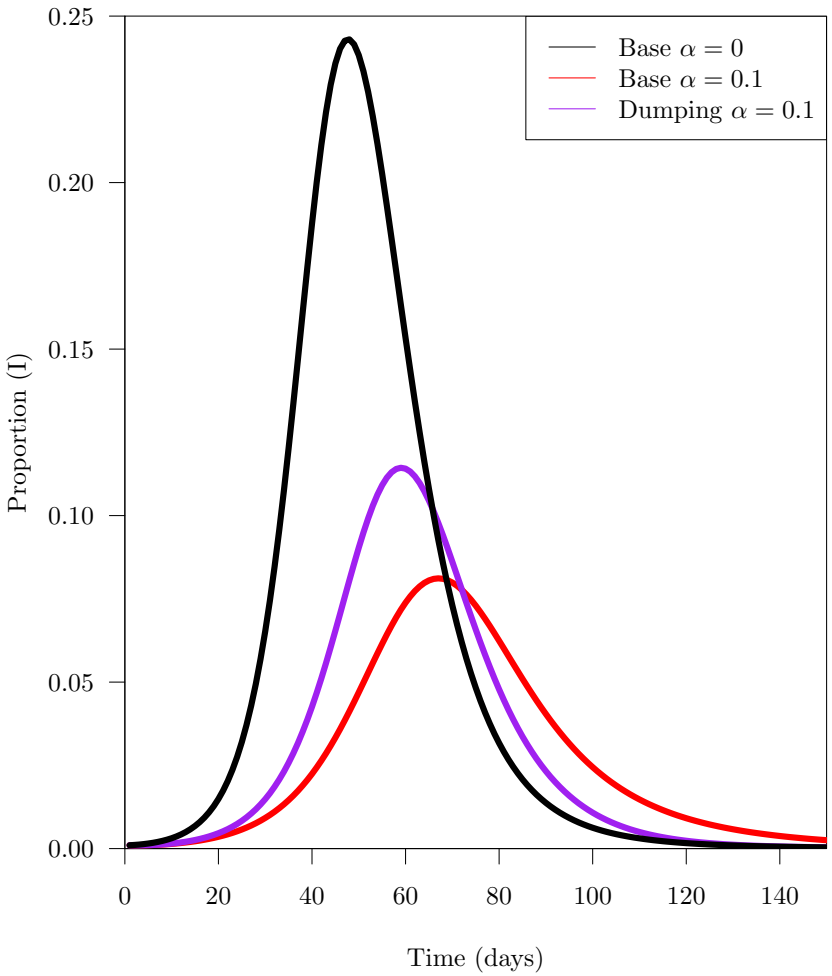


- Estimated death rate of cholera in the 19th century ranges from a small percentage up to fifty percent
- Including disease induced death is "beneficial" if death rate by cholera is high (Why?)
- Improper sanitation increases peak prevalence

Multi-Patch Model

$$\frac{dS_i}{dt} = \mu N - \mu S_i - \beta_i S_i I_i - \phi \beta_i S_i \sum_j^n I_j - \beta_w S_i W_i - \psi \beta_w S_i \sum_j^n W_j$$

$$\frac{dI_i}{dt} = \beta_i S_i I_i + \beta_i \phi S_i \sum_j^n I_j + \beta_w S_i W_i + \beta_i \psi S_i \sum_j^n W_j - I_i(\gamma + \mu + \alpha)$$

$$\frac{dR_i}{dt} = \gamma I_i - \mu R_i$$

$$\frac{dW_i}{dt} = \xi I_i + \beta_i \psi I_i \sum_j^n W_j - \sigma W_i$$

Multi-Patch Model Assumptions

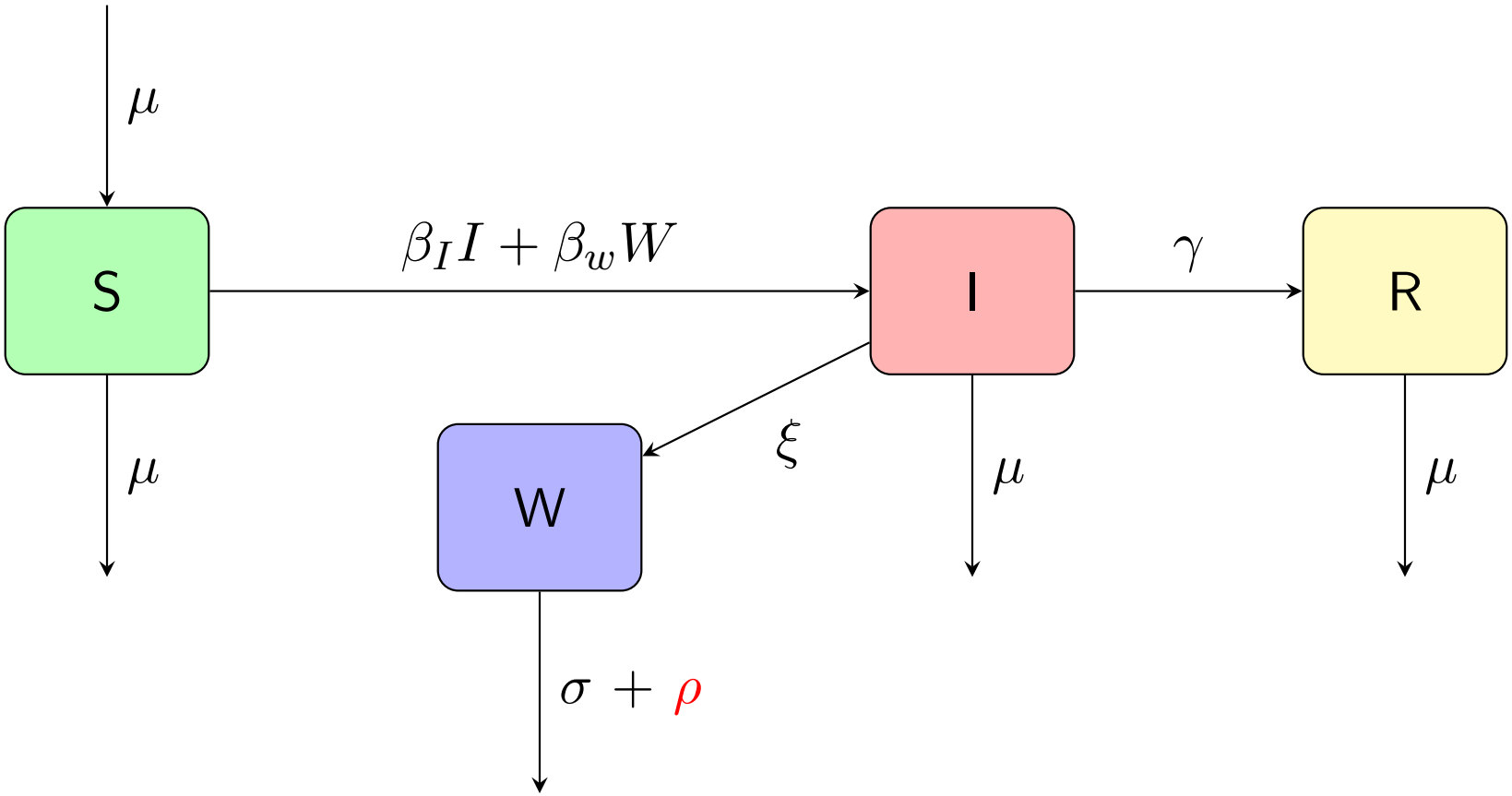
- No dispersal of individuals
- Infected individuals can infect the susceptible in neighboring patches
- All patches neighbouring i have the same transmission rate to patch i

Multi-Patch Model Simulation

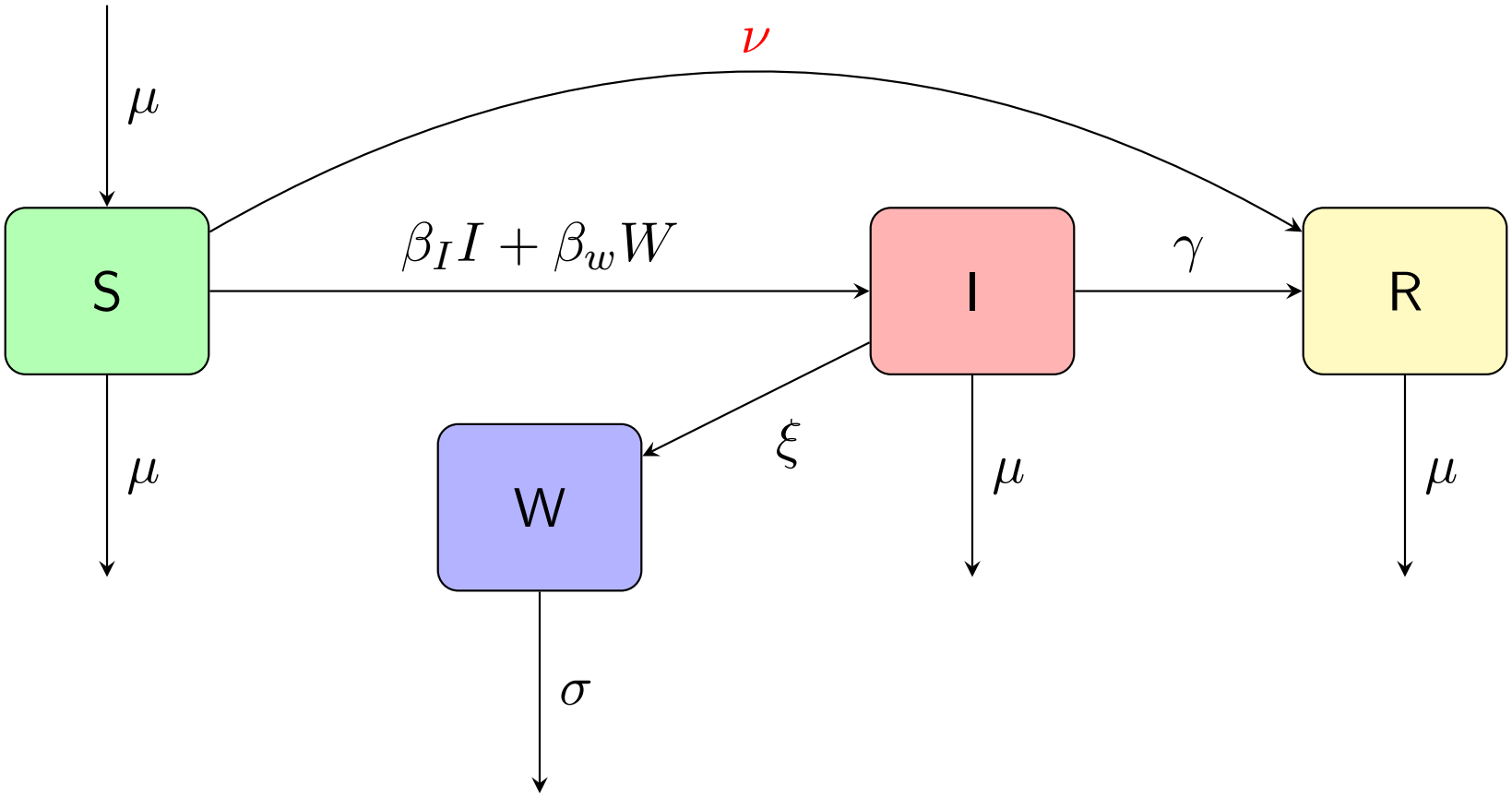
Treatment Strategies For Cholera

1. Sanitation of Water
2. Vaccinations
3. Antibiotics

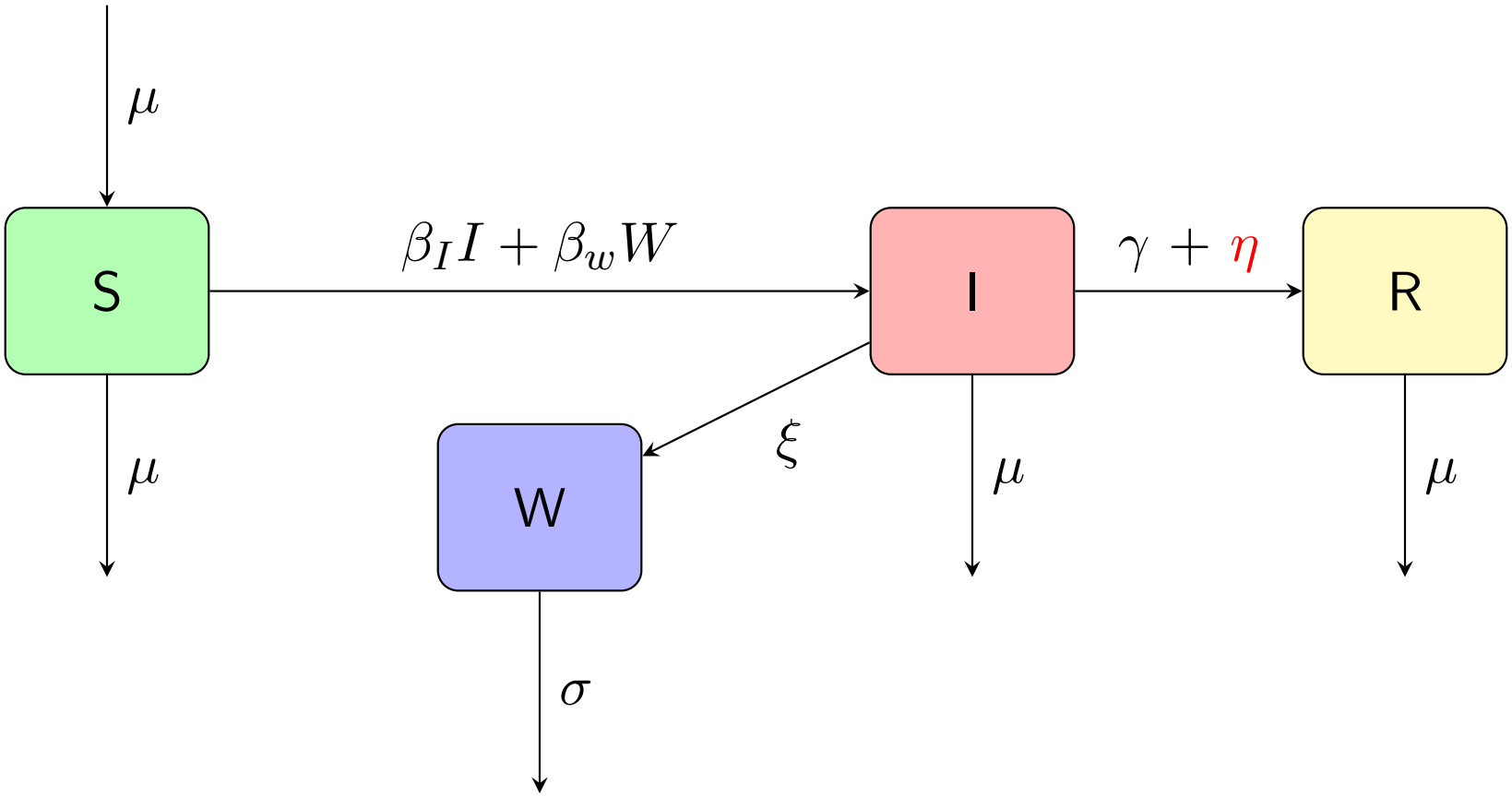
Sanitation of Water



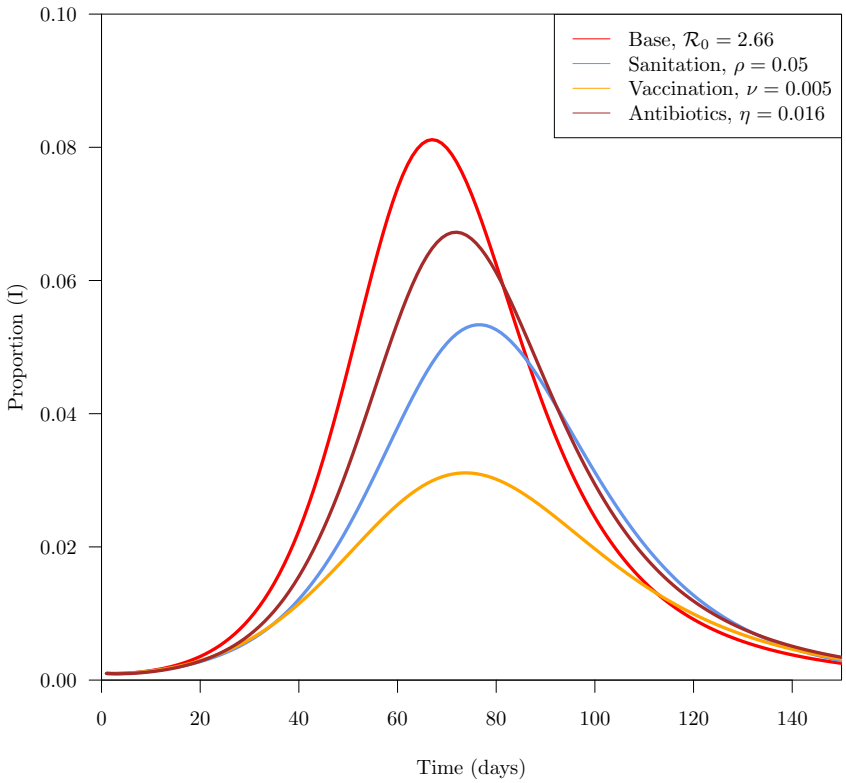
Vaccinations



Antibiotics



Comparing the Treatment Strategies



- Parameters chosen from literature
- Lowest peak prevalence for antibiotic treatment
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Conclusions and Further Research

- 19th century outbreaks
- Significance of the using multi-patch model
- Our treatment simulations suggest...
- Further research on the spread of water borne diseases like cholera can be done in areas like...

Thank you!

References I