

Let us start by using the Lyapunov function $L = I$. $L = 0$ when $(S, I) = (1, 0)$. Since $R_0 \leq 1$ we can conclude that $\beta \leq \gamma + \mu$ or $\beta - \gamma - \mu \leq 0$. Calculating \dot{L} we have:

$$\begin{aligned}
\dot{L} &= \frac{dL}{dI} \frac{dI}{dt} + \frac{dL}{dS} \frac{dS}{dt} \\
&= \beta SI - \gamma I - \mu I + \mu I - \beta SI^2 - \mu SI \\
&= I(\beta S - \gamma - \mu S - \beta SI) \\
&\leq I(\beta S - \gamma S - \mu S - \beta SI) \\
&= I[S(\beta - \gamma - \mu) - \beta SI] \\
&< 0 \forall S, I \in (0, 1)
\end{aligned}$$

$L(1, 0) = 0$ and $\dot{L} < 0 \forall S, I \in (0, 1)$. Hence, L is negative definite on the interval $(0, 1)$. L is therefore a strict Lyapunov function for the DFE on the entire biologically relevant space, which implies that the DFE is globally asymptotically stable.