Examining Treatment Strategies for Cholera Incorporating Spatial Dynamics

Group: Plague Doctors Jessa Mallare, Sid Reed, Daniel Segura, Aref Jadda

McMaster University

Instructor: Dr. David Earn

April 8, 2019

Introduction

• Treatments have not always gone as planned in history

Cholera

Some Biology on Cholera

- Vibrio cholerae
- Colonize small intestines
- 10
- Causes dehydration



Outbreaks in London (19^{th} Century)

- 1832, 1849, 1854, 1866
- Miasma Theory
- John Snow



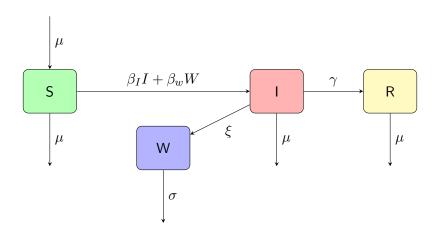
Single-Patch Model

- Entire population (N) included
- 3 Compartments : S, I, R
- Compartment values are proportional
- Environment (Water)

SIRW Model Assumptions

- Birth Rate = Natural Death Rate and is constant
- Homogenous susceptibility to cholera across population
- No waning immunity
- No latency period
- Only infected individuals can infect the water sources
- Water source is still

SIRW Model



SIWR Model Phase Portrait

R_0 Calculation

 Using the method of Next Generation Matrix (van den Driessche and Watmough, 2002)

$$F = \begin{pmatrix} \beta_i & \beta_w \\ 0 & 0 \end{pmatrix}$$
$$V = \begin{pmatrix} \frac{1}{\gamma + \mu + \alpha} & 0 \\ \frac{1}{\gamma + \mu + \alpha} & \frac{1}{\sigma} \end{pmatrix}$$

• R_0 is computed as the spectral radius of FV^{-1} :

$$\mathcal{R}_0 = \rho(FV^{-1})$$
$$= \frac{\beta_i + \beta_w}{\gamma + \mu}$$

Equilibria and Stability

Two equilibria:

1 DFE:
$$(S, I, R) = (1, 0, 0)$$

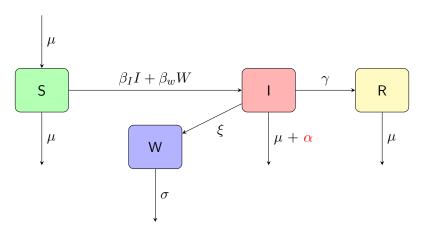
2 EE:
$$(s^*, i^*, r^*) = (\frac{1}{R_0}, \frac{\mu}{\gamma + \mu}(1 - s^*), i^*)$$

- The DFE is stable when $\mathcal{R}_0 < 1$
- The EE is stable when $\mathcal{R}_0 > 1$

Final Size

Effects of the 19th Century Treatments

• Added parameter death caused by cholera (α)



Effects of the 19th Century Treatments

Multi-Patch Model

$$\frac{dS_i}{dt} = \mu N - \mu S_i - \beta_i S_i I_i - \phi \beta_i S_i \sum_{j=1}^{n} I_j - \beta_w S_i W_i - \psi \beta_w S_i \sum_{j=1}^{n} W_j$$

$$\frac{dI_i}{dt} = \beta_i S_i I_i + \beta_i \phi S_i \sum_{j=1}^n I_j + \beta_w S_i W_i + \beta_i \psi S_i \sum_{j=1}^n W_j - I_i (\gamma + \mu + \alpha)$$

$$\frac{dR_i}{dt} = \gamma I_i - \mu R_i$$

$$\frac{dW_i}{dt} = \xi I_i + \beta_i \psi I_i \sum_{j=1}^{n} W_j - \sigma W_i$$



Multi-Patch Model Assumptions

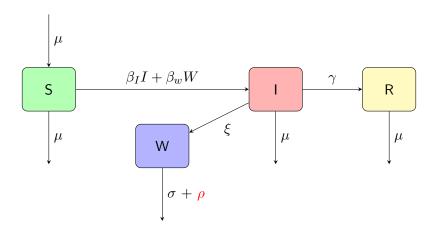
- No dispersal of individuals
- Infected individuals can infect the susceptible in neighboring patches
- All patches neighbouring i have the same transmission rate to patch i

Multi-Patch Model Simulation

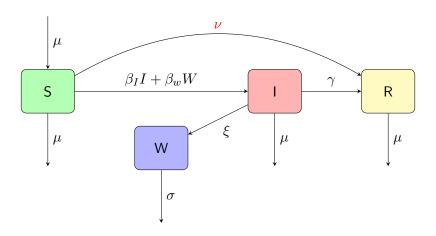
Treatment Strategies For Cholera

- Sanitation of Water
- Vaccinations
- Antibiotics

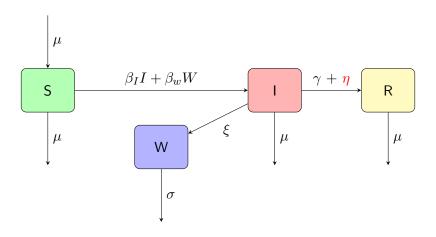
Sanitation of Water



Vaccinations



Antibiotics



Comparing the Treatment Strategies

Comparing the Treatment Strategies

Conclusions and Further Research

- 19th century outbreaks
- Significance of the using multi-patch model
- Our treatment simulations suggest. . .
- Further research on the spread of water borne diseases like cholera can be done in areas like. . .

Thank you!