Examining Control Strategies for Cholera Incorporating Spatial Dynamics

Group Name: The Plague Doctors

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This assignment is **due in class** on **Wednesday March 27 2019 at 10:30am**.

Abstract

We solve everything because we're really smart

Contents

1 Introduction

It's time for a theory of everything. Since we're all really smart, we've created one.

2 Single Patch Models

2.1 Single Patch SIR Model With A Water Compartment

$$\frac{dS}{dt} = \mu N - \mu S - \beta_i SI - \beta_w SW$$
$$\frac{dI}{dt} = \beta_i SI + \beta_w SW - I(\gamma + \mu + \alpha)$$
$$\frac{dR}{dt} = \gamma I - \mu R$$
$$\frac{dW}{dt} = \xi I - \sigma W$$

- μ = natural death rate
- β_i = transmission rate between S and I class
- β_w = transmission rate between I and W class
- γ = recovery rate (I to R class)
- α = death rate from cholera
- ξ = Shedding rate of cholera from I to W class
- σ = Removal rate of cholera from W class (depends on what we define as our water source)
- This model assumes that you start off with low intensity symptoms (lower rate of shedding) and the symptoms reach a high intensity with a greater rate of shedding.
- α_i = death rate by cholera in low or high intensity
- δ = rate at which symptoms increase in severity

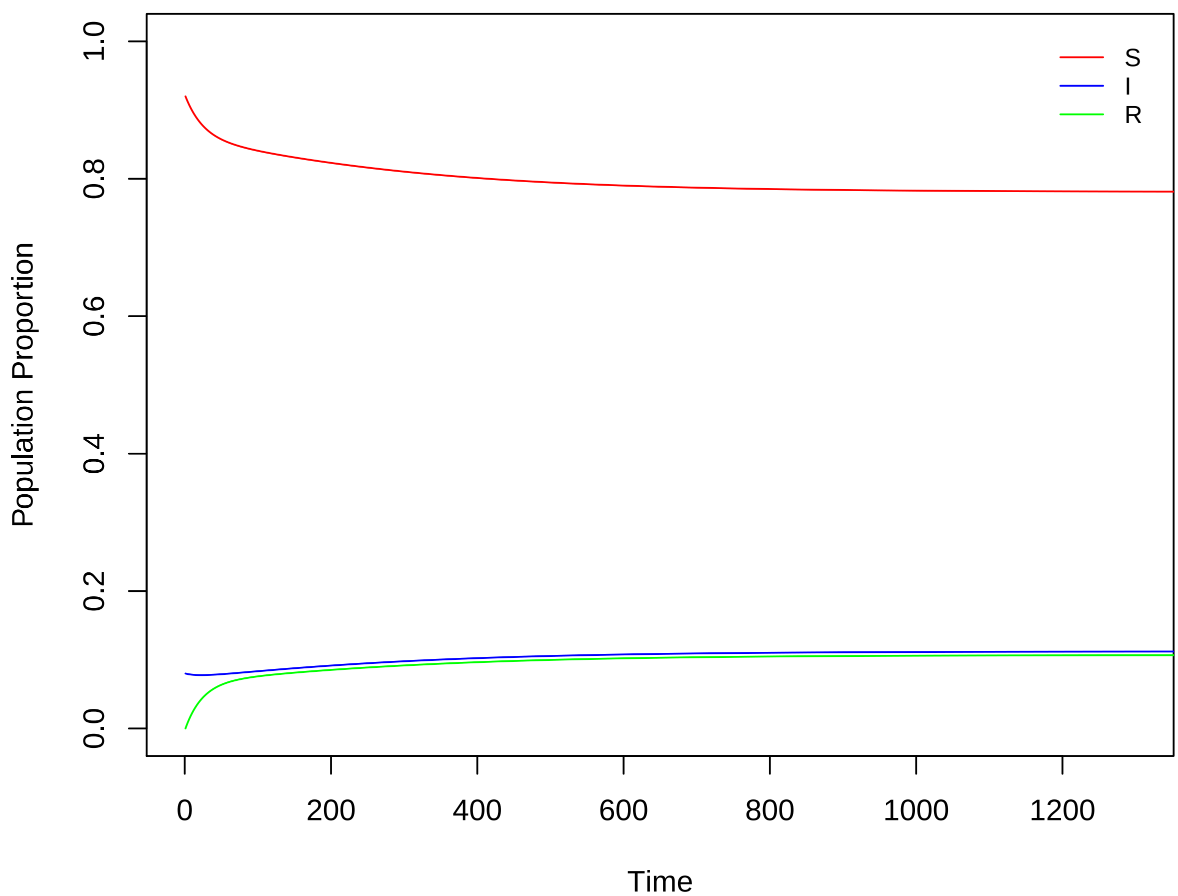

Figure 1: Plot of the SIRW model for a single patch. Parameters are $\mu = 0.15$ $\beta_i = 0.06$ $\gamma = 0.14$ $\sigma = 0.07$ $\beta_w = 0.15$ $\alpha = 0$.Further the initial conditions for the model were $S_0 = 0.92$ $I_0 = 0.08$ $R_0 = 0$

2.2 Equilibrium and $\mathcal{R}_0$ Of The Single Patch Model

The basic reproductive number $\mathcal{R}_0$ is defined as the number of secondary infections as a result of a single infective during a time step. $\mathcal{R}_0$ can be computed as the spectral radius (i.e. the eigenvalue with the largest absolute value) of the next generation matrix at the disease free equilibrium. The next generation matrix FV^1, where the entry F_{ij} of the matrix F is the rate at which infected individuals in compartment j produce new infections in compartment i, and the entry of V_{ij} of the matrix V is the mean time spent in compartment j after moving into j from compartment k. For our model, we have

$$F = \begin{pmatrix} \beta_i & \beta_w \\ 0 & 0 \end{pmatrix}$$
$$V = \begin{pmatrix} \frac{1}{\gamma+\mu+\alpha} & 0 \\ \frac{1}{\gamma+\mu+\alpha} & \frac{1}{\theta} \end{pmatrix}$$

The basic reproductive number is computed as the spectral radius of FV^{-1} as seen in Tien & Earn, 2010, which is

$$\begin{aligned} \mathcal{R}_0 &= \rho(FV^{-1} \\ &= \frac{\beta_i + \beta_w}{\gamma + \mu} \end{aligned}$$

This singla patch model has a disease-free equillibrium at (S,I,R)=(1,0,0) when $\mathcal{R}_0 < 1$. It also has an endemic-equillirbium when $\mathcal{R}_0 > 1$

2.3 Single Patch With Low And High Shedding Compartments

$$\begin{aligned} \frac{dS}{dt} &= \mu N - \mu S - \beta_L S I_L - \beta_H S I_H - \beta_w S W \\ \frac{dI_L}{dt} &= \beta_i S(I_L + I_H) + \beta_w S W - I_L(\mu + \delta + \alpha_L) \\ \frac{dI_H}{dt} &= \delta I_L - I_H(\gamma + \mu + \alpha_H) \\ \frac{dR}{dt} &= \gamma I_H - \mu R \\ \frac{dW}{dt} &= \xi_L I_L + \xi_H I_H - \sigma W \end{aligned}$$

- This model assumes that you start off with low intensity symptoms (lower rate of shedding) and the symptoms reach a high intensity with a greater rate of shedding.
- α_i = death rate by cholera in low or high intensity
- δ = rate at which symptoms increase in severity

3 Multi Patch Model

$$\frac{dS}{dt} = \mu N - \mu S - \beta_i SI - \beta_w SW$$
$$\frac{dI}{dt} = \beta_i SI + \beta_w SW - I(\gamma + \mu + \alpha)$$
$$\frac{dR}{dt} = \gamma I - \mu R$$
$$\frac{dW}{dt} = \xi I - \sigma W$$

- μ = natural death rate
- β_i = transmission rate between S and I class
- β_w = transmission rate between I and W class
- γ = recovery rate (I to R class)
- α = death rate from cholera
- ξ = Shedding rate of cholera from I to W class
- σ = Removal rate of cholera from W class (depends on what we define as our water source)

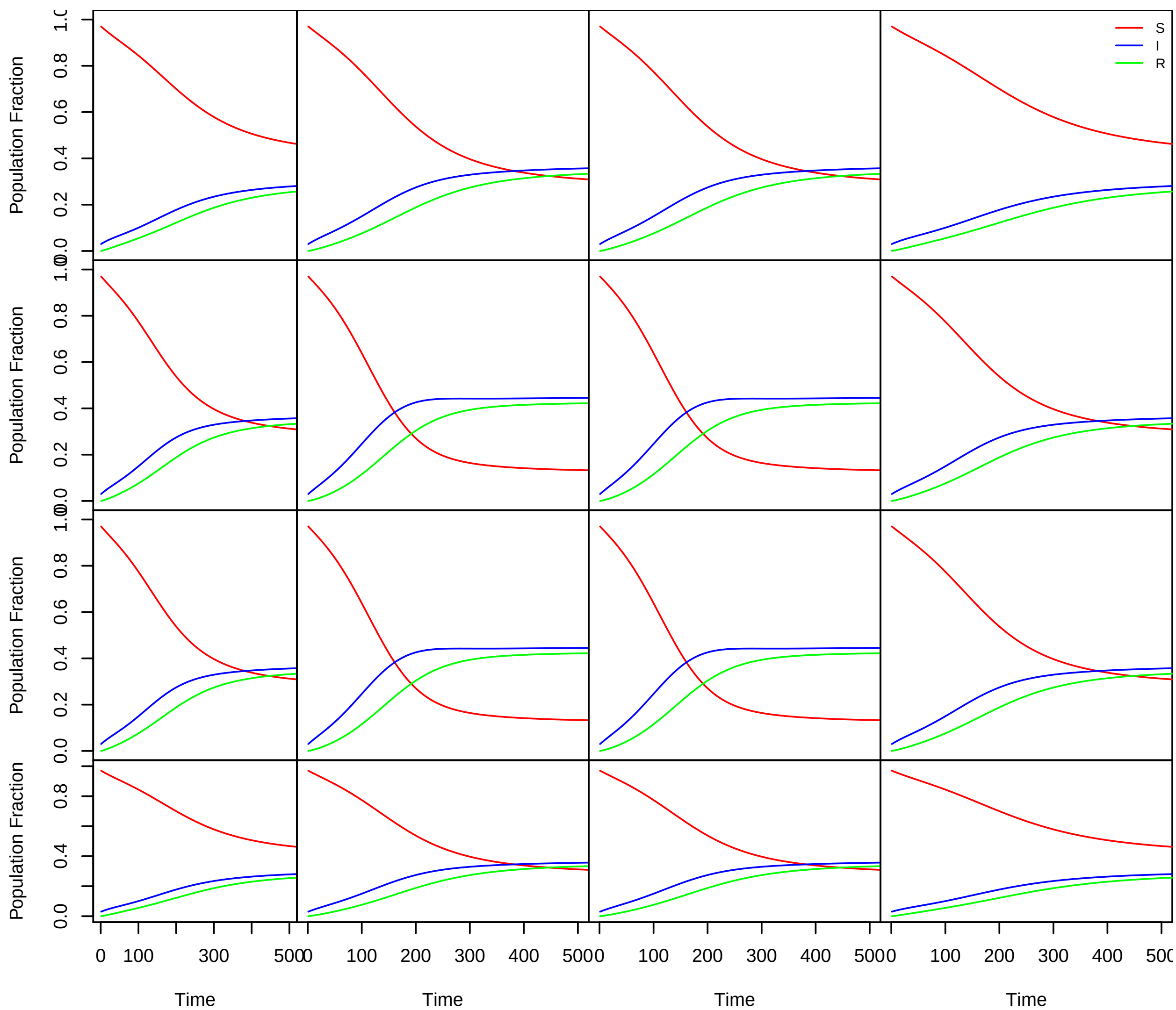

Figure 2: Plot of the SIRW model for all patches in a multi patch model. Parameters are $\mu = 0.15$ $\beta_i = 0.06$ $\gamma = 0.14$ $\sigma = 0.07$ $\beta_w = 0.15$ $\alpha = 0$. The initial conditions for the model were $S_0 = 0.97$ $I_0 = 0.03$ $R_0 = 0$. The influence of neighbouring patches is 0.15.

4 Treatment Strategies For Cholera

4.1 Treatment Plan 1: Sanitation of water over time

$$\frac{dS}{dt} = \mu N - \mu S - \beta_i SI - \beta_w SW$$
$$\frac{dI}{dt} = \beta_i SI + \beta_w SW - I(\gamma + \mu + \alpha)$$
$$\frac{dR}{dt} = \gamma I - \mu R$$
$$\frac{dW}{dt} = \xi I - \sigma W - \rho(I)W$$

- $\rho(I) = \begin{cases} \lambda & I \geq 0.1 \\ 0 & 0 \leq I \leq 0.1 \end{cases}$
  Represents the sanitation (increased removal of cholera) rate of λ, implemented at certain threshold of infected (in this case the threshold is based on I but can be based on W (i.e. testing water levels for cholera)

4.2 Treatment Plan 2: Vaccinations on Base Model

$$\frac{dS}{dt} = \mu N - \mu S - \beta_i SI - \beta_w SW - \nu S$$
$$\frac{dI}{dt} = \beta_i SI + \beta_w SW - I(\gamma + \mu + \alpha)$$
$$\frac{dR}{dt} = \gamma I - \mu R + \nu S$$
$$\frac{dW}{dt} = \xi I - \sigma W$$

- $\nu =$ is vaccination rate on S class

4.3 Treatment Plan 3: Antibiotics on Base Model

$$\frac{dS}{dt} = \mu N - \mu S - \beta_i SI - \beta_w SW$$
$$\frac{dI}{dt} = \beta_i SI + \beta_w SW - I(\gamma + \eta + \mu + \alpha)$$
$$\frac{dR}{dt} = (\gamma + \eta)I - \mu R$$
$$\frac{dW}{dt} = \xi I - \sigma W$$

- $\eta =$ is antibiotic rate on I class

5 Comparing Treatment Strategies For Cholera

— END OF PROJECT—

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References

1. Tien, J. H. & Earn, D. J. Multiple transmission pathways and disease dynamics in a waterborne pathogen model. *Bull. Math. Biol.* **72,** 1506–1533 (2010).