Examining Treatment Strategies for Cholera Incorporating Spatial Dynamics

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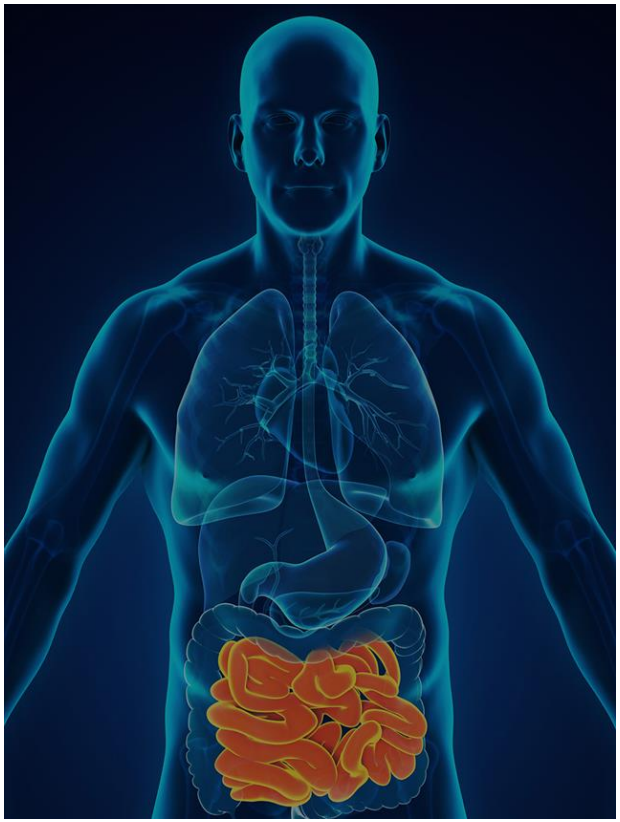
Introduction

- Treatments have not always gone as planned in history
- Cholera

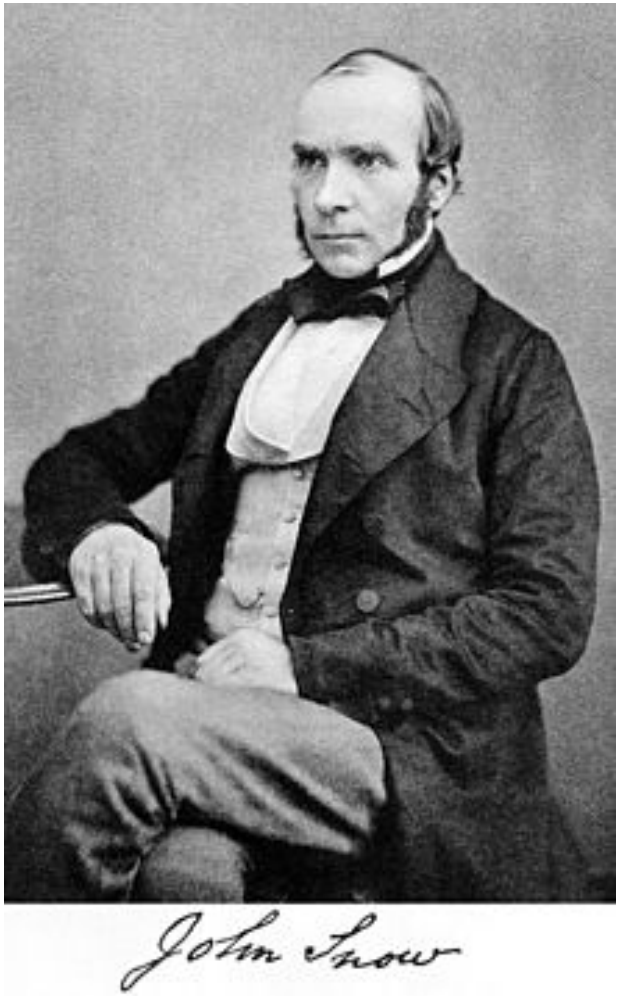
Some Biology on Cholera

- Vibrio cholerae
- Colonize small intestines
- 10 percent of infected develop symptoms
- Causes dehydration



Outbreaks in London (19^{th} Century)

- 1832, 1849, 1854, 1866
- Miasma Theory
- John Snow



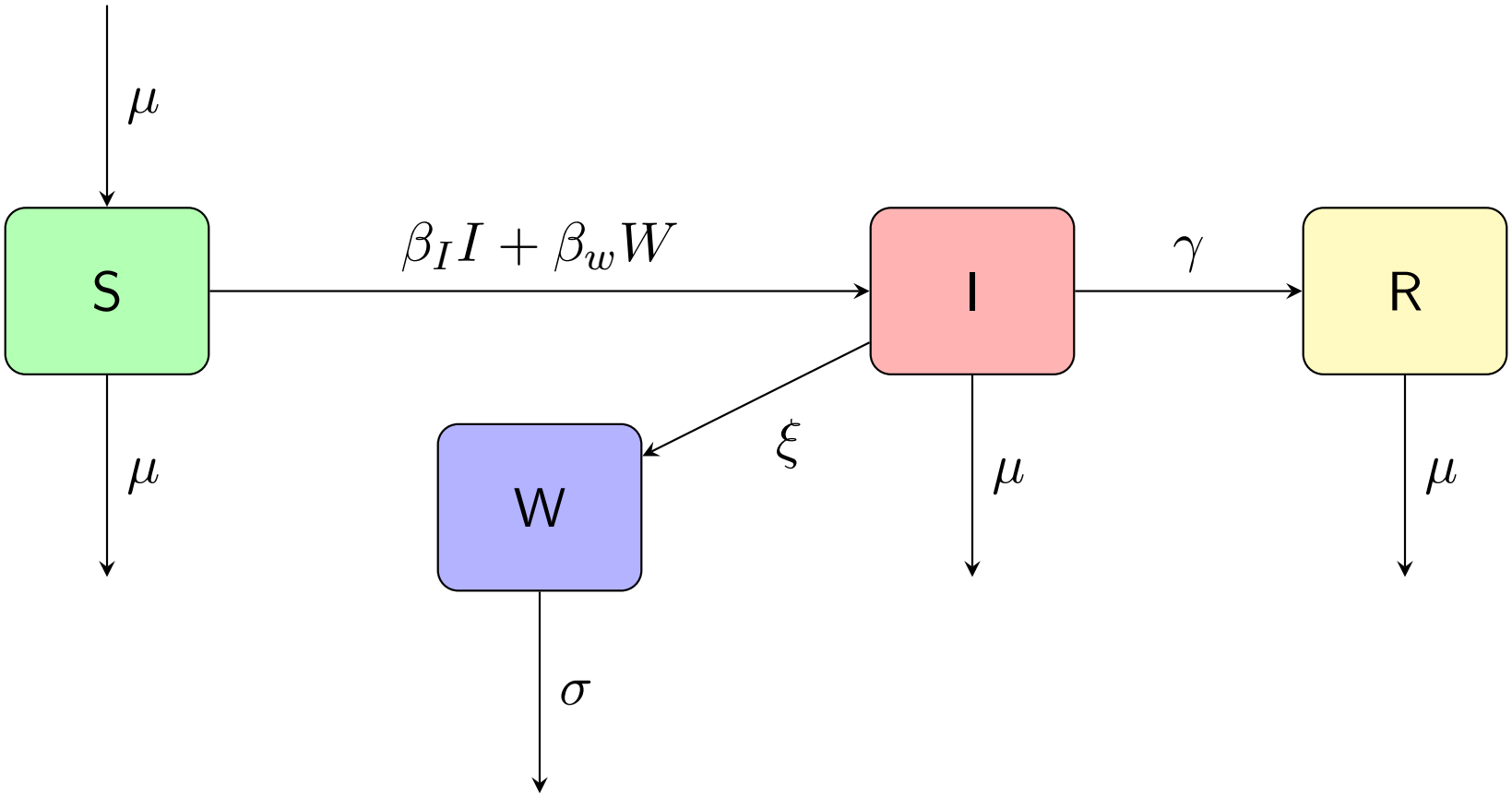
Developing a Single-Patch Model

- Entire population (N) included
- 3 Compartments : S, I, R
- Compartment values are proportional
- Environment (Water)

SIRW Model Assumptions

- Birth Rate = Natural Death Rate and is constant
- Homogenous susceptibility to cholera across population
- No waning immunity
- No latency period
- Only infected individuals can infect the water sources
- Water source is still

SIRW Model



SIWR Model Phase Portrait

R_0 Calculation

- Using the method of Next Generation Matrix (van den Driessche and Watmough, 2002)

$$F = \begin{pmatrix} \beta_i & \beta_w \\ 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{1}{\gamma+\mu+\alpha} & 0 \\ \frac{1}{\gamma+\mu+\alpha} & \frac{1}{\sigma} \end{pmatrix}$$

- R_0 is computed as the spectral radius of FV^{-1}:

$$\begin{aligned} \mathcal{R}_0 &= \rho(FV^{-1}) \\ &= \frac{\beta_i + \beta_w}{\gamma + \mu} \end{aligned}$$

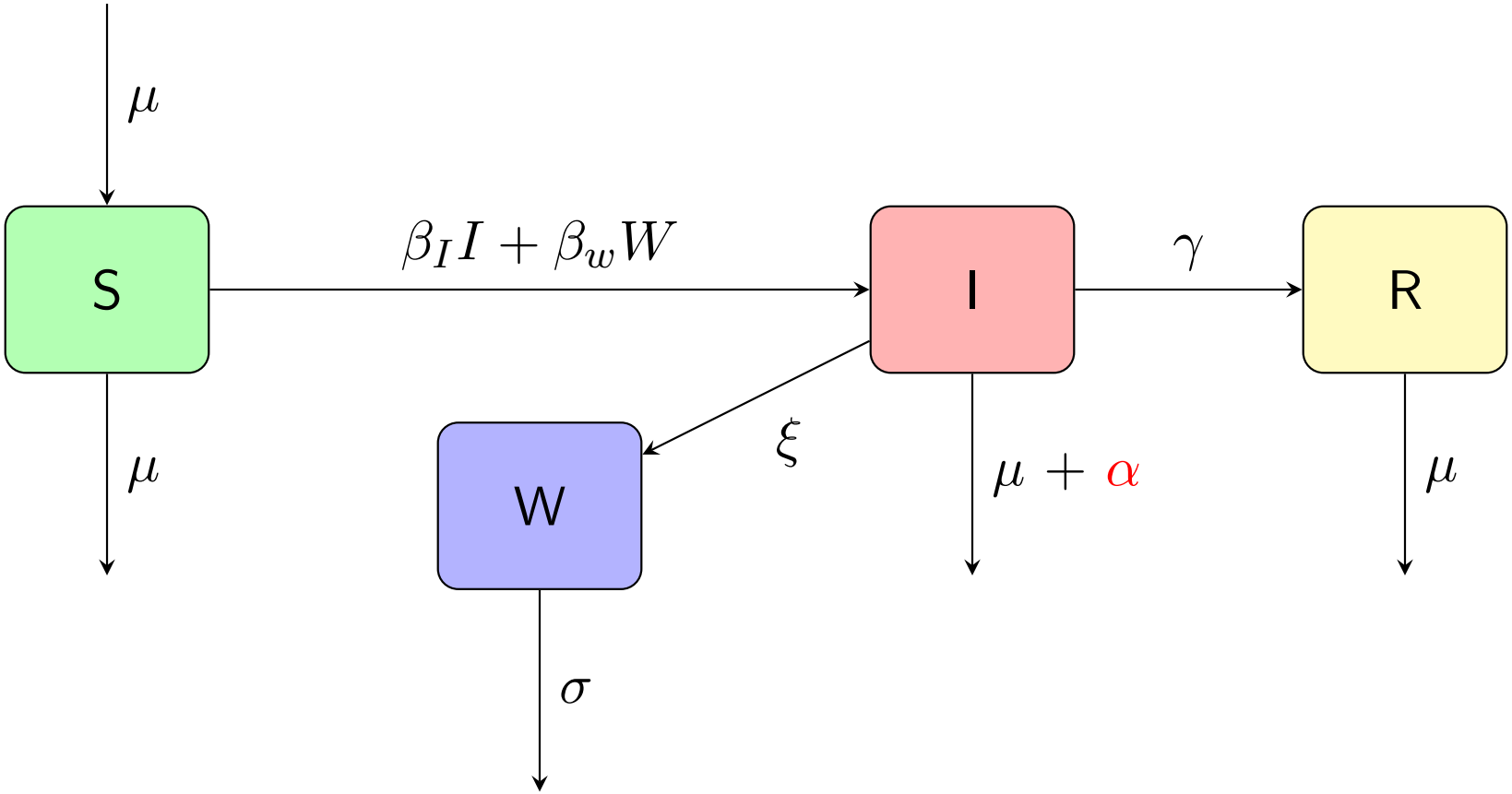
Equilibria and Stability

- Two equilibria:
  1. DFE: $(S, I, R) = (1, 0, 0)$
  2. EE: $(s^*, i^*, r^*) = (\frac{1}{R_0}, \frac{\mu}{\gamma+\mu}(1 - s^*), i^*)$
- The DFE is stable when $\mathcal{R}_0 < 1$
- The EE is stable when $\mathcal{R}_0 > 1$

Final Size

Effects of the 19th Century Treatments

- Added parameter death caused by cholera (α)



Effects of the 19th Century Treatments

Multi-Patch Model

$$\frac{dS_i}{dt} = \mu N - \mu S_i - \beta_i S_i I_i - \phi \beta_i S_i \sum_j^n I_j - \beta_w S_i W_i - \psi \beta_w S_i \sum_j^n W_j$$

$$\frac{dI_i}{dt} = \beta_i S_i I_i + \beta_i \phi S_i \sum_j^n I_j + \beta_w S_i W_i + \beta_i \psi S_i \sum_j^n W_j - I_i(\gamma + \mu + \alpha)$$

$$\frac{dR_i}{dt} = \gamma I_i - \mu R_i$$

$$\frac{dW_i}{dt} = \xi I_i + \beta_i \psi I_i \sum_j^n W_j - \sigma W_i$$

Multi-Patch Model Assumptions

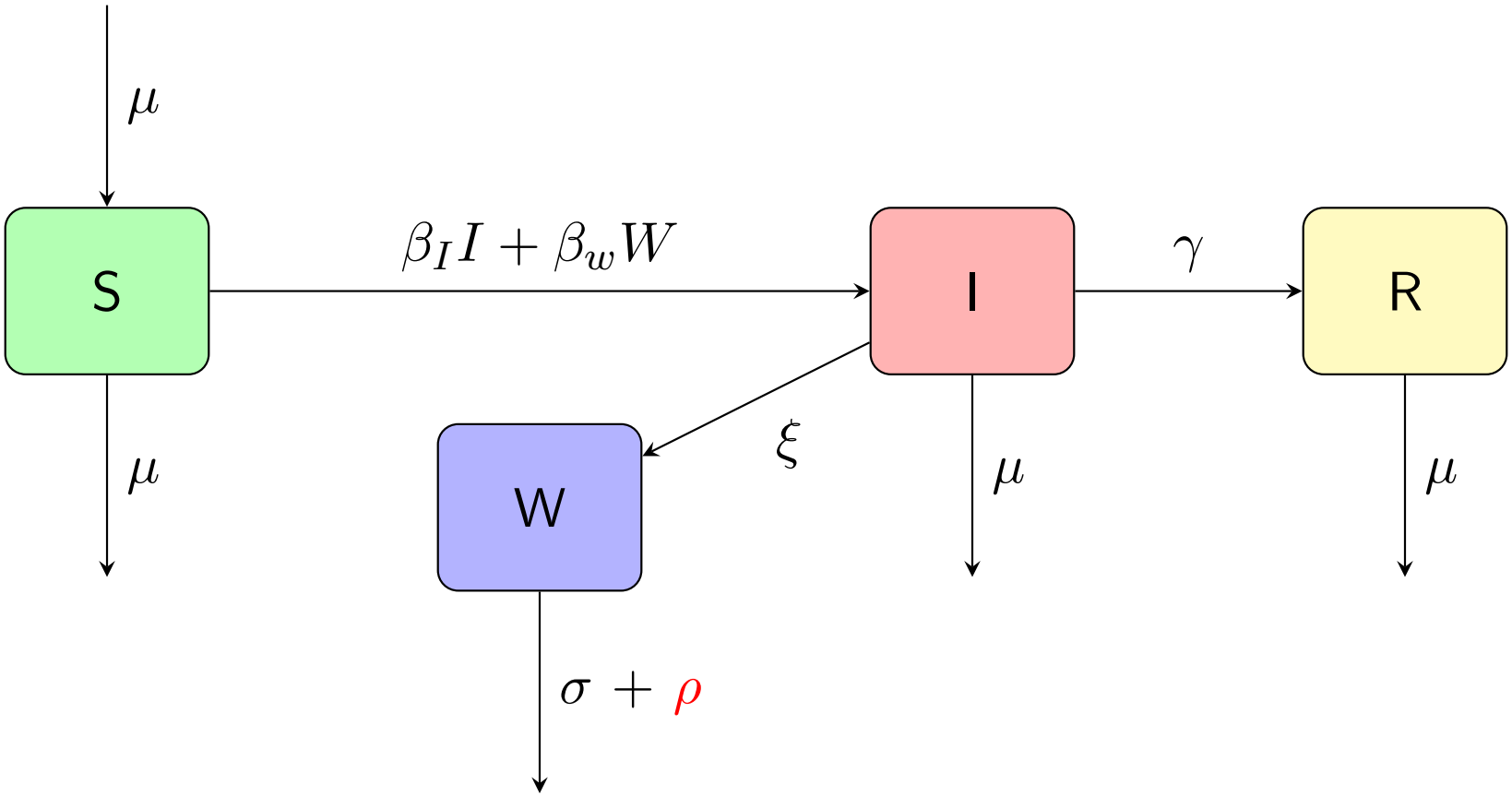
- No dispersal of individuals
- Infected individuals can infect the susceptible in neighboring patches
- All patches neighbouring i have the same transmission rate to patch i

Multi-Patch Model Simulation

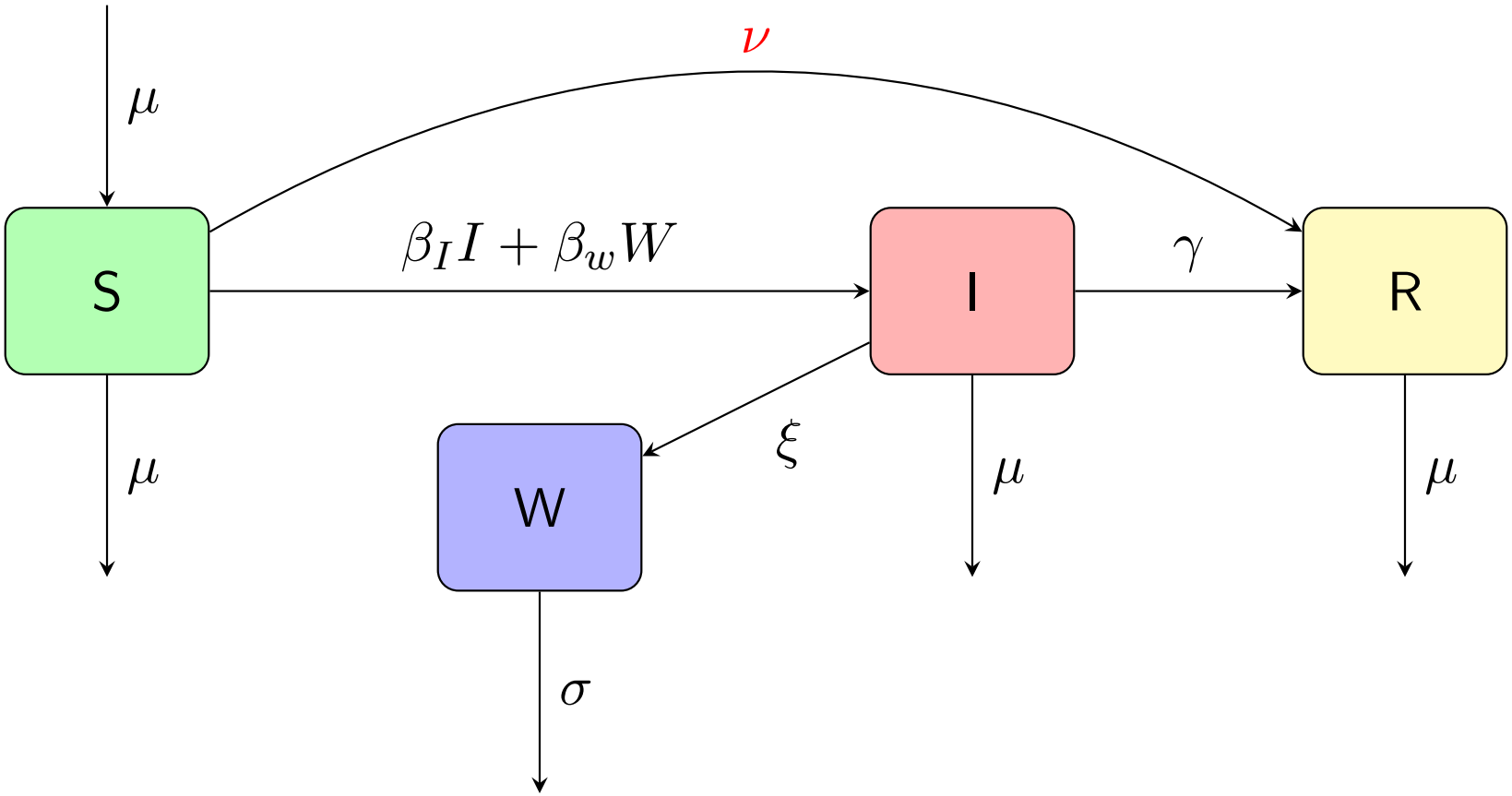
Treatment Strategies For Cholera

1. Sanitation of Water
2. Vaccinations
3. Antibiotics

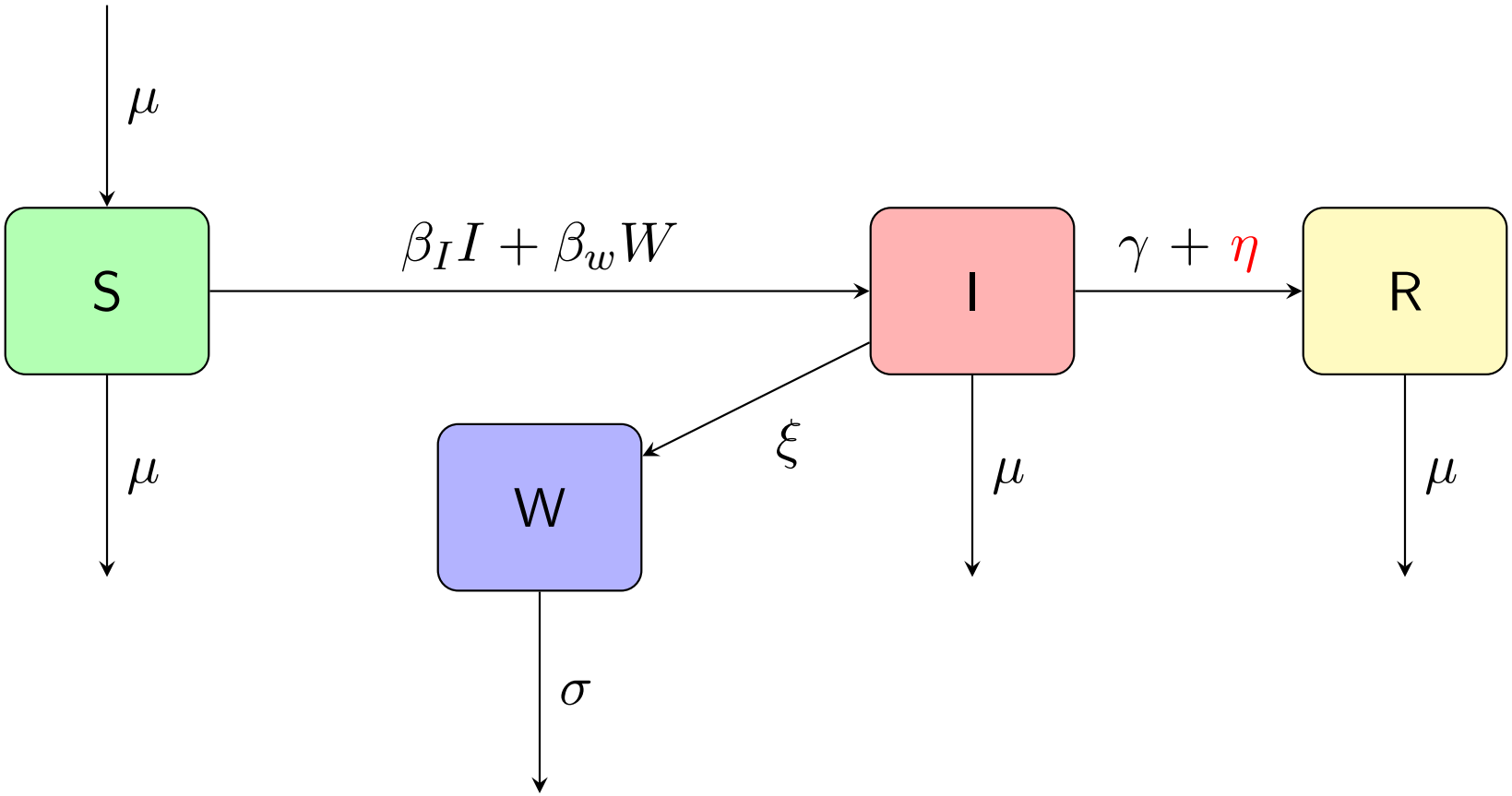
Sanitation of Water



Vaccinations



Antibiotics



Comparing the Treatment Strategies

Comparing the Treatment Strategies

Conclusions and Further Research

- 19th century outbreaks
- Significance of the using multi-patch model
- Our treatment simulations suggest...
- Further research on the spread of water borne diseases like cholera can be done in areas like...

Thank you!